Exercises corresponding to Appendix of book: Mathematics with applications in micro economics

Exercise 1.1 (point of intersection)

a) Sketch the graphs of the following two functions $y(x) = 3x+2$ and $y(x) = -5x+4$,

b) Determine the point of intersection of these functions.

Exercise 1.2 (break-even point)

Consider the revenue function $R(x) = px, p > 0$ and the cost function $C(x) = c + vx, c > 0, 0 < v < p$. Determine the break-even point, i.e. $R(x) = C(x)$.

Exercise 1.3 (zero points quadratic function)

Determine the zero points of the following quadratic functions:

a) $y(x) = x^2 + 7x + 6$,

b) $y(x) = 4x^2 + 2x + 1$

c) $y(x) = 2x^2 + 12x + 18$.

Exercise 1.4 (x-intercepts of quadratic function)

Determine p such that the following quadratic functions have two x-intercepts:

a) $y(x) = x^2 + px + 3$,

b) $y(x) = -2x^2 2 - x + p$,

c) $y(x) = p\,x + 2px + 1$.

Exercise 1.5 (graph of quadratic function)

a) Sketch the graphs of the following two functions $y(x) = x^2 + 4x + 3$ and $y(x) = -x^2 + 6$,

b) Calculate the points of intersection of the graphs of these functions.

Exercise 1.6 (comparing functions)

a) Consider the functions $f(x) = 2x + 4$ and $g(x) = 2x^2 + 3x + 4$. Determine all x such that $f(x) \geq g(x)$. 2 2

b) Consider the functions $f(x) = x + 4x + 3$ and $g(x) = -x + 6$. Determine all x such that $f(x) \geq g(x)$.

Exercise 1.7 (intersection functions)

Consider the functions $y_1(x) = \frac{1}{4}x^2 - 5x$ and $y_2(x) = 3x + p$. Determine all p such that the two functions have no point in common.

Exercise 1.8 (solving equations by factorizing)

Solve the following equations:

a) $x^2 + x - 2 = 0$,

b) $2x^3 - 2x^2 + x - 1 = 0$,

c) $x^3 + 2x - 4x^2 - 8 = 0$,

d) $2x^4 - 2x^3 + x - x = 0$.

Exercise 1.9 (comparing functions)

a) Consider the functions $f(x) = \frac{2-x}{3+x}$ and $g(x) = -x+1$. Determine all $x > -3$ such that $f(x) \geq g(x)$

b) Consider the functions $f(x) = \frac{1-x^2}{3+x}$ and $g(x) = x + 1$. Determine all $x > -3$ such that $f(x) \leq g(x)$

Exercise 1.10 (using properties power functions)

Determine m and n if the following expression are simplified into $x^m y^n$

a) $x^2x^5yy^2$

b) $\frac{xx^{\frac{1}{3}}y^2}{x^{-\frac{2}{3}}y^{-1}}$

c) $(x^{-1}y^4)^2$

d) $x^{\frac{10}{6}}\sqrt[3]{x}$

Exercise 1.11 (power calculations)

Consider the functions $y_1(x) = \sqrt{2x + 3}$ and $y_2(x) = x$.

a) Draw the graphs of $y_1(x)$ and $y_2(x)$ in one figure

b) Calculate the points of intersection of the graphs of these two functions

c) Solve the inequality $y_1(x) < y_2(x)$.

Exercise 1.12 (using properties exponential functions) Rewrite the following expressions as a power of 2:

a) 8
b) $8^{\frac{4}{3}}$
c) $\sqrt{32}$
d) $64^{-\frac{1}{2}}$

Exercise 1.13 (using properties exponential functions)
Solve the following equations:

a) $2^x = 4^{4x+6}$
b) $27^2 = (\frac{1}{3})^{-x+2}$
c) $(\frac{1}{4})^{x^2-1} = 1$

Exercise 1.14 (intersection functions)

Consider the functions $y_1(x) = 3^{x+2}$ and $y_2(x) = 24 + 3^x$. Determine the point of intersection of the graphs of these two functions.

Exercise 1.15 (using properties logarithmic function) Express the following expressions in a single logarithm

a) $\log x + 2\log y$
b) $\log x + \log(\frac{1}{y}) - \log z$

Exercise 1.16 (solving equation)
Solve the following equations

a) $\ln(x + 7) + \ln(x + 3) = 0$
b) $({}^3\log x)^2 + 6 = 5 \cdot {}^3\log x$

Exercise 1.17
Determine a such that the equation

$$\frac{x}{3} - \frac{a}{x} = 2$$

has precisely one solution.

Exercise 1.18
The demand of a good is given by $q = 60 - 10p$. The fixed cost of the production of this good is 25 Euro and the variable costs are 2 Euro per unit. The total revue are $TR(p) = pq$. Determine the break even point.

Exercise 1.19 Solve the inequality

$$x^3 + 2x \leq 3x^2.$$

Exercise 1.20

Consider the functions $y_1(x) = {}^2\log(x - 2)$ and $y_2(x) = 2 - {}^2\log(x + 4)$.

a) Determine all x such that $y_1(x) > 3$.
b) Determine all x such that $y_1(x) < y_2(x)$.

Exercise 2.21 (difference quotient)

The difference quotient of the function $y(x) = x^2+5x+3$ in $x = a$ at a change of $\Delta x = 3$ is equal to 3. Determine a.

Exercise 2.22 (difference quotient)

The average increase of the demand $q(p) = -p^2 + 4p + 7$ in $p = 3$ at a change of Δp is equal to $\frac{1}{2}$.

Determine Δp.

Exercise 2.23 (slope of line)

Determine the slope of the line that intersects the graph of the function $y(x) = x^2+5x+6$ in the points $(1,y(1))$ and $(3,y(3))$.

Exercise 2.24 (slope of line)

A line intersects the graph of the function $y(x) = x^2+7$ in the points $(a,y(a))$ and $(b,y(b))$. The slope of this line is equal to 5 and $b - a = 3$. Determine a and b.

Exercise 2.25 (difference quotient and derivative)

Consider the function $y(x) = \frac{x^2}{2x+1}$.

a) Calculate the difference quotient in $x = 2$ and $\Delta x = 3$
b) Determine the number the difference quotient approaches if $\Delta x \to 0$ in $x = 2$

Exercise 2.26 (tangent line)

Determine the tangent line in $(1,y(1))$ at the graph of the following functions:

a) $y(x) = x^{\frac{1}{2}}x^{\frac{1}{3}}$
b) $y(x) = 2^{-x}$

Exercise 2.27 (slope of line)

The slope of the tangent line at the graph of the function $y(x) = x^2+3x+4$ at the point $(x_0,y(x_0))$ is equal to the slope of the line through the points $(0,4)$ and $(2,14)$. Determine x_0.

Exercise 2.28

Determine the tangent line at the graph of the function $y(x) = 2x^2+2$ that intersects the x-axis in $x = 1$.

Solutions

Week 1

Exercise 1.1

a) p.m.;

b) $(x, y) = (\frac{1}{4}, 2\frac{3}{4})$.

Exercise 1.2

$x = \frac{c}{p-v}$.

Exercise 1.3

a) $x = -6, x = -1$;

b) no zero points;

c) $x = -3$.

Exercise 1.4

a) $p > \sqrt{12}, p < -\sqrt{12}$;

b) $p > -1/4$;

c) no intersection point for any p.

Exercise 1.5

a) p.m.;

b) $(-1 - \frac{1}{2}\sqrt{10}, 2\frac{1}{2} - \sqrt{10}), (-1 + \frac{1}{2}\sqrt{10}, 2\frac{1}{2} + \sqrt{10})$.

Exercise 1.6

a) $-1/2 \leq x \leq 0$;

b) $x \leq -1 - 1/2\sqrt{10}, x \geq -1 + 1/2\sqrt{10}$.

Exercise 1.7 $p \leq -58$.

Exercise 1.8

a) $x = 1, x = -2$;

b) $x = 1$;

c) $x = 2, x = -2$;

d) $x = 0, x = 1$.

Exercise 1.9

a) $-3 < x < -\frac{1}{2} - \frac{1}{2}\sqrt{5}, x > -\frac{1}{2} + \frac{1}{2}\sqrt{5}$;

b) $x \geq -1$.

Exercise 1.10

a) $m = 7, n = 3$;
b) $m = 2, n = 3$;
c) $m = -2, n = 8$;
d) $m = 2, n = 0$.

Exercise 1.11

a) p.m.;
b) (3,3);
c) $x > 3$.

Exercise 1.12

a) 3;
b) 4;
c) $2^{\frac{1}{2}}$;
d) −3.

Exercise 1.13

a) $x = -12/7$;
b) $x = -2/5$;
c) $x = -1, x = 1$.

Exercise 1.14

(1,27).

Exercise 1.15 a) $\log(xy^2)$;
b) $\log(x/(yz))$;

Exercise 1.16

a) $x = -5 + \frac{1}{2}\sqrt{\ }20$;
b) $x = 9, x = 27$;

Exercise 1.17 $a = -3$.

Exercise 1.18

$x = p = 2 + \frac{3}{2}\sqrt{6}$.

Exercise 1.19 $x \leq 0, 1 \leq x \leq 2$.

Exercise 1.20

a) $x > 10$;
b) $2 < x < -1 + \frac{1}{2}\sqrt{52}$.

Exercise 2.21
$a = -\frac{15}{6}$.

Exercise 2.22 $\Delta p = -2.5$.

Exercise 2.23
9.

Exercise 2.24 $a = 1, b = 4$.

Exercise 2.25

a) $\frac{\frac{5^2}{11} - \frac{4}{5}}{3} = \frac{27}{55}$;
b) $\frac{12}{25}$.

Exercise 2.26

a) $y = \frac{5}{6}x + \frac{1}{6}$;
b) $y = -\frac{1}{2}(\ln 2)x + (\frac{1}{2} + \frac{1}{2}\ln 2)$.

Exercise 2.27 $x_0 = 1$.

Exercise 2.28
$y = 4(1 - \sqrt{2})x - 4(1 - \sqrt{2})$, $y = 4(1 + \sqrt{2})x - 4(1 + \sqrt{2})$