

Strategic Complementarity, Fragility, and Regulation

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We show how the balance sheet composition of a financial intermediary, precisions of public and private information, and market stress impinge on fragility through the degree of strategic complementarity of investors' strategies. We show how a solvency and a liquidity ratio are required to control the probabilities of insolvency and illiquidity: With more competition the solvency requirement has to be strengthened, and with higher fire sales penalties and more conservative fund managers the liquidity requirement has to be strengthened while the solvency one relaxed. Higher disclosure may aggravate fragility and require an increase in the liquidity ratio; the regulator should set together disclosure and prudential policy.

Keywords: crises, illiquidity risk, insolvency risk, prudential policy, leverage ratio, liquidity ratio, disclosure, transparency, opaqueness, panic, run, derivatives market, shadow banking, sovereign debt. JEL Codes G21, G28.

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1. Introduction

In a crisis situation, and the present financial crisis is a good example, things seem to go wrong at the same time and an adverse shock is magnified by the actions and reactions of the investors.¹ In particular, liquidity evaporates while short-term investors rush for the exit, and a solvency problem may arise. The increased reliance on market funding of financial intermediaries, investment banks in particular but also commercial banks, has been blamed for the increased fragility. The demise of Northern Rock in 2007, and Bear Stearns and Lehman Brothers in 2008, or of IKB or Hypo Real State in Germany, are cases in point where the short-term leverage of the institutions was revealed as a crucial weakness of their balance sheet.² In this context it has proved difficult to disentangle liquidity from solvency risk. The influence of the opaqueness of financial products, the impact of public news (such as those provided by the ABX index on residential-based mortgage securities, public statements about the health of banks,³ or stigma associated to borrowing from the discount window which becomes known),⁴ as well as the stabilizing or de-stabilizing influence of derivative markets are also debated. The crisis has put regulatory reform in the agenda. Policy makers and regulators are struggling with how to reform capital requirements, introduce liquidity requirements, and control markets for derivatives.⁵

Lack of attention to liquidity issues is believed to be part of the inadequate policy response during the Great Depression. Friedman and Schwartz (1963) in their analysis of the Great Depression argued forcefully that many bank failures arose out of panics, that is, because of liquidity rather than solvency problems. This would be consistent

¹ See Brunnermeier (2009) and Krishnamurthy (2009).

² For example, in June 2007 wholesale funds represented about 26% of liabilities in Northern Rock (Shin (2009)) and short term financing represented an extremely high percentage of total liabilities in Lehman Brothers before the crisis (Adrian and Shin (2010)). Washington Mutual faced a withdrawal of \$16.5 billion of large deposits just in the two weeks before its collapse (according to the Office of Thrift Supervision). See also the evidence in Ivashina and Scharfstein (2010).

³ Such as the case of the run on IndyMac Bancorp in June 2008 which followed shortly after the public release of letters by Senator Schumer of the Banking Committee.

⁴ See Armantier et al. (2011).

⁵ See, for example, FSA (2009) and BIS (2009). The Dodd-Frank Act (2010) introduces a leverage limitation for financial holding companies above a certain size. BIS (2009) proposed two new liquidity ratios: a liquidity coverage ratio to cover short term cash outflows with highly liquid assets and a net stable funding ratio to cover required stable funding with available stable funds. It is worth noting that Bear Stearns was regulated by the SEC and was in fact subject to a liquidity requirement which proved ineffective in the crisis.

with the self-fulfilling view of crises of Bryant (1980) and Diamond and Dybvig (1983). This view of crisis has been disputed by Gorton (1985, 1988) and others.⁶ During the Great Recession started by the subprime mortgage crisis in 2007 a lot of attention has been turned to liquidity issues. For example, it has been claimed that the sovereign debt crisis in the euro area is driven by a self-fulfilling panic.⁷ However, it is increasingly clear that in the euro area there are important solvency problems also. In general, the evidence points at both solvency and liquidity issues being intertwined in any crisis.⁸ The corollary of this fact is that both solvency and liquidity regulation are needed. This paper presents a first attack on how those regulations should be related.

In the model considered investors have to decide whether to keep the investment or run. The investment may be in a currency, bank, or short-term debt. A financial intermediary will be our leading example. The model is based on the theory of games with strategic complementarities with incomplete information (an example of which are the “global games” of Carlsson and van Damme (1993) and Morris and Shin (1998); see Vives (2005)).⁹ We provide a general framework, which bridges the panics and fundamentals view of crises, characterizes illiquidity and insolvency risk, and nests among others the models of Morris and Shin (1998, 2004), Rochet and Vives (2004), and Bebhuk and Goldstein (2011). The paper makes the following incremental contributions to the literature: (i) it shows how the degree of strategic complementarity of the actions of investors is the crucial parameter to characterize

⁶ Gorton (1988) disputed the view that crises are panic driven with a study of crises in the National Banking Era concluding that crises were predictable (see also Schotter and Yorulmazer (2009)). The information view of crises has been developed, among others by Chari and Jagannathan (1988), Jacklin and Bhattacharya (1988), and Allen and Gale (1998). Postlewaite and Vives (1987) present a model with incomplete information about the liquidity shocks suffered by depositors featuring a unique Bayesian equilibrium with a positive probability of bank runs. In their model there is no uncertainty about the fundamental value of the banks’ assets and no solvency problems.

⁷ See De Grauwe and Ji (2012).

⁸ Calomiris and Mason (2003) dispute also the analysis of Friedman and Schwartz (1963) and conclude that some episodes of banking crises in the 1930s in the US can be explained by deteriorating fundamentals while others could be explained by a panic component dominating (as the crises in January and February of 1933). Starr and Yilmaz (2007) claim that both fundamentals and panic elements coexist in the explanation of the dynamics of bank runs in Turkey.

⁹ The importance of strategic complementarities in macro models has been highlighted by Cooper and John (1988). Liu and Mello (2011) apply the global games methodology to study the behavior of hedge funds in the crisis.

equilibrium and fragility (understood as equilibrium sensitivity to small changes in parameters and possibility of discrete jumps), and how it relates to the deep parameters in the model; (ii) delivers predictions both in the case where there is a unique equilibrium and where there are multiple equilibria; (iii) typifies how a regulator has to set solvency and liquidity requirements together with the degree of transparency to control the probabilities of insolvency and illiquidity; and (iv) applies the model to interpret the 2007 run on SIVs. The model can also be applied to sovereign debt crises and currency attacks.

We characterize how the degree of strategic complementarity depends on the balance sheet composition of a financial intermediary, parameters of the information structure of investors (precisions of public and private information), and the level of stress indicators in the market. Strategic complementarity increases with a weaker balance sheet (higher leverage) and with competitive pressure for funding, the level of the fire-sale penalty of early asset liquidation as well as with a higher precision of public information. A weaker balance sheet or an increase in stress indicators raises the probability of a crisis, and the range of fundamentals for which there is coordination failure from the point of view of the institution attacked (that is, when the institution is solvent but illiquid). Importantly, the impact of bad news, say a public signal about weak fundamentals, is magnified when strategic complementarity is high. This means that public signals, coming for example from a derivatives market, may be destabilizing (over and above the strict content of the news), and the more so the more precise they are. The results are consistent with the 2007 run on SIVs (structured investment vehicles) and ABCP (asset-backed commercial paper) conduits.

The policy message that follows from the analysis is that a regulator, with the tools available, needs to pay attention to the composition of the balance sheet of financial intermediaries, in particular to the ratio of cash to unsecured short-term debt and to the short-term leverage ratio (ratio of unsecured short-term debt to equity or, more in general, stable funds). Those two ratios, together with the required return of short-term debt and parameters of the information structure, are crucial determinants of the probabilities of insolvency and illiquidity. Often minimum ratios on solvency (inverse of short-term leverage) and liquidity will be sufficient to control the probabilities of

insolvency and illiquidity. It is found that in a more competitive environment (with higher return on short-term debt) the solvency requirement has to be strengthened. This means that financial liberalization should go together with increased solvency requirements and contrast with the practice that led to banking crises like the US S&Ls in the 1980s. The expectation of turbulent environments, with high fire sales penalties and conservative investors, requires that the liquidity requirement be strengthened while the solvency one relaxed. Finally, when it is expected that potent public signals will be available, such as the introduction of a derivatives market or a new disclosure requirement, illiquidity may increase and an enhanced liquidity requirement may be necessary. The results have also implications for the design of prudential policy with respect to sovereign debt.

The paper proceeds as follows. Section 2 sets up the framework of analysis with the basic model, and characterizes the equilibrium and its links with strategic complementarity, as well as its comparative statics properties. It specializes also the model to a bank run and illustrates the results in this leading example. Section 3 deals with regulation of a financial intermediary. Section 4 applies and extends the model to the 2007 run on SIV and Section 5 deals with the literature connections and evidence. Section 6 considers other applications, including a sovereign debt run. Concluding remarks close the paper. The Appendix collects the proofs of the propositions.

2. A stylized crisis model: the link between strategic complementarity and fragility

This section presents the general framework of analysis with the game of investors, and the characterization of equilibrium and of its comparative static properties.

2.1 The investors' game

Consider the following binary action game among a continuum of investors of mass one. The action set of player i is $\{0,1\}$, with $y_i = 1$ interpreted as “acting” and $y_i = 0$ “not acting”. To act may be to attack a currency, refuse to roll over debt, run on a bank or SIV (Structured Investment Vehicle) or not renew a certificate of deposit in the interbank market.

Let $\pi^1 = \pi(y_i = 1, y; \theta)$ and $\pi^0 = \pi(y_i = 0, y; \theta)$ denote, respectively, the payoffs to acting and not acting where y is the fraction of investors acting and θ is the state of the world. The differential payoff to acting is $\pi^1 - \pi^0 = B > 0$ if $y \geq h(\theta)$, and $\pi^1 - \pi^0 = -C < 0$ if $y < h(\theta)$, where $h(\theta)$ is the resistance function or critical fraction of investors above which it pays to act. We have that

$\pi^1 - \pi^0$	$y > h(\theta)$ $B > 0$	$y \leq h(\theta)$ $-C < 0$
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Let $\gamma \equiv C/(B + C)$ be the critical success probability of the collective action such that it makes an agent indifferent between acting and not acting. This is the ratio of the cost of acting to the differential incremental benefit of acting in case of success in relation to failure. An investor will “act” if his assessed probability of successful mass action is larger than γ . It is assumed that $h(\cdot)$ is strictly increasing, smooth on $(\underline{\theta}, \tilde{\theta})$, and crossing 0 at $\theta = \underline{\theta}$ ($\lim_{\theta \uparrow \underline{\theta}} h(\theta) = 0$) and 1 at $\theta = \tilde{\theta}$.¹⁰

The investors’ game is of strategic complementarities since $\pi^1 - \pi^0$ is increasing in y .¹¹ It follows from these payoffs, if the state of the world is known, that if $\theta < \underline{\theta}$ then it is a dominant strategy to act; if $\theta > \tilde{\theta}$ then it is a dominant strategy not to act; and for $\theta \in (\underline{\theta}, \tilde{\theta})$ there are multiple equilibria. Both everyone acting and no one acting are equilibria. Since the game is a game of strategic complementarities there is a largest and a smallest equilibrium. That is, there are extremal equilibria. The largest equilibrium is $y_i = 1$ for all i if $\theta \leq \tilde{\theta}$, and $y_i = 0$ for all i if $\theta > \tilde{\theta}$, and it is (weakly) decreasing in θ . This is a consequence of $\pi^1 - \pi^0$ being decreasing in θ .

¹⁰ Note that this allows the function $h(\cdot)$ to be discontinuous at $\theta = \underline{\theta}$ with $h(\underline{\theta}) > 0$ and $h(\theta) < 0$ for $\theta < \underline{\theta}$.

¹¹ In a game of strategic complementarities the marginal return of the action of a player is increasing in the level of the actions of rivals. This leads to best replies being monotone increasing. See Vives (2005).

From now on I consider an incomplete information version of the game where investors have a Gaussian prior on the state of the world $\theta \sim N(\mu_\theta, \tau_\theta^{-1})$ and investor i observes a private signal $s_i = \theta + \varepsilon_i$ with Gaussian i.i.d. distributed noise $\varepsilon_i \sim N(0, \tau_\varepsilon^{-1})$.¹² It is worth noting that the prior mean μ_θ of θ can be understood as a public signal of precision τ_θ . With this interpretation as a conditional expectation of θ for a given realization of a public signal, μ_θ can be negative.

The model encompasses several crisis situations studied in the literature: currency attacks (Morris and Shin (1998)), loan foreclosure (Morris and Shin (2004)), credit freezes (Bebchuk and Goldstein (2011)), and bank runs (Rochet and Vives (2004)). In the following section I present the bank run example, which will serve as the basis of the policy exercise in Section 3.

2.2 The bank run model

We consider a bank run model which fits in the general framework introduced (and is based on Rochet and Vives (2004)). We examine other applications of the framework in Section 6. Traditional bank runs were the outcome of massive deposit withdrawal by individual depositors. Modern bank runs are the outcome of non-renewal of short-term credit in the interbank market, like in the case of Northern Rock, the 2007 run on SIV, or the 2008 run by short-term creditors in the case of Bear Stearns or Lehman Brothers.

Consider a market with three dates: $t = 0, 1, 2$. At date $t = 0$, the bank has own funds E (taken to include stable resources: equity, long-term debt and even insured deposits) and uninsured short term debt (e.g. uninsured wholesale deposits, certificates of deposit (CDs)) in amount $D_0 \equiv 1$. These funds are used to finance risky investment I and cash reserves M . The balance sheet constraint at $t = 0$ is thus $E + D_0 = I + M$. The returns θI on these assets are collected at date $t = 2$ and if the

¹² This is referred to in the literature as a “global game”. Those games were introduced by Carlsson and van Damme (1993) as games of incomplete information with types determined by each player observing a noisy signal of the underlying state. The goal is to select an equilibrium with a perturbation in a complete information game with multiple equilibria. The basic idea is that players entertain the “global picture” of slightly different possible games being played. Each player has then a noisy signal of the game being played.

bank can meet its obligations, the short-term debt is repaid at face value D , and the equityholders of the bank obtain the residual (if any). Investors are also entitled to the face value D if they withdraw in the interim period $t = 1$. Let $m \equiv M/D$ be the liquidity ratio, $\ell = D/E$ be the short-term leverage ratio, and $d = D/D_0$ the return of the short-term debt.¹³

A continuum of fund managers makes investment decisions in the short-term debt market. At $t = 1$ each fund manager, after the observation of a private signal about the future realization of θ , decides whether to cancel ($y_i = 1$) or renew his position ($y_i = 0$). If $y_i D > M$ then the bank has to sell some of its assets¹⁴ in a secondary market to meet payments. The early sale value of the assets of the bank involves a fire sales penalty $\lambda > 0$ (retrieving only $\theta/(1 + \lambda)$ for each unit invested).¹⁵

A fund manager is rewarded for taking the right decision (that is, withdrawing if and only if the bank fails). The cost of canceling the investment is C and the benefit for getting the money back or canceling when the bank fails is $\hat{B} = B + C$. Again, $\gamma \equiv C/(B + C)$ is likely to be small since $B + C$ is the benefit to make the right decision. What is crucial is that investors/fund managers, whatever the reason, adopt a behavioral rule of the type: cancel the investment if and only if the probability that the bank fails is above threshold γ . This rule will arise also if investors expect a fixed return when withdrawing, nothing if they withdraw and the bank fails, and there is a

¹³ The distinction of stable funds within liabilities is made also in the BIS (2009) document dealing with liquidity risk. The two ratios proposed in the BIS document are in fact equivalent in our simple formulation. The BIS short-term funds ratio would correspond to m and the BIS net stable funds ratio to E/I . Note also that leverage $D/(D + E)$ equals $(1 + \ell^{-1})^{-1}$ and is monotone in ℓ .

¹⁴ Or borrow against collateral in the repo market.

¹⁵ The parameter λ could be related to the Libor-OIS spread (a measure of the difference between interbank rates and the rates paid on instruments which are not exposed to the default risk of intermediaries) which increased substantially both in August 2007 (start of the crisis) and September 2008 (Lehman Brothers' collapse). In case of secured collateral λ relates to the haircut required. Haircuts on asset backed securities rose dramatically after the collapse of Lehman Brothers (see, e.g., Gorton and Metrick (2010)). It is important to remark that the liquidity requirements on broker-dealers in the US were related to unsecured funding while, for example, the demise of Bear Stearns in the end happened because of its failure to renew its secured funding. (See SEC's Oversight of Bear Stearns and Related Entities (SEC, 2008).)

(small) cost of withdrawing. Note that a larger γ is associated to a less conservative investor. Risk management rules may therefore influence γ .¹⁶

Let $\underline{\theta}$ be the solvency threshold of the bank.¹⁷ Using the balance sheet constraint we have $\underline{\theta} \equiv (D - M)/I = (1 - m)/(\ell^{-1} + d^{-1} - m)$. If $\theta < \underline{\theta}$ the bank fails even if all fund managers renew credit to the bank; and $\tilde{\theta} \equiv (1 + \lambda)\underline{\theta}$ the “supersolvency” threshold, such that a bank does not fail even if no fund manager renews his CDs. Under these conditions the bank fails if

$$y \geq h(\theta) = m + \frac{\ell^{-1} + d^{-1} - m}{\lambda}(\theta - \underline{\theta})$$

for $\theta \geq \underline{\theta}$, and $h(\theta) = 0$ if $\theta < \underline{\theta}$. We have that $h(\underline{\theta}) = m > 0$ and h is decreasing in λ , d . (See Figure 1.) If $1 - \ell^{-1} - d^{-1} < 0$, noting that $\text{sign}\{\partial \underline{\theta}/\partial m\} = \text{sign}\{1 - \ell^{-1} - d^{-1}\}$, h is increasing in m .¹⁸

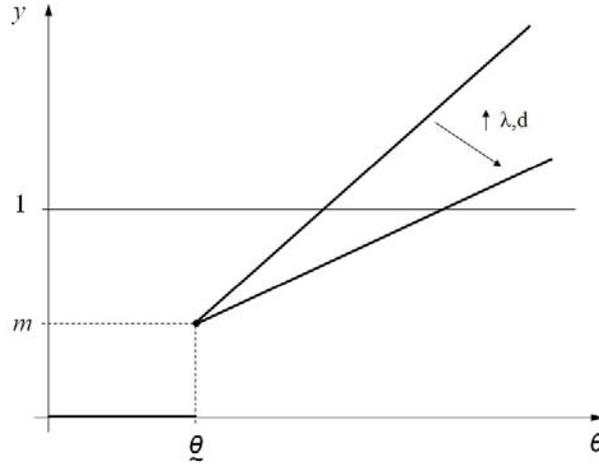


Figure 1. The resistance function in the bank runs model:

$h(\theta) = m + (\ell^{-1} + d^{-1} - m)\lambda^{-1}(\theta - \underline{\theta})$ if $\theta \geq \underline{\theta}$, and $h(\theta) = 0$ if $\theta < \underline{\theta}$. Above the solid line $y \geq h(\theta)$ and the bank fails.

¹⁶ The degree of conservatism of investors is an important determinant of short-term lending behavior according to the review of Krishnamurthy (2010) of debt markets during the crisis.

¹⁷ It is worth noting that the solvency threshold is related to what the FDIC calls the “Net Non-core Funding Dependence Ratio”, as part of the CAMELS assessment, computed as non-core liabilities, less short-term investments divided by long-term assets.

¹⁸ We have that $\partial h/\partial m = \lambda^{-1}(1 + \lambda - \theta) > 0$ since $\theta < (1 + \lambda)\underline{\theta} < 1 + \lambda$ as $1 - \ell^{-1} - d^{-1} < 0$ implies that $\underline{\theta} < 1$.

In the balance sheet of a financial intermediary typically we have $1 - \ell^{-1} - d^{-1} < 0$. Indeed, the ratio of (uninsured) short-term debt over stable funds (equity, long term debt and insured deposits) $\ell = D/E$ is below 1 for commercial banks, and although it is above 1 for investment and wholesale banks typically $d^{-1} \geq .9$ (with an interest rate of at most 10%), and therefore we would need $\ell^{-1} < .1$ or $\ell > 10$ to have $1 - \ell^{-1} - d^{-1} > 0$.¹⁹ For a typical SIV we have also that $\ell < 1$.²⁰ We assume henceforth, unless otherwise stated, that $1 - \ell^{-1} - d^{-1} < 0$.

An alternative interpretation of the bank model would be to consider the aggregate banking sector of a country with θ being a macroeconomic fundamental, with the investors being those outside the banking sector. In this interpretation the ratios are the aggregate ratios, we abstract from any externalities among banks, and a crisis is a crisis of the banking sector.

2.3. Equilibrium and strategic complementarity

In this section we study equilibria of the investors' game and the factors influencing the degree of strategic complementarity. We will look for a symmetric equilibrium in threshold strategies in which an investor will run (or act) if and only if he receives a signal below a certain threshold. It can be shown that these are the only equilibria possible in the game.

In order to gain some intuition into the structure of the game let us think in terms of the best reply of a player $r(\cdot)$ to the (common) signal threshold \hat{s} used by the other players. An equilibrium then is an s^* such that $s^* = r(s^*)$. This signal threshold will imply a critical state-of-the-world $\theta^* \in [\underline{\theta}, \tilde{\theta}]$ below which the acting mass is successful.

¹⁹ See Figure A in the Appendix for data on some US banks.

²⁰ According to the Global Financial Stability Report of the IMF (April 2008), the typical funding profile of SIV in October 2007 was 27% in asset-backed commercial paper (ABCP) and the rest in medium-term notes and capital notes. This would mean that $\ell = .34$.

The best reply can be computed in two steps. Given a signal threshold at \hat{s} which other investors run the investor computes the failure threshold $\hat{\theta}$ such that the institution fails if and only if $\theta \leq \hat{\theta}$: $Pr(s \leq \hat{s} / \hat{\theta}) = h(\hat{\theta})$. This yields the failure curve $\hat{\theta} = \theta_f(\hat{s})$, which is increasing in \hat{s} , for which the fraction of acting players $y = Pr(s \leq \hat{s} / \hat{\theta})$ equals the critical fraction above which it pays to act $h(\hat{\theta})$. Then given a failure threshold $\hat{\theta}$ the investor computes the signal threshold \hat{s} below which it is optimal to run: $Pr(\theta \leq \hat{\theta} / \hat{s}) = \gamma$. This yields the optimal threshold curve $\hat{s} = s_T(\hat{\theta})$, which is linear with slope $\frac{\tau_\theta + \tau_\varepsilon}{\tau_\varepsilon}$ and increasing in $\hat{\theta}$, and for which the expected payoff of acting and not acting is the same. (See Figure 2).

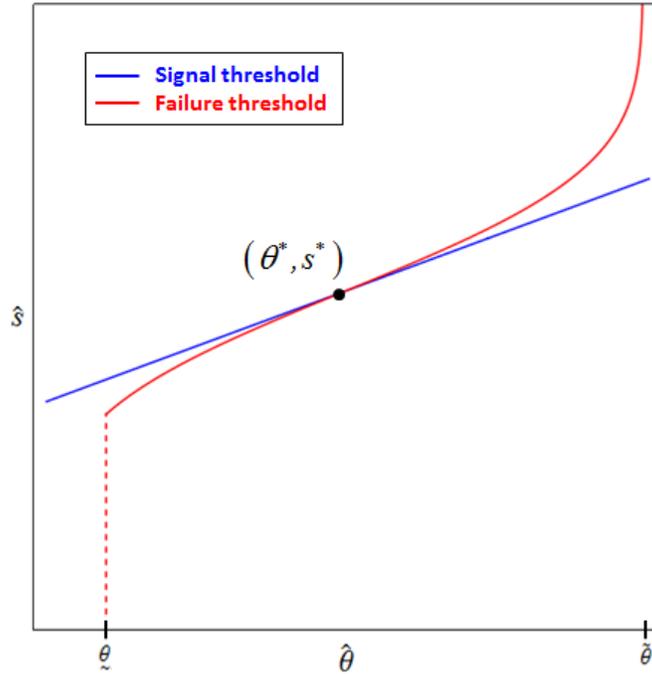


Figure 2: Equilibrium characterization. Parameters: $\tau_\varepsilon = 0.05$, $\gamma = 0.5$, $m = 0.1$, $\ell^{-1} = 0.2$, $d^{-1} = 0.9$, $\lambda = 1$, $\theta = 0.9$, $\tilde{\theta} = 1.8$, $\tau_\theta^L = 0.05$, $\tau_\theta = 0.5$, $\mu_\theta = 1.3$.

The best response function is the composition of the two thresholds: $r(\hat{s}) = s_T(\theta_f(\hat{s}))$, which is increasing. The game is, indeed, of strategic complementarities: a higher threshold \hat{s} by others induces a player to use also a higher threshold. (See Figure 3.) Equilibria are fixed points of this best response or, equivalently, given by the intersection points of the two curves in space of failure and

signal thresholds: $\hat{\theta} = \theta_F(\hat{s})$ and $\hat{s} = s_T(\hat{\theta})$.

The following proposition provides the equilibrium characterization and generalizes the results in the literature. Let h_1 denote the smallest slope of $h(\cdot)$ on $(\underline{\theta}, \tilde{\theta})$.

Proposition 1.

- (i) An equilibrium is characterized by two thresholds (s^*, θ^*) with s^* yielding the signal threshold below which an investor acts and $\theta^* \in [\underline{\theta}, \tilde{\theta}]$ the state-of-the-world critical threshold, below which the acting mass is successful. The probability of a crisis conditional on $s = s^*$ is γ .
- (ii) There is a critical $\bar{h}_0 \in (0, 1)$ such that there is an equilibrium with $\theta^* = \underline{\theta}$ if $h(\underline{\theta}) \geq \bar{h}_0$. For $h(\underline{\theta}) < \bar{h}_0$ we have that $\theta^* > \underline{\theta}$ for any equilibrium and the equilibrium is unique if $(\tau_\theta / \sqrt{\tau_\varepsilon}) \leq h_1 \sqrt{2\pi}$. If $h(\cdot)$ is linear and $(\tau_\theta / \sqrt{\tau_\varepsilon}) > h_1 \sqrt{2\pi}$, then there is a range of γ for which there are three equilibria.

The complete proof of the proposition is in the Appendix and it exploits the monotonicity properties of the game of strategic complementarities with incomplete information. The uniqueness condition $\tau_\theta / \sqrt{\tau_\varepsilon} \leq h_1 \sqrt{2\pi}$ implies that $r'(\hat{s}) = \frac{\tau_\theta + \tau_\varepsilon}{\tau_\varepsilon} \theta'_F(\hat{s}) \leq 1$. It is worth noting that r' is increasing in τ_θ since $\theta'_F(\hat{s})$ is independent of τ_θ . This ensures that $r(\cdot)$ crosses the 45° line only once and that the equilibrium is unique. A sufficient condition to have multiple equilibria (necessary also for regular equilibria for which $r'(\hat{s}) \neq 1$) is that $r'(s) > 1$ for $r(s) = s$.

In summary, *a necessary condition for multiple equilibria is that strategic complementarity be strong enough*; a sufficient condition is *that strategic complementarity be strong enough at relevant points (candidate equilibria)*. Indeed, when *strategic complementarity* is moderate always (case $r'(\cdot) \leq 1$) then there is a unique equilibrium, when it is not there may be multiple equilibria (see Figure 3). When $h(\underline{\theta}) < \bar{h}_0$ and h_1 is large enough there is a unique equilibrium. In the linear

case, for an intermediate range of γ , as h_1 , the slope of $h(\cdot)$, decreases we have multiple equilibria (generically three), and if h_1 decreases further we go back to a unique equilibrium since then strategic complementarity is strong in an irrelevant range (see Figure 3).²¹

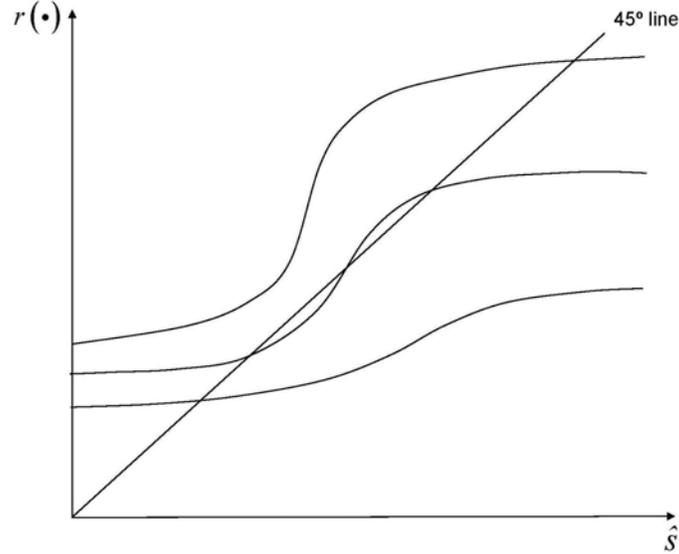


Figure 3: Possible best responses of a player to the threshold strategy \hat{s} used by rivals for an intermediate range of γ . The lower (upper) branch corresponds to h_1 high (low), the intermediate to h_1 intermediate.

The strength of the strategic complementarity among the actions of the players depends on the slope of the best response. The maximal value of the slope is increasing in h_1^{-1} and in τ_θ , and decreasing and then increasing in τ_ε . The degree of strategic complementarity will be higher whenever h is less sensitive to θ (larger h_1^{-1}) and/or the prior is more precise (τ_θ large) and/or signals more imprecise (τ_ε low). When h_1^{-1} is large a change in fundamentals θ changes little the critical threshold $h(\theta)$. This implies that a change in the strategy threshold \hat{s} used by other investors leads to a larger optimal reaction since the induced change in the conditional

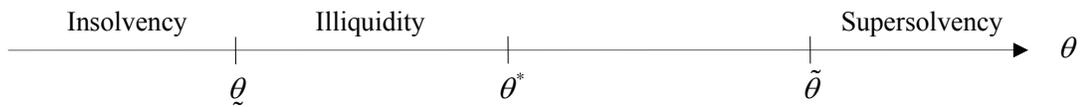
²¹ A similar situation arises with movements in γ , but then the best reply moves vertically (with $(\tau_\theta/\sqrt{\tau_\varepsilon}) > h_1\sqrt{2\pi}$, for example, when $h(\theta) = \theta/10$, $\tau_\theta = 1$, $\tau_\varepsilon = 5$, $\bar{\theta} = 5$ and $\gamma \in \{.1, .5, .9\}$).

probability that the acting investors succeed is larger. When noise in the signals is large a player faces little uncertainty about the behavior of others and the strategic complementarity is increased. In the limit cases $\tau_\epsilon \rightarrow +\infty$ (or, equivalently, a diffuse prior $\tau_\theta \rightarrow 0$), investors face maximal strategic uncertainty (then the distribution of the proportion of acting players \tilde{y} is uniformly distributed over $[0,1]$ conditional on $s_i = s^*$). In contrast, at any of the multiple equilibria with complete information when $\theta \in (\underline{\theta}, \tilde{\theta})$, investors face no strategic uncertainty (e.g. in the equilibrium in which everyone acts, an investor has a point belief that all other investors will act).

In the bank run case $h_1^{-1} = \lambda / (\ell^{-1} + d^{-1} - m)$ and therefore strategic complementarity among investors is increasing in short-term leverage ℓ , face value of short-term debt d , fire sales penalty λ as well as the liquidity ratio m .

2.4. Coordination failure, illiquidity risk, and insolvency risk

At equilibrium with threshold θ^* , there is a crisis when $\theta < \theta^*$. In the range $[\theta^*, \tilde{\theta})$ there is coordination failure from the point of view of investors, because if all of them were to act then they would succeed. In the range $[\underline{\theta}, \theta^*)$ there is coordination failure from the point of view of the institution attacked. In the bank model in the range $[\underline{\theta}, \theta^*)$ the bank is solvent but illiquid, that is, the bank would have no problem if only investors would renew their short-term debt, but in the range they do not and the bank is illiquid.



The risk of illiquidity is therefore given by $Pr(\underline{\theta} \leq \theta < \theta^*)$ and the risk of insolvency by $Pr(\theta < \underline{\theta}) = \Phi(\sqrt{\tau_\theta}(\underline{\theta} - \mu_\theta))$ where Φ denotes the standard Normal cumulative distribution. $Pr(\theta < \underline{\theta})$ is the probability that the bank is insolvent when there is no coordination failure from the point of view of the bank.

The overall probability of a “crisis” is $Pr(\theta < \theta^*) = \Phi(\sqrt{\tau_\theta}(\theta^* - \mu_\theta))$. Note that $Pr(\underline{\theta} \leq \theta < \theta^*) = Pr(\theta < \theta^*) - Pr(\theta < \underline{\theta})$. A crisis occurs for low values of the fundamentals. In contrast, in the complete information model there are multiple self-fulfilling equilibria in the range $(\underline{\theta}, \tilde{\theta})$.

2.5 Comparative statics

We develop the comparative statics properties of equilibria, both when the equilibrium is unique or multiple. The comparative statics results that follow hold when the equilibrium is unique, and when there are multiple equilibria for the extremal equilibria (the largest and the smallest). Furthermore, the results hold for any equilibrium, even the middle unstable one, if out-of-equilibrium adjustment is adaptive. With best-reply dynamics at any stage after the parameter perturbation from equilibrium, a new state of the world θ is drawn independently and a player responds to the strategy threshold used by other players at the previous stage. Then a parameter change that moves monotonically the best reply will induce a monotone adjustment process with an unambiguous prediction. For instance, if we are at the higher equilibrium of the middle branch in Figure 3, an increase in h_1 may induce a movement to the lower branch and best reply dynamics would settle at the unique (and lower) equilibrium.²² The following proposition states the results. The comparative statics results are not restricted to parameter configurations where the equilibrium is unique as in the received literature.

In the applications it is useful to parameterize the resistance function $h(\theta; \alpha)$ with a parameter α representing an index of vulnerability or stress (with $\partial h / \partial \alpha < 0$). A larger α means more vulnerability or a more stressful environment for the institution attacked since the threshold for the attack to be successful is lower. For example, in the bank run we have that $h(\theta; \alpha) = m > 0$ and we can identify the parameter α with λ , d , ℓ or m^{-1} when $1 - \ell^{-1} - d^{-1} < 0$, since h is decreasing in these variables.

²² See Echenique (2002) and Vives (2005).

Proposition 2. Comparative statics. Let $h(\varrho) < \bar{h}_0$. At extremal equilibria (or under adaptive dynamics):

- (i) Both θ^* , s^* , the probability of crisis $Pr(\theta < \theta^*)$ are decreasing in γ (i.e., with less conservative investors) and in the expected value of the state of the world μ_θ , and increasing in stress indicator α .
- (ii) A release of a public signal μ_θ has a multiplier effect on equilibrium thresholds (i.e. over and above the impact on the best response of an investor), which is enhanced if τ_θ is higher.
- (iii) Let $\gamma < 1/2$. If μ_θ is low enough (weak fundamentals with $\mu_\theta \leq \varrho$ is sufficient) then a more precise public signal increases θ^* , $Pr(\theta < \theta^*)$ and the range $[\varrho, \theta^*)$ while a more precise private signal reduces it. If μ_θ is high enough (strong fundamentals with $\mu_\theta > \tilde{\theta} - (\tau_\theta + \tau_\varepsilon)^{-1/2} \Phi^{-1}(\gamma)$ is sufficient), the results are reversed.

Remark 1: It is immediate from (i) that the range $[\varrho, \theta^*)$ is decreasing in γ and μ_θ .

Remark 2: The region of potential multiplicity $(\tau_\theta/\sqrt{\tau_\varepsilon}) > h_1\sqrt{2\pi}$ is enlarged with an increase in payoff complementarity (decrease in h_1) and/or an increase in the precision of the public signal in relation to the private one $\tau_\theta/\sqrt{\tau_\varepsilon}$.

Remark 3: When the equilibrium is unique it can be checked that (iii) can be strengthened as follows: There are thresholds $\hat{\mu}_\theta$ and $\widehat{\mu}_\theta$ with $\hat{\mu}_\theta < \widehat{\mu}_\theta$ such that θ^* increases with τ_θ if and only if $\mu_\theta < \hat{\mu}_\theta$ and with τ_ε if and only if $\mu_\theta > \widehat{\mu}_\theta$.²³

²³ We have that $\hat{\mu}_\theta$ is the unique solution in μ_θ to $\partial\varphi/\partial\tau_\theta(\theta^*(\mu_\theta), \mu_\theta) = 0$ or $\theta^*(\mu_\theta) = \mu_\theta + (\tau_\theta + \tau_\varepsilon)^{-1/2} \Phi^{-1}(\gamma)/2$ and that $\widehat{\mu}_\theta$ is the unique solution in μ_θ to $\partial\varphi/\partial\tau_\varepsilon(\theta^*(\mu_\theta), \mu_\theta) = 0$ or $\theta^*(\mu_\theta) = \mu_\theta + (\tau_\theta + \tau_\varepsilon)^{-1/2} \Phi^{-1}(\gamma)$. Note that $\hat{\mu}_\theta < \widehat{\mu}_\theta$ since $\theta^*(\mu_\theta)$ is decreasing and $\Phi^{-1}(\gamma) < 0$ since $\gamma < 1/2$.

Remark 4: Releasing more public information is not necessarily good according to result (iii). To start with it will not be good if fundamentals are weak (μ_θ low enough). In this case a more precise public signal is a coordinating device for investors to act (attack) since each one knows that others will put more weight to the public signal. The effect is reinforced with a lower precision of private information since then the value of the public signal is enhanced. (Note that strategic complementarity is maximized for high τ_θ and low τ_ε .) The opposite happens when fundamentals are good.

Let us consider illustrate the comparative static results in the bank run model (see Claim 2 in the Appendix). First of all, in this model \bar{h}_0 is given by a liquidity ratio $\bar{m} \in (0,1)$. We have then that for $m < \bar{m}$, $\theta^* > \underline{\theta}$ in equilibrium. The comparative statics of λ , d , ℓ , or m^{-1} when $1 - \ell^{-1} - d^{-1} < 0$ follow from those of α in Proposition 2(i). It is immediate that

- The probability of insolvency $Pr(\theta < \underline{\theta})$ is decreasing in the liquidity ratio m and in the expected return on the bank's assets μ_θ , increasing in the short-term leverage ratio ℓ and the face value of debt d , and independent of the fire-sales penalty λ , and the critical withdrawal probability γ .
- At extremal equilibria (or under adaptive dynamics), the probability of failure $Pr(\theta < \theta^*)$ (and the critical θ^*) and the range of illiquidity $\theta^* - \underline{\theta}$ are decreasing in m , γ , and μ_θ , and increasing in λ , ℓ , and d . The probability of illiquidity $Pr(\underline{\theta} \leq \theta < \theta^*)$ is decreasing in γ and increasing in λ and ℓ .

The limit case where $\tau_\varepsilon \rightarrow \infty$ allows for a closed-form solution. Then it is easy to see that

$$s^* = \theta^* = \underline{\theta} \left(1 + \frac{\lambda}{1-m} (\max\{1-\gamma-m, 0\}) \right)$$

and $\bar{m} = 1 - \gamma$.²⁴ Indeed, when $m < 1 - \gamma$, both θ^* and $\theta^* - \underline{\theta}$ are decreasing in γ and in m (provided that $1 - \ell^{-1} - d^{-1} < 0$), and increasing in λ , ℓ , and d .

²⁴ Note that in this case θ^* and s^* are independent of μ_θ and τ_θ .

The following table summarizes the results. Note that the qualitative comparative statics of the probability of failure and of the range of illiquidity are identical.

Table 1. Comparative statics of solvency and liquidity risk for $m < \bar{m}$.

	$\Pr(\theta < \underline{\theta})$ (Insolvency)	$\Pr(\theta < \theta^*)$ (Failure)	$\theta^* - \underline{\theta}$ (Range of illiquidity)
m (Liquidity ratio)	(-)	(-)	(-)
μ_θ (Strength of fundamentals)	(-)	(-)	(-)
ℓ (Leverage ratio)	(+)	(+)	(+)
d (Cost of funds)	(+)	(+)	(+)
λ (Fire sales penalty)	0	(+)	(+)
γ^{-1} (Investor conservatism)	(0)	(+)	(+)

An increase in balance sheet stress (lower m whenever $1 - \ell^{-1} - d^{-1} < 0$ or higher ℓ), or market stress (higher d , the return of deposits, which can be interpreted as an increase in competitive pressure, or higher λ , the level of fire-sales penalty for early liquidation, or lower γ , more conservative investors), increases the probability of failure. This probability also increases with worse fundamentals (lower μ_θ) and more precise public information (higher τ_θ) when fundamentals are bad according to Proposition 2.

Figure 4 illustrates how an increase in public precision raises strategic complementarity and helps investors coordinate on a run when fundamentals are weak.

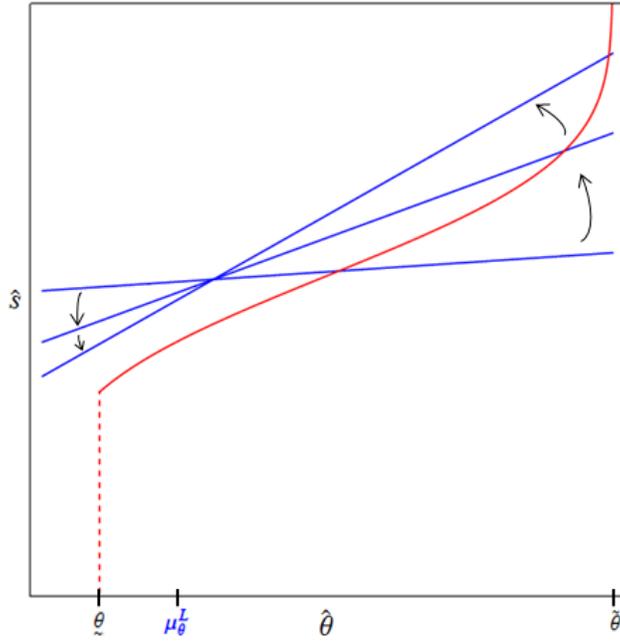


Figure 4. Increase in public precision with weak fundamentals (low public signal). Red: critical failure threshold for given signal threshold. Blue: optimal signal threshold given failure threshold. Parameters: $\tau_\varepsilon = 0.05$, $\gamma = 0.5$, $m = 0.1$, $\ell^{-1} = 0.2$, $d^{-1} = 0.9$, $\lambda = 1$, $\underline{\theta} = 0.9$, $\tilde{\theta} = 1.8$, $\tau_\theta^L = 0.05$, $\tau_\theta^M = 0.5$, $\tau_\theta^H = 0.8$, $\mu_\theta^L = 1.1$.

Figure 5 illustrates how the effect of bad news (when the public signal goes from μ_θ^H to μ_θ^L) has a very different impact when public precision is low or high. In the latter case strategic complementarity is high since τ_θ is high.

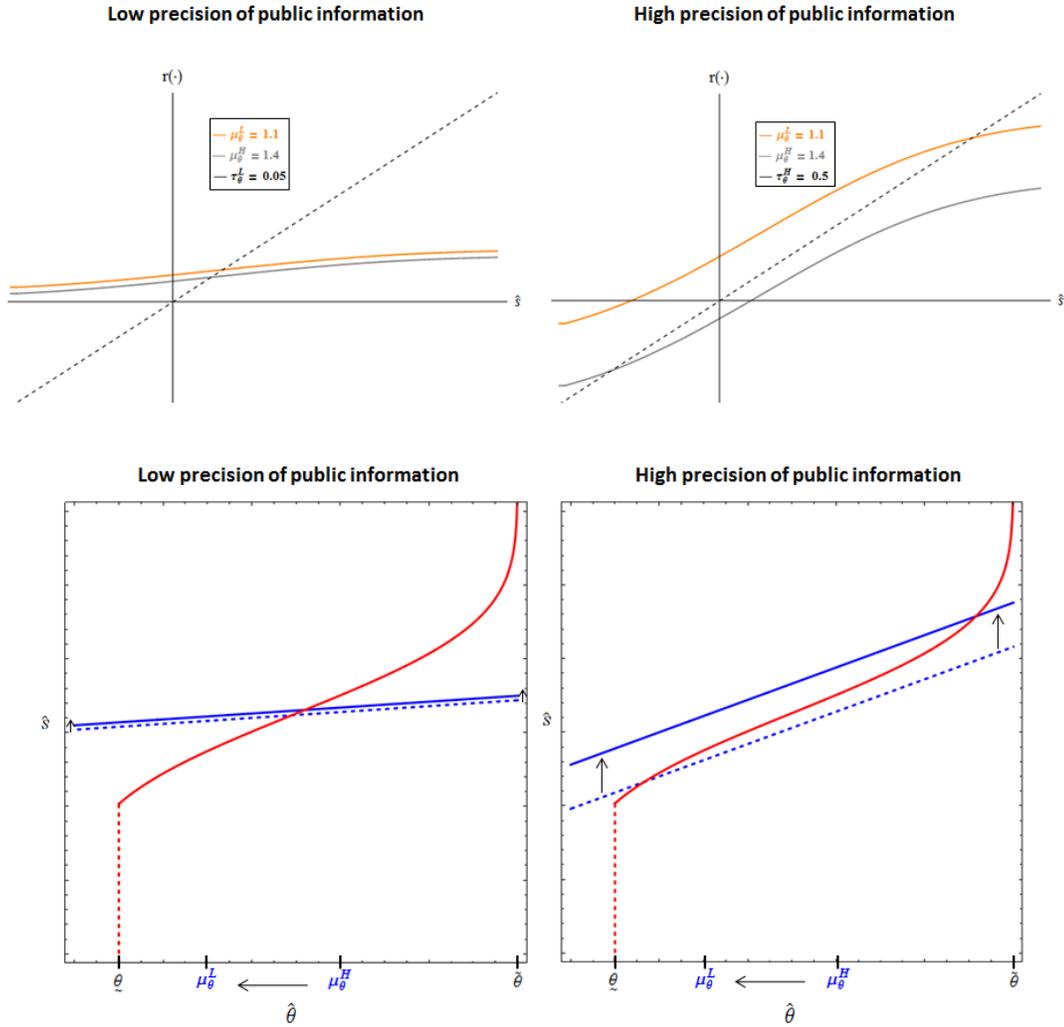


Figure 5. The effect of bad news. Parameters: $\tau_\varepsilon = 0.05$, $\gamma = 0.5$, $m = 0.1$, $\ell^{-1} = 0.2$, $d^{-1} = 0.9$, $\lambda = 1$, $\theta = 0.9$, $\tilde{\theta} = 1.8$, $\tau_\theta^L = 0.05$, $\tau_\theta^H = 0.5$, $\mu_\theta^L = 1.1$, $\mu_\theta^H = 1.4$.

Let us turn now to regulation in the bank runs model and how the likelihood of crises can be controlled.

3. Liquidity and solvency regulation of financial intermediaries

Two common objectives of regulators are the control of the probabilities of insolvency and of illiquidity (see, e.g., Freixas and Rochet (2008)). The control of potential insolvency is grounded typically on the excessive incentives to take risk on banks with limited liability. The control of illiquidity is grounded on alleviating fragility and the coordination problems in the financing of banks with their associated contagion problems. In short, the second objective tries to avoid that a solvent

institution fails and the first is the prudential requirement that provides a lower bound for solvency. The liquidity concern can be addressed with the lender of last resort facility of a central bank or by liquidity regulation. In the present paper we will assume those objectives for the regulator which will use as instruments leverage and liquidity requirements.

Suppose thus that the regulator wants to control the maximum probabilities of insolvency q and of failure p , and therefore the probability of illiquidity. This can be rationalized assuming that the regulator has a loss function which is increasing in q and p which is minimized subject to an efficiency constraint (in terms of the expected value of bank assets). The control is possible using liquidity and leverage ratios. For the sake of simplicity let us restrict attention to the parameter range where equilibrium is unique.

If the regulator wants that $p = q$ then this can be done, in the parameter region with a unique equilibrium, setting $m \geq \bar{m}$ and inducing $\theta^* = \underline{\theta}$. In general this will be too costly (in terms of foregone returns), because of the reduction in the level of investment in the risk asset, and the regulator will set $p > q$.

Since both $\underline{\theta}$, and θ^* are decreasing in m and ℓ^{-1} , we can make sure that $Pr(\theta < \underline{\theta}) \leq q$ and $Pr(\theta < \theta^*) \leq p$ choosing m and ℓ^{-1} large enough so that

$$\underline{\theta} \leq \underline{\theta}_q \equiv \mu_\theta + \Phi^{-1}(q)/\sqrt{\tau_\theta}, \quad (\text{S})$$

and

$$\theta^* \leq \theta_p^* \equiv \mu_\theta + \Phi^{-1}(p)/\sqrt{\tau_\theta}. \quad (\text{L})$$

The boundaries of both constraints can be shown to be linear and downward sloping in the space (m, ℓ^{-1}) provided that $\underline{\theta}_q < 1$:

$$\ell^{-1} \geq (\underline{\theta}_q)^{-1} - d^{-1} - \left((\underline{\theta}_q)^{-1} - 1 \right) m, \quad (\text{S})$$

and

$$\ell^{-1} \geq \frac{1+k\lambda}{\theta_p^*} - d^{-1} - \left(\frac{1+\lambda}{\theta_p^*} - 1 \right) m. \quad (L)$$

where $k \in (0,1)$ is a function of p, γ and $\tau_\theta/\tau_\varepsilon$ (see Figure 6).

Note that since from (S) we have that $\ell^{-1} \geq 1 - d^{-1} - \left((\theta_q)^{-1} - 1 \right) (m-1)$ we have that $0 > \left((\theta_q)^{-1} - 1 \right) (m-1) \geq 1 - \ell^{-1} - d^{-1}$ as desired for $\theta_q < 1$ and $m < 1$.

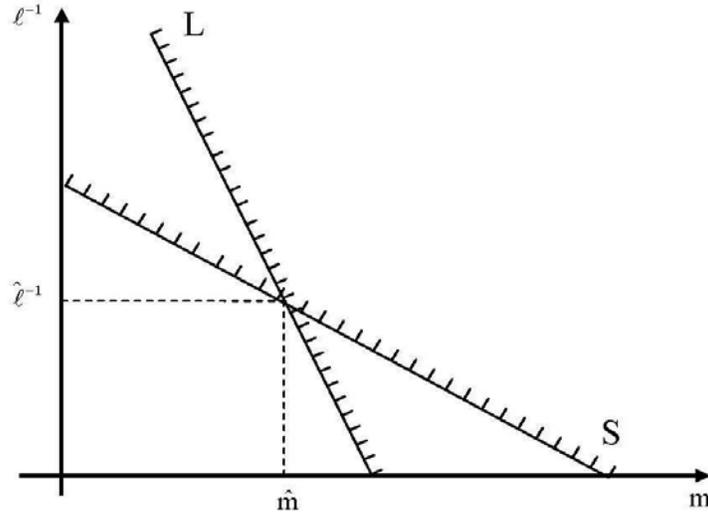


Figure 6: Solvency (S) and liquidity (L) constraints to control probabilities of insolvency and crisis with a short-term leverage ratio ($\ell = D/E$) and a liquidity ratio ($m = M/D$).

We have, therefore, that the regulator can control the maximum probabilities of insolvency q and of a crisis p , and therefore the probability of illiquidity, by an appropriate induced choice of liquidity m and leverage ℓ ratios. The regulator has to propose therefore a region in (m, ℓ^{-1}) space where the ratios of a bank have to lie. This region is the upper contour of the constraints S and L , limited by a kinked downward sloping schedule reflecting the (partial) *substitutability* between m and ℓ^{-1} (Figure 6). The bank will choose then the least cost combination (m, ℓ^{-1}) which will necessarily lie on the frontier of one of the constraints. It is worth noting that often there will be no loss of efficiency if the regulator sets minimum levels for (m, ℓ^{-1}) given the kink in the constraint set. This is so since the optimization of the financial intermediary will lead to the kink in the constrained set for an ample range of given

prices for liquidity and capital. The minimal $(\hat{m}, \hat{\ell}^{-1})$ ratios will be given by the intersection of the boundaries of the solvency and the liquidity constraints. When $\lambda = 0$ the two constraints collapse into one and in equilibrium $\theta^* = \underline{\theta}$. If we require $p = q$, then $\hat{m} = k = \bar{m}$ and we can control q with a requirement on ℓ^{-1} .

The following Result provides a summary of the main conclusions of our analysis while Proposition 3 provides the technical statement of the results (see the proof in the Appendix).

Result. The regulator has to act on the solvency ℓ^{-1} and liquidity m ratios to control for the probabilities of insolvency and of a crisis. With $\underline{\theta}_q < 1$ the minimal regulatory ratios \hat{m} and $\hat{\ell}^{-1}$ move in opposite directions in response to parameter changes: The liquidity requirement \hat{m} has to be increased and the solvency one $\hat{\ell}^{-1}$ decreased with increased fire-sales penalty λ , more conservative fund managers γ^{-1} , and better fundamentals μ_θ , as well a more precise public information τ_θ under certain conditions. More funding pressure for intermediaries calls for an increased $\hat{\ell}^{-1}$ and a constant \hat{m} .

Proposition 3.

(i) Both a solvency (S) and a liquidity (L) linear constraints, acting on the ratios ℓ^{-1} and m , have to be fulfilled to control for the probabilities of insolvency (q) and of a crisis (p).

(ii) Let $\underline{\theta}_q < 1$. The solvency constraint S becomes tighter when d increases and is independent of γ and λ . The liquidity constraint L becomes tighter when d , λ increase or γ decreases. Both constraints are more relaxed with a higher μ_θ , while (provided $1/2 > p$) a higher τ_θ has a relaxing effect on S and also on L as long as $\tau_\varepsilon \rightarrow \infty$ or $p > \gamma$ and τ_ε is small.

(iii) There is a constant $k \in (0,1)$ such that the minimal regulatory ratios for liquidity and solvency are, respectively,

$$\hat{m} = \max \left\{ 1 - (1 - k) \left(1 - \lambda^{-1} \left(\frac{\theta_p^*}{\theta_q} - 1 \right) \right)^{-1}, 0 \right\}$$

and

$$\hat{\ell}^{-1} = (\theta_q)^{-1} - d^{-1} - \left((\theta_q)^{-1} - 1 \right) \hat{m} > 1 - d^{-1}.$$

If $p > q$ and $\lambda k > \left(\frac{\theta_p^*}{\theta_q} - 1 \right)$ then $k > \hat{m} > 0$ and the comparative statics of the regulatory ratios is given in Table 2. When $\tau_\varepsilon \rightarrow \infty$ we have that $k = 1 - \gamma$.

Table 2. Comparative statics of regulatory ratios when $p > q$, and λk not too small.

	λ (Fire sales penalty)	γ^{-1} (Investor conservatism)	d (Cost of funds)	μ_θ (Strength of fundamentals)	τ_θ (Precision of public information)
\hat{m} (Liquidity ratio)	(+)	(+)	0	(+)	(+)*
$\hat{\ell}^{-1}$ (Solvency ratio)	(-)	(-)	(+)	(-)	(-)*

* Provided $p < 1/2$ and either $\tau_\varepsilon \rightarrow \infty$ or τ_ε small enough and $p > \gamma$.

Remark 5: The constant k is independent of μ_θ , increasing in p , decreasing in γ , and with ambiguous dependence on $\tau_\theta/\tau_\varepsilon$; k is precisely the liquidity ratio \bar{m} which would eliminate the illiquidity region when it is required that $p = q$ and it is induced that $\theta^* = \theta$.

A higher return on short-term debt d will increase the solvency requirement and leave unaffected the liquidity requirement. This is so since both constraints are tightened in a vertical way when d increases (see Figure 7.a). This means that liberalization that increases competition should go together with increased solvency requirements. This did not happen, or happened only partially, in several liberalization episodes that lead to subsequent crises such as the Savings & Loans in the US in the 1980s, the 1990s crisis in the Nordic countries, and the banking liberalization in Spain that started in the late 1970s.

A higher fire sales penalty λ and more conservative investment managers (lower γ) will increase the liquidity requirement and decrease the solvency one. This is so since those changes do not affect the solvency constraint and tighten the liquidity constraint (see figures 7.b and 7c.). This means that in a more competitive environment (higher d) the solvency requirement has to be strengthened, and in an environment where λ is high and γ is low the liquidity requirement has to be strengthened while the solvency one relaxed. It is worth to remark that intermediaries with higher funding costs (higher d) should have a higher solvency requirement and the same liquidity requirement.

An increase in return prospects μ_θ calls for a higher liquidity requirement and a lower solvency one. Both constraints are more relaxed when μ_θ increases. However, for $p > q$, θ_p^*/θ_q is decreasing in μ_θ and therefore L is relaxed relatively less than S, implying that \hat{m} increases (indeed, \hat{m} is decreasing in θ_p^*/θ_q) and $\hat{\ell}^{-1}$ decreases with μ_θ (since k is independent of μ_θ). The same is true for an increase in τ_θ whenever $p \leq \gamma < 1/2$ or in the case $\tau_\varepsilon \rightarrow \infty$ (for which $k = 1 - \gamma$). (See Figure 7.d.) In general, a sufficient condition for L to be relaxed less than S (if at all) when τ_θ increases is that k increases with τ_θ (and this happens if τ_ε is small enough and $p > \gamma$). The condition requires that the regulator is more conservative than investors. Then $\partial \hat{m} / \partial \tau_\theta > 0$ and $\partial \hat{\ell}^{-1} / \partial \tau_\theta < 0$. It is also possible that $\partial \hat{m} / \partial \tau_\theta < 0$ and increasing τ_θ leads then to decreases in both regulatory ratios.²⁵

It is worth noting that our policy prescription that with higher return prospects (higher μ_θ) solvency requirements should be relaxed does not contradict the BIS macroprudential recommendation that in good times capital requirements should be tightened. The results of our static model should be understood to hold on average during the cycle.

²⁵ For example, when $p = .2, \gamma = .29, q = .1, \lambda = 2.1$, and $\mu_\theta = d = 1.1$, we have that $\partial \hat{m} / \partial \tau_\theta < 0$ for a wide range of τ_ε and τ_θ .

The results imply that the regulator should set together disclosure and prudential policy. This is so since prudential requirements depend on the level of the precision τ_θ of the public signal. For example, when $p < 1/2$, and either $\tau_\varepsilon \rightarrow \infty$ or τ_ε is small enough and $p > \gamma$ (with very conservative investors), we have that \hat{m} increases and $\hat{\ell}^{-1}$ decreases with τ_θ . Requiring more disclosure has to go together then with an increased liquidity requirement and a decreased solvency one. As we will see in Section 4 introducing a derivatives market may imply that a public signal of high precision becomes available and this should be taken into account by prudential regulation.

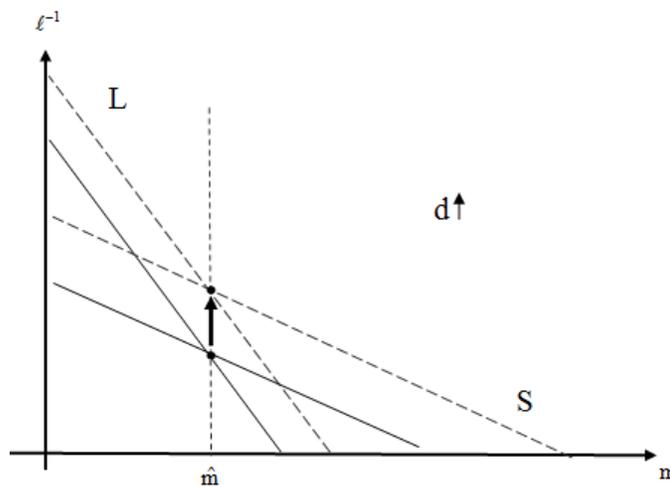


Figure 7.a

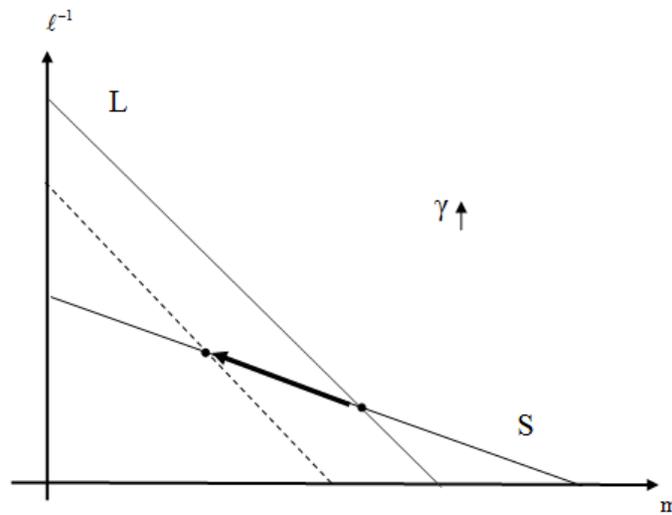


Figure 7.b

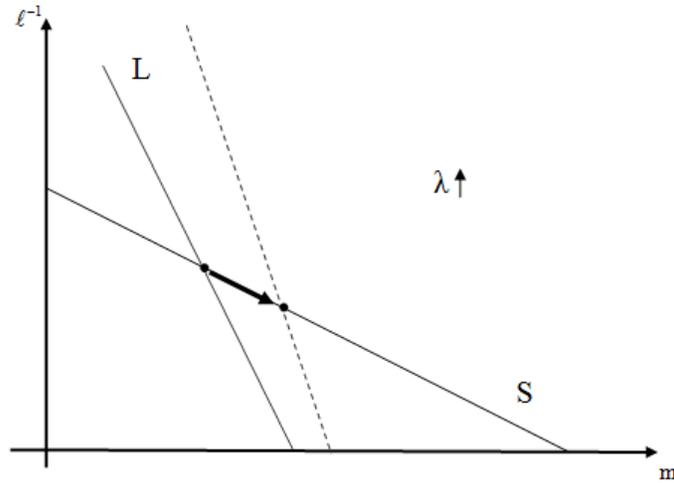


Figure 7.c

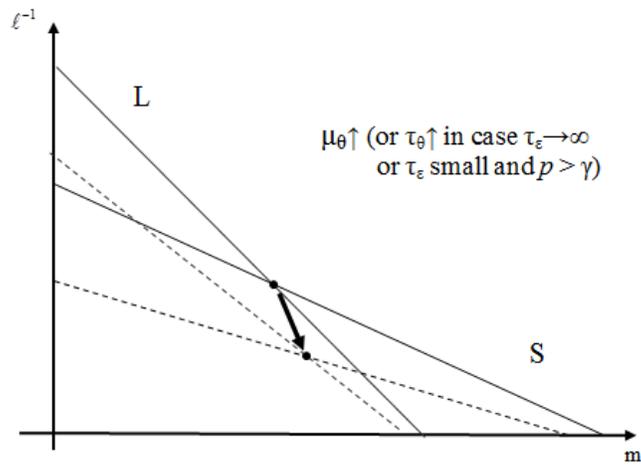


Figure 7.d

Remark 6: It is worth exploring what happens in the extreme case when $1 - \ell^{-1} - d^{-1} > 0$. Banks with intense investment banking or wholesale activity have $1 - \ell^{-1} > 0$, since $\ell = D/E$ is generally above 1, and therefore $1 - \ell^{-1} - d^{-1} > 0$ becomes possible. If leverage is high enough, and $1 - \ell^{-1} - d^{-1} > 0$, then $\partial\theta/\partial m > 0$, $\theta > 1$, and $\partial\theta^*/\partial m > 0$ if γ or λ are small. In this case increasing the liquidity ratio leads to a higher likelihood of insolvency and crisis. Suppose that the regulator requires that $\theta_q > 1$ (assume also that $(\theta_q)^{-1} - d^{-1} > 0$ and

$(1 + k\lambda)(\theta_p^*)^{-1} - d^{-1} > 0$). We have then that the solvency constraint is upward sloping, and the liquidity constraint will be also upward sloping if $\theta_p^* > 1 + \lambda$. If the liquidity constraint is downward sloping then the potential choices of the bank when faced with the constraints are $(m, \ell^{-1}) = (0, (1 + k\lambda)(\theta_p^*)^{-1} - d^{-1})$ and $(m, \ell^{-1}) = (\hat{m}, \hat{\ell}^{-1})$. If both constraints are upward sloping then only the first choice survives. This means that when $1 - \ell^{-1} - d^{-1} > 0$ it may be optimal (to keep the probabilities of insolvency and illiquidity under control) to induce the intermediary to choose to keep no liquid reserves and just impose a leverage limit. In short, for a very highly leveraged institution it is better not to keep liquid reserves.

4. An interpretation of the 2007 run on SIV

A slowdown in house prices and tightening of monetary policy led to increasing doubts about subprime mortgages that were reflected in a sharp decline in 2007 in the asset-based securities index ABX. This index had been launched in January 2006 to track the evolution of residential mortgage-based securities (RMBS). The index is a credit derivative based on an equally weighted index of 20 RMBS tranches, and there are also subindexes of tranches with different rating, for different vintages of mortgages. The ABX index has provided two important functions: information about the aggregate market valuation of subprime risk, and an instrument to cover positions in asset-based securities, for example by shortening the index itself (Gorton (2008, 2009)).²⁶ The decline in the ABX index during 2007 seems to have played a major role in the unfolding of the crisis and the run on SIV and ABCP conduits in particular. Indeed, at the end of 2006 sub-indexes for triple-B securities moved somewhat downward after trading at par, and dropped dramatically in 2007 (see Figure 8).²⁷ Something similar happened to the CMBX, a synthetic index corresponding to the ABX including 25 credit default swaps on commercial mortgages.

²⁶ In fact, trading in the ABX indices (by Paulson & Co. and by Goldman Sachs) has delivered two of the largest payouts in the history of financial markets. See Stanton and Wallace (2011) who argue also that ABX prices are imperfect measures of subprime security values.

²⁷ The index starts trading at par except in the case of the 2007-2 index which opened significantly below par.

ABX BBB- Prices

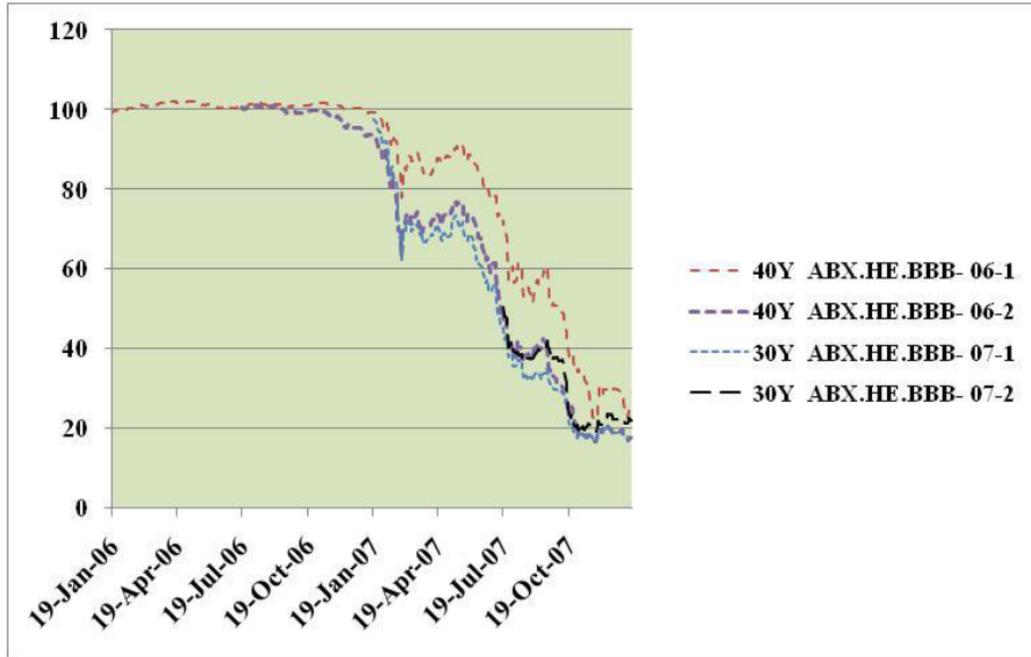


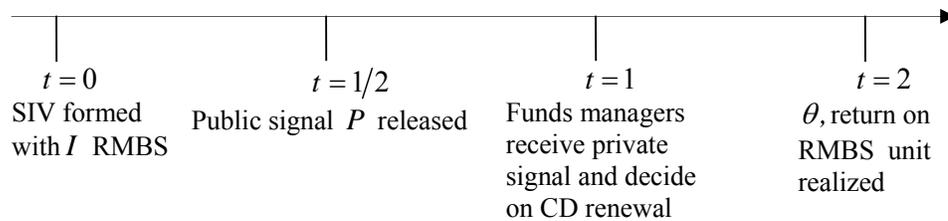
Figure 8: Prices of the 2006-1, 2006-2, 2007-1 and 2007-2 vintages of the ABX index for the BBB- tranche. (Source: Gorton (2008).)

These indexes were highly visible and had a strong influence on markets and the evolution of the indexes went together with a sequence of bad news on subprime mortgages (bankruptcies and earning warnings for originators, downgrading of ratings for RMBS bonds and CDO –collateralized debt obligations, and large losses for hedge funds) from January to August 2007. The accumulated bad news in the ABX indexes culminated in the panic of August 2007 when BNP Paribas froze a fund because of a complete evaporation of liquidity in some segments of the US securitized market. A spike in the overnight spread in ABCP as well as in the Libor-OIS spread followed and the ABCP outstanding plummeted.

The runs began on ABCP conduits and SIV which had some percentage of securities backed by subprime mortgages. These vehicles were funded with short maturity paper and the run amounted to investors not rolling over the paper. While for a typical SIV, ABCP liabilities were 27% of the total at the end of 2007, for a typical conduit

they were at 100%.²⁸ Those vehicles need not have a high proportion of assets directly contaminated by the subprime mortgages but they had a large indirect exposure through a large share of assets issued by the financial sector. As short-term financing dried up, bank sponsors intervened and absorbed many of these vehicles onto their balance sheets.²⁹ It is important to distinguish between the conduits which were motivated by regulatory arbitrage, and which were fully insured by large commercial banks, and those motivated by risk transfer off balance sheet considerations. There is little scope for investor strategic complementarities in the first but substantial in the second. Correspondingly, there was a larger decline in ABCP conduits of the second type relative to those of the first type starting in August 2007.³⁰

Consider the following time line in the basic banking model. At time $t = 0$ mortgage loans are awarded and securitized. At $t = 0$ a SIV is formed and holds I loans and M reserves financed by equity E and short-term debt (CDs) D_0 . At $t = 1/2$ a public signal p about θ is released. At $t = 1$ each fund manager, after receiving a private signal about θ , decides whether to cancel ($y_i = 1$) or renew his CD ($y_i = 0$). At $t = 2$ the returns θI on the RMBS assets are collected, if the bank can meet its obligations, the CDs are repaid at their face value D , and the equityholders of the SIV obtain the residual (if any).



The public signal P may be the value of the ABX index or the price of a derivatives' market with RMBS as underlying asset. Denote by τ its precision. In the simplest scenario neither the SIV nor the fund managers in the short-term debt market

²⁸ See the Global Financial Stability Report of the IMF (April 2008).

²⁹ See Acharya and Schnabl (2010) and Covitz et al. (2012) for evidence on the runs in the ABCP market.

³⁰ See Acharya, Schnabl and Suarez (2011).

participate in the derivatives market. In this case the introduction of the ABX index implies a discrete increase in the public precision together with (public) bad news. This will lead to a higher probability of a crisis, a higher θ^* , both because of the direct effect of bad news ($E(\theta|P)$ low) and of increased public precision (with $E(\theta|P)$ low).

A high level of noise in the signals will push in the same direction also (Proposition 2 (iii)). Recall that a lower τ_ϵ increases strategic complementarity when τ_ϵ is already low (the maximal slope of $r(\cdot)$ tends to infinity as $\tau_\epsilon \rightarrow 0$). Imprecise signals of SIV investors are likely given the opaqueness of the structured subprime products and distance from loan origination.³¹ In this case we may expect $\tau/\sqrt{\tau_\epsilon}$ to be large also. The reason is that the precision of the signal of the fund managers (investors in the SIV) τ_ϵ may be low (think of the German Landesbank investing in structured subprime products) and much lower than the precision of the private signals of the sophisticated traders in the derivatives market (think of investment banks such as Goldman Sachs or hedge funds such as Paulson&Co) which influence positively the public precision τ (together with their risk tolerance).³²

The impact of the bad news is magnified when short-term leverage ℓ , the cost of funds d , and fire sales penalty λ for early asset sales are high (all those factors make h_1^{-1} and strategic complementarity high). This was the situation in the crisis. In fact, the fire-sale penalties increased dramatically with the market becoming practically illiquid. Those SIV then had to be absorbed back by the parent banking institution.

The model of section 3 applies and therefore both solvency and liquidity requirements will be necessary for shadow banks (such as SIVs). In particular, according to Proposition 3 and Table 2, when τ_ϵ is low and investors are conservative (with $\gamma < p$), higher precision of the public signal should lead to an increased liquidity

³¹ See Pagano and Volpin (2012) for a model where issuers of structured bonds choose opaque ratings to enhance the liquidity of their primary market at the cost of diminishing (perhaps drastically) the liquidity of the secondary market.

³² See Angeletos and Werning (2006), Tarashev (2007), and Vives (Section 4.4, 2008) for related models.

requirement. That is, the introduction of a strong public signal like the one provided in a derivatives market should be accompanied with a tightened liquidity obligation.

The presence of a derivative market may have other consequences. First of all, it may allow hedging the risk associated to the subprime products. This can be done by the SIV itself, by the fund managers with exposure to SIV by providing short-term financing, and by other investors with subprime exposure.³³ The SIV by shorting the index may reduce exposure to subprime risk and increase reserves for potential non-renewal of CDs at $t = 1$ at the cost of not profiting from the full appreciation potential of the subprime investment I . If the SIV hedges completely its position then it is completely safe but the expected return is low, a partial hedge will increase the reserves to diminish the failure probability. Interestingly, the private information of the SIV hurts its hedging possibilities creating adverse selection in the derivatives market.³⁴

5. Literature connections, empirical implications, and evidence

Previous literature has obtained results related to the ones presented and a growing body of evidence is consistent with them. Let us deal with both issues in turn.

5.1. Received results

Insolvency risk and illiquidity risk. Morris and Shin (2009) also study how insolvency risk and illiquidity risk vary with the balance sheet composition of a financial institution in a model where future fundamental uncertainty interacts with the strategic uncertainty of the present. In their model there would be no illiquidity risk if there was no future insolvency risk (ex post uncertainty) since it is assumed that partial liquidation of assets has no long run effect. The authors show that illiquidity risk is (i) decreasing in the ratio of cash plus interim realizable assets to short term

³³ Public precision of the price in the derivatives market will decrease in the degree of risk aversion of informed investors in the market and in the sensitivity of the hedger demand to their endowment shock (Vives (Section 4.4, 2008)).

³⁴ When the SIV has market power and precise information in relation to the prior, and the hedgers have a endowment shocks very correlated then the derivatives market dries up (Medrano and Vives (2004)). This is likely to be the case in a crisis situation.

liabilities; (ii) increasing in the "outside option ratio" (opportunity cost of the funds of short run debt holders); and (iii) increasing in the ex post variance of the asset portfolio ("fundamental risk ratio"). The results and the broad message of the paper are consistent with ours: regulation needs to pay attention to the balance sheet composition of a financial intermediary. Liu and Mello (2011) explain how the fragile capital structure of hedge funds, due a coordination problem in redemptions, limits their arbitrage capabilities. They show that this fragility induces hedge funds to invest relatively more in cash assets and in using leverage ex ante. These predictions are borne also in our model when a regulator wants to control the probabilities of insolvency and illiquidity since a decrease in stable funds (E) leads then to more cash holdings (M).

Competitive pressure and fragility. The links between competitive pressure and fragility have been analyzed in several papers. Chang and Velasco (2001) in a model of financial crisis in emerging markets in the Diamond and Dybvig (1983) tradition find that financial liberalization increases the expected welfare of depositors but may increase also fragility. In Matutes and Vives (1996), in a model which combines the banking model of Diamond (1984) with a differentiated duopolistic structure à la Hotelling, an increase in rivalry does increase the probability of failure in an interior equilibrium of the depositor's game where banks have positive market shares. Cordella and Yeyati (1998) find that disclosure of a risk exposure of a bank (which is not controlled by the bank manager) may increase fragility by increasing the deposit rates demanded by investors. Goldstein and Pauzner (2005) also show how increasing the deposit rate increases the probability of a run of depositors in a model of the global games type. In our model increasing the deposit rate increases both the probability of insolvency and of illiquidity.

Negative feedback loops. In the bank crisis model the fire-sales penalty will be related to adverse selection. Vives (2010) shows how the asset fire-sale penalty is increasing in the noise in the signals of the bidders and in the amount auctioned, and decreasing in the number of bidders. In a crisis scenario it is plausible to expect noisier signals, an increased amount auctioned, and fewer bidders. All this will mean that when the bank tries to sell more assets because it is in distress it will face a larger discount, and this, in turn, will induce more sales to face the commitments, and further discounts. In

the extreme the market may collapse because adverse selection is very severe in relation to the number of bidders. Similar phenomena may happen with the face value of debt since when a bank in distress needs refinancing it will be offered worse terms, and this aggravates in turn the fragility of the bank.³⁵

Liquidity regulation. It is worth mentioning also that liquidity requirements may have unintended consequences aggravating adverse selection and drying up markets for liquidity (Malherbe (2012)). The reason is that when intermediaries hoard liquidity their selling behavior is suspicious that they want to get rid of lemon assets. Perotti and Suarez (2011) study how liquidity requirements and Pigouvian taxes can help internalizing the systemic externality induced by the short-term funding of intermediaries. In our model the regulator implicitly minimizes a loss function which includes a cost for illiquidity related to systemic (macro-prudential) reasons.

Large players and market power. In the interbank market example market power may either facilitate liquidity provision (because liquidity is a public good and then sound banks may have an incentive to provide liquidity to a bank in trouble to avoid contagion (Allen and Gale (2004), Sáez and Shi (2004)) or may impede it (as banks with surplus funds underprovide lending strategically to induce fire-sales of bank-specific assets of needy intermediaries (Acharya et al. (2011))).³⁶

Strategic complementarity and the "many to fail" problem. The literature has also found strategic complementarities in the decisions of individual banks that correlate their decisions to force the central bank to bail them out collectively. Farhi and Tirole (2012) show how private leverage choices of financial intermediaries display strategic complementarities through the response of monetary policy. This makes optimal for banks to adopt a risky balance sheet. Acharya and Yorulmazer (2007) show how the regulator has ex post incentives to bail out failed banks when many fail. This provides

³⁵ See Brunnermeier and Pedersen (2009) for a model of a liquidity spiral combining market and funding liquidity, and Bernardo and Welch (2004) for a model where the fear of having to liquidate assets after a crisis may contribute to the frenzy by incentivating sales in the middle of the run. See also Eisenbach (2010) for a feedback model of short-term debt and rollover risk with endogenous fire-sale penalties.

³⁶ See also Corsetti et al. (2004, 2006) for other effects of the presence of large players.

incentives to banks, particularly small ones, to herd in their investment policies and increase the risk of collective failure.³⁷

5.2 Empirical implications and evidence

The model has a rich array of empirical implications. To start with it is a model consistent with crises being driven both by fundamentals and panic components. It predicts also the presence of a multiplier effect of public information, which is increasing in its precision. Table 1 provides testable predictions that link the probability of failure and of insolvency of intermediary with the strength of fundamentals (μ_θ), balance sheet ratios (m and ℓ), and market stress parameters (λ , γ and d). We have that the strength of strategic complementarities among investors, with its associated multiplier effect on comparative statics, increases with ℓ , λ and d .

There is evidence that in banking crises are driven by both solvency and liquidity issues. Gorton (1988) in his study of crises in the US National Banking Era (1865-1914) concluded that panics were triggered when the "fundamentals" (a leading indicator of recession) reached a certain level. Calomiris and Mason (2003) find that some episodes of banking crises in the 1930s in the US can be explained by deteriorating fundamentals while others are open to being interpreted as the panic component dominating (as the crises in January and February of 1933). Starr and Yilmaz (2007) study bank runs in Turkey and conclude that both fundamentals and panic elements coexist in the explanation of the dynamics of the crises. Schotter and Yorulmazer (2009) find in an experimental study that the severity of bank runs depends on the fundamentals (the state of the economy).

Identifying and estimating strategic complementarities is not an easy task since the strategic aspect must be disentangled from responses of investors to common shocks. Chen et al. (2010) identify strategic complementarities in mutual fund investment by

³⁷ Gala and Volpin (2010) show how public information may be welfare damaging by correlating the investment decisions of agents and exacerbating the negative externality that one borrower imposes on others in the presence of credit rationing.

relying precisely on the fact that illiquid funds (those with a higher fire sales penalty λ in our model) generate larger strategic complementarities for investors than liquid ones since redemptions impose larger costs in the former funds. Hertzberg et al. (2011)) use a natural experiment with a credit registry expansion in Argentina to identify complementarities.

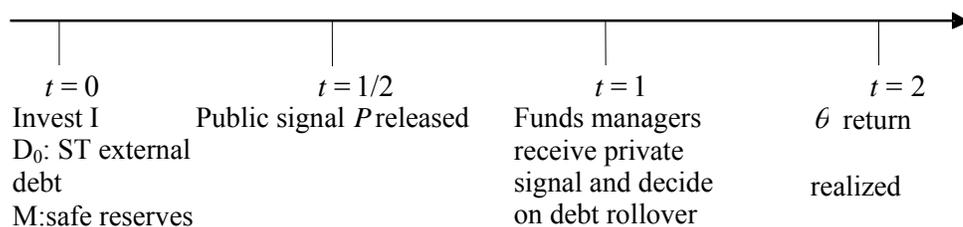
Consistently with result Remark 1, there is experimental evidence that bank runs occur less frequently when banks face less stress (a lower α in our model) in the sense of a larger number of withdrawals being necessary to induce insolvency (Madies (2006) and Garratt and Keister (2009)). There is also evidence of the multiplier effect of public information (Proposition 2 (iii)) in the credit registry expansion in Argentina (Hertzberg et al. (2011)) and with discount window stigma (Armantier et al. (2011), Acharya and Merrouche (2012)). The latter refers to the reluctance of banks to borrow from the discount window because of fear that the bad news will become publicly known. Armantier et al. (2011) find that during the crisis banks borrowed from the Term Auction Facility, in which the borrowing bank is one of many, at higher rates than those available at the discount window, and that this spread was increasing with more stressed conditions in the interbank market. The spread indicates how much a bank is willing to pay to avoid the release of a public bad signal. This is consistent with the result of having a higher publicity multiplier associated to a higher fire sales penalty λ in the interbank market.

In accordance with the results in Corollary 1 among the 72 largest commercial banks in OECD countries, those which relied less on wholesale funding, and had higher capital cushions and liquidity ratios, fared better during the crisis (in the sense of having smaller equity value declines and being subject to less government intervention, see Ratnovski and Huang (2009)). Berger and Bouwman (2012) also find that capital improves the performance of banks during crisis periods. Finally, Fahlenbrach et al. (2011) find that banks with more short term funding performed worse both in the 1998 (as a result of Russia's default on its debt) and in the 2007 crises.

6. Other applications

We deal in this section with three more illustrations of our approach: a sovereign debt crisis, a currency attack, and a credit freezes model.

Sovereign debt crisis. The bank run model of Section 2.2 can be reinterpreted as a sovereign debt run on, say a country belonging to a monetary union. At $t = 0$, now the country has some local resources E and D_0 is external short-term debt. Those resources are invested in entrepreneurial projects I and foreign cash reserves M (to be understood in a strong currency). As before, investors in short-term debt can withdraw in an interim period $t = 1$ and entitled to a face value D .



The public signal now can be interpreted as a credit rating. Now $m \equiv M/D$ is the reserves liquidity ratio, $\ell = D/E$ be the short-term external leverage ratio, and $d = D/D_0$ the risk premium on the short-term debt. The regions of insolvency and illiquidity identified in Section 3.2 apply.

We can derive some prudential policy recommendations out of our regulatory results. Namely, if the objective of the international regulator is to limit the risk of country insolvency and illiquidity, constraints must be put on short-term external leverage and the ratio of reserves. In a country with a higher risk premium the solvency requirement should be higher but if fire sales penalties are higher (say on real state assets), then the liquidity requirement should be higher and the solvency one lower. Obviously, if both are higher it may well be that both requirements should be set higher. Those prudential constraints have to be adjusted in the presence of sovereign credit ratings which should imply strengthened reserves requirements.

Currency attacks. A streamlined version of the currency attacks model of Morris and Shin (1998) where θ represents the reserves of the central bank (with $\theta \leq 0$ meaning that reserves are depleted) fits our general model. Each speculator has one unit of resources to attack the currency ($y_i = 1$) at a cost C . Letting $h(\theta; \alpha) = \alpha^{-1}\theta$, where $\alpha > 0$ is the mass of attackers or α^{-1} the proportion of uncommitted reserves of the central bank, the attack succeeds if $y \geq \alpha^{-1}\theta$. Still α could be interpreted as the wealth available to a fixed mass of speculators. (See Figure 9a.) The capital gain if there is depreciation is fixed and equal to $\hat{B} = B + C$. We have that $\gamma \equiv C/(B + C)$ is likely to be small.

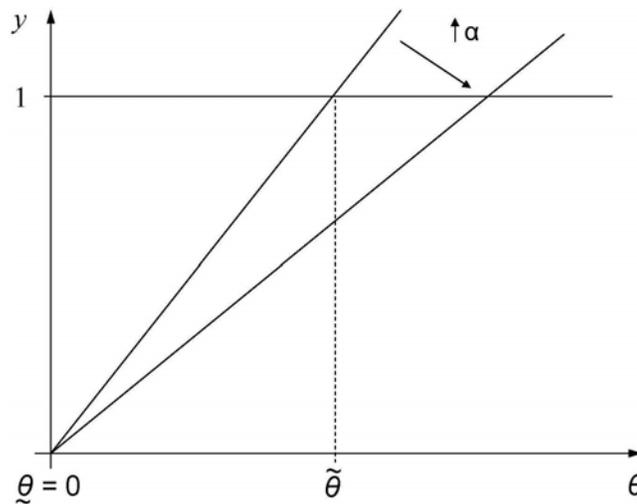


Figure 9a. The function $h(\theta; \alpha) = \alpha^{-1}\theta$ in the currency crises model.

In the range $[\theta^*, \tilde{\theta})$ if currency speculators were to coordinate their attack then they would succeed, but in fact the currency holds. From Proposition 2 (i) we have immediately that the probability of a currency crisis is decreasing in the relative cost of the attack γ and in the expected value of the reserves of the central bank μ_θ ; the probability of illiquidity $Pr(\underline{\theta} \leq \theta < \theta^*)$ increases with α while the probability of insolvency $Pr(\theta < \underline{\theta})$ is unaffected by α .³⁸

³⁸ When $h(\theta; \alpha) = \alpha^{-1}\theta$ we have always that $\underline{\theta} < \theta^*$ and $Pr(\underline{\theta} \leq \theta < \theta^*)$ increases with α ($\underline{\theta} = 0$ and θ^* increases with α).

Credit market freezes. Consider the model of self-fulfilling freezes in credit markets of Bebchuk and Goldstein (2011). This is a variation of the loan foreclosure model by Morris and Shin (2004)).³⁹ In this case θ are the fundamentals of the firms and y the proportion of banks not renewing credit to firms. Firms with good projects, at which banks can invest, have a return above the risk-free rate only if $\theta \geq \underline{\theta}(\alpha) + \alpha^{-1}y$ where $\underline{\theta}(\alpha) = \tilde{\theta} - \alpha^{-1}$ with $\tilde{\theta} > 0$. Firms with bad projects return noting and banks can detect good projects from bad ones. The parameter α is to be interpreted as the inverse of the product of the mass of banks and a complementarity parameter that explains the performance of firms. We have that $h(\theta; \alpha) = \alpha(\theta - \underline{\theta}(\alpha))$ (see Figure 9b.) Note that in this case the increase in fragility, $\partial h / \partial \alpha = -\tilde{\theta} < 0$, goes together with a *decrease* in the degree of strategic complementarity, $\partial h_1 / \partial \alpha > 0$. The parameter $1 - \gamma$ equals the ratio of the gross risk-free return to the gross return of good projects. The authors use the model to assess the responses of the government in the present financial crisis. The comparative static results in Proposition 2 apply.

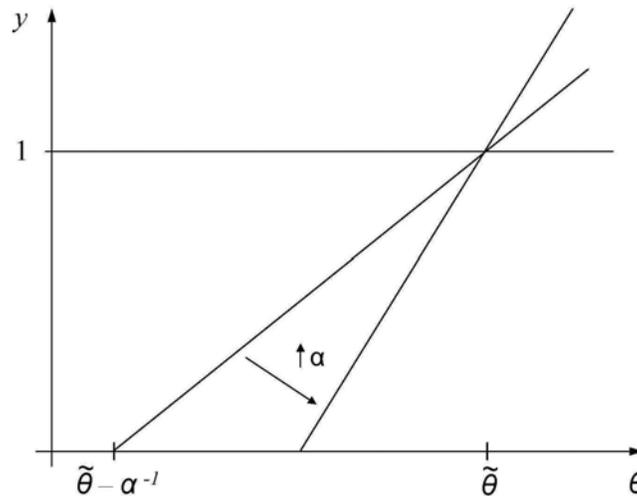


Figure 9b. The function $h(\theta; \alpha) = \alpha(\theta - \underline{\theta}(\alpha))$ in the credit market freezes model.

³⁹ The loan foreclosure model is formally equivalent to the currency attacks model. Now θ is the ability of the firm to meet short-term claims (where $\theta \leq 0$ means no ability). The action is to foreclose a loan. In this case $h(\theta; \alpha) = \alpha^{-1}\theta$ where $\alpha > 0$ is the mass of creditors (or α^{-1} proportion of uncommitted liquid resources of the firm) and the project fails if $y \geq \alpha^{-1}\theta$. The face value of the loan is L , the value of collateral (at interim liquidation) is $K < L$ and let $B = K$ and $C = L - K$. We have that $\gamma \equiv 1 - K/L$.

7. Concluding remarks

This paper presents a stylized model of a financial crisis which characterizes solvency and liquidity risk and highlights how a high degree of strategic complementarity among actions of investors enhances fragility (understood as high equilibrium sensitivity to small changes in parameters and the possibility of discrete jumps in the presence of multiple equilibria). The degree of strategic complementarity increases with short-term leverage, competition for funds, fire sales penalties and with the precision of public information. A high degree of opaqueness on the asset side of a financial intermediary together with a strong public signal (say from a derivatives market) will increase the degree of strategic complementarity and potential fragility. I characterize also how the probability of failure or crisis depend on the balance sheet structure (leverage and liquidity), market stress parameters (degree of competition, the fire sales penalty of early liquidation of investments), and the informativeness of public and private signals.

The main general policy conclusion on regulatory reform is that a piecemeal approach will not work. The regulator will need to pay attention to the composition of the balance sheet of a financial intermediary and to the level of disclosure to control the probabilities of insolvency and illiquidity. In order to do so a leverage limitation and a liquidity requirement are needed, they are partially substitutable, and have to be set together taking into account the level of transparency. Indeed, in an environment with high market illiquidity and conservative investors the liquidity requirement has to be tightened while the solvency one relaxed; prudential constraints may have to be modified with higher disclosure levels (e.g. with stricter liquidity and relaxed solvency requirements). Competition policy and prudential regulation are not independent: In a more competitive situation leverage limits have to be strengthened.⁴⁰

The analysis has several important limitations. First of all, the balance sheet of the intermediary is exogenous and, correspondingly, the objectives of the regulator are also exogenous. Both could be endogenized introducing, for example, a moral hazard

⁴⁰ This theme is developed in Vives (2011).

o commitment problem on the part of the intermediary which would rationalize the short-term debt structure and indicate an optimal closure policy for the regulator.⁴¹ Second, the analysis is basically static while we are trying to capture dynamic phenomena.⁴² Third, the investors are symmetric⁴³; and finally, the analysis focuses on a single institution or a consolidated banking sector and takes market parameters as given (e.g., the fire sales penalty); consequently, it does not take into account externalities among banks and systemic effects.⁴⁴ These issues are left for further research.

⁴¹ See Calomiris and Kahn (1991), Diamond and Rajan (2000), Gale and Vives (2002) and Rochet and Vives (2004) for related approaches to the issue.

⁴² See, for example, the dynamic analysis of panic debt runs in He and Xiong (2012).

⁴³ Sákovics and Steiner (2012) study a global game example where players are ex ante asymmetric.

⁴⁴ See Acharya and Yorulmazer (2007) and Farhi and Tirole (2012).

Appendix

$$D / E = (\text{Deposits (Uninsured)} + \text{Short Term Debt} + \text{Other Liabilities}) / (\text{Equity} + \text{Long Term Debt} + \text{Deposits (Insured)})$$

(Sep. 30, 2008)

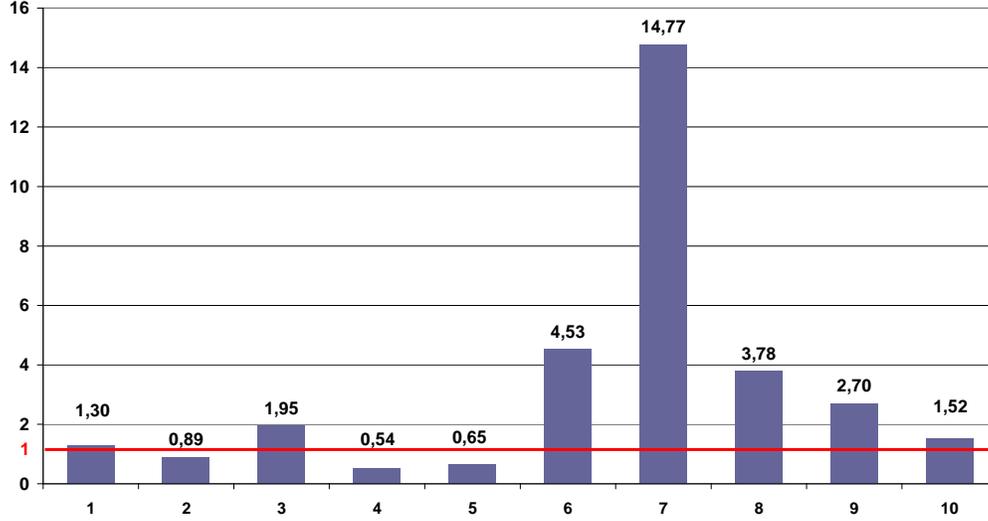


Figure A: Short-term leverage ratio of US banks at September 30, 2008: $\ell = D/E$, D = Deposits (Uninsured) + Short Term Debt + Other Liabilities, E = Equity + Long Term Debt + Deposits (Insured). Source: Veronesi and Zingales (2010), (data for Goldman Sachs and Morgan Stanley as of 08/31/2008).⁴⁵

Claim 1: Characterization of the best reply. The best reply is given by

$$r(\hat{s}) = \frac{\tau_\theta + \tau_\varepsilon}{\tau_\varepsilon} \theta_F(\hat{s}) - \frac{\tau_\theta}{\tau_\varepsilon} \mu_\theta - \frac{\sqrt{\tau_\theta + \tau_\varepsilon}}{\tau_\varepsilon} \Phi^{-1}(\gamma).$$

The maximal value of the slope is

$$\bar{r}' \equiv \frac{\tau_\theta + \tau_\varepsilon}{\tau_\varepsilon + h_1 \sqrt{2\pi\tau_\varepsilon}},$$

which is increasing in h_1^{-1} and in τ_θ , and first decreasing and then

increasing in τ_ε ($\text{sign}\{\partial \bar{r}' / \partial \tau_\varepsilon\} = \text{sign}\{(1/2)h_1 \sqrt{2\pi\tau_\varepsilon} (1 - \tau_\theta \tau_\varepsilon^{-1}) - \tau_\theta\}$). If

$$\left(\tau_\theta / \sqrt{\tau_\varepsilon}\right) \leq h_1 \sqrt{2\pi} \text{ then } r'(\hat{s}) = \frac{\tau_\theta + \tau_\varepsilon}{\tau_\varepsilon} \theta_F'(\hat{s}) \leq 1.$$

⁴⁵ Only State Street Corp. would go above $\ell = 10$ with $\ell = 14.77$ (which would yield $1 - \ell^{-1} - d^{-1} = 0$ with an interest rate of 7.26%). Lehman Brothers had $\ell = 3.76$ at the end of 2007 (derived from Adrian and Shin (2010)).

Proof: From the equation $\Phi(\sqrt{\tau_\varepsilon}(\hat{s} - \theta)) = h(\theta)$ we can solve for \hat{s} as a function of θ and obtain $s_F(\theta) = \theta + (1/\sqrt{\tau_\varepsilon})\Phi^{-1}(h(\theta))$ with derivative $s'_F = 1 + (1/\sqrt{\tau_\varepsilon})h'(\phi(\Phi^{-1}(h(\theta))))^{-1}$, where ϕ is the density of the standard normal. Since ϕ is bounded above by $1/\sqrt{2\pi}$ and h_1 is the smallest slope of $h(\cdot)$, it follows that s'_F is bounded below: $s'_F \geq 1 + \sqrt{2\pi/\tau_\varepsilon}h_1$. Hence, $\theta'_F(\hat{s}) \leq (1 + \sqrt{2\pi/\tau_\varepsilon}h_1)^{-1}$, with strict inequality, except possibly when $h(\theta) = 1/2$ because then $\Phi^{-1}(1/2) = 0$ and ϕ attains its maximum: $\phi(0) = 1/\sqrt{2\pi}$. It follows that the maximal value of the slope is

$$\bar{r}' \equiv \frac{\tau_\theta + \tau_\varepsilon}{\tau_\varepsilon + h_1\sqrt{2\pi\tau_\varepsilon}},$$

which is increasing in h_1^{-1} and in τ_θ , and first decreasing and then increasing in τ_ε . Note that $\bar{r}' \leq 1$ if and only if $(\tau_\theta/\sqrt{\tau_\varepsilon}) \leq h_1\sqrt{2\pi}$. We have that $\bar{r}' \rightarrow \infty$ as $\tau_\varepsilon \rightarrow 0$ and $\bar{r}' \uparrow 1$ as $\tau_\varepsilon \rightarrow \infty$. It is easily checked that $\text{sign}\{\partial\bar{r}'/\partial\tau_\varepsilon\} = \text{sign}\{(1/2)h_1\sqrt{2\pi\tau_\varepsilon}(1 - \tau_\theta\tau_\varepsilon^{-1}) - \tau_\theta\}$, in particular $\partial\bar{r}'/\partial\tau_\varepsilon < 0$ for $\tau_\varepsilon < \tau_\theta$. ■

Proof of Proposition 1: The game is “monotone supermodular” (i.e., of strategic complementarities with a monotone information structure) since $\pi(y_i, y; \theta)$ has increasing differences in $(y_i, (y, -\theta))$, that is, the differential payoff to act $\pi^1 - \pi^0$ is increasing in the aggregate action and the negative of the state of the world $(y, -\theta)$, and signals are affiliated. This means that extremal equilibria exist, are symmetric (because the game is symmetric), and are in monotone (decreasing) strategies in type (Van Zandt and Vives (2007)). Since there are only two possible actions, the strategies must then be of the threshold form: $y_i = 1$ if and only if $s_i < \hat{s}$ where \hat{s} is the threshold. It follows also that the extremal equilibrium thresholds, denoted \bar{s} and \underline{s} , bound the set of strategies which are the outcome of iterated elimination of strictly dominated strategies. If $\bar{s} = \underline{s}$ the game is dominance solvable and the equilibrium is unique. An equilibrium will be characterized by two thresholds (s^*, θ^*) with s^* yielding the signal threshold to act and θ^* the state-of-the-world

critical threshold, below which the acting mass is successful and an acting player obtains the payoff $B - C > 0$. In equilibrium the fraction of acting players $y(\theta^*, s^*) = Pr(s \leq s^* / \theta^*) \equiv \Phi(\sqrt{\tau_\varepsilon}(s^* - \theta^*))$ must be no larger than the critical fraction above which it pays to act $h(\theta^*)$. Note that $\theta^* \in [\underline{\theta}, \tilde{\theta}]$ since $\lim_{\theta \uparrow \underline{\theta}} h(\theta) = 0$ and $h(\tilde{\theta}) = 1$. Furthermore, at the critical signal threshold the expected payoff of acting and not acting should be the same:

$$\begin{aligned} & E[\pi(1, y(\theta, s); \theta) - \pi(0, y(\theta, s); \theta) / s = s^*] \\ &= Pr(\theta \leq \theta^* / s^*)B + Pr(\theta > \theta^* / s^*)(-C) = 0; \end{aligned}$$

or
$$Pr(\theta \leq \theta^* / s^*) \equiv \Phi\left(\sqrt{\tau_\theta + \tau_\varepsilon}\left(\theta^* - \frac{\tau_\theta \mu_\theta + \tau_\varepsilon s^*}{\tau_\theta + \tau_\varepsilon}\right)\right) = \gamma, \quad \text{where}$$

$$\gamma \equiv C / (B + C) < 1.$$

If $\Phi(\sqrt{\tau_\varepsilon}(s^* - \underline{\theta})) \leq h(\underline{\theta})$ then $\theta^* = \underline{\theta}$. When $s^* \leq \underline{\theta} + \Phi^{-1}(h(\underline{\theta})) / \sqrt{\tau_\varepsilon}$ and by

replacing $s^* = \underline{\theta} + \Phi^{-1}(h(\underline{\theta})) / \sqrt{\tau_\varepsilon}$ in $\Phi\left(\sqrt{\tau_\theta + \tau_\varepsilon}\left(\theta^* - \frac{\tau_\theta \mu_\theta + \tau_\varepsilon s^*}{\tau_\theta + \tau_\varepsilon}\right)\right) = \gamma$, it

follows that for $h(\underline{\theta}) \geq \bar{h}_0$, where $\bar{h}_0 = \Phi\left(\frac{\tau_\theta}{\sqrt{\tau_\varepsilon}}(\underline{\theta} - \mu_\theta) - \sqrt{1 + \frac{\tau_\theta}{\tau_\varepsilon}}\Phi^{-1}(\gamma)\right) > 0$, we

have that at the equilibrium $\theta^* = \underline{\theta}$. (Indeed, when $h(\underline{\theta}) \geq \bar{h}_0$ then $s_F(\underline{\theta}) \geq s_T(\underline{\theta})$.)

For $h(\underline{\theta}) < \bar{h}_0$ we have that $\theta^* > \underline{\theta}$ at any equilibrium. Indeed, then

$\Phi(\sqrt{\tau_\varepsilon}(s^* - \theta^*)) = h(\theta^*)$ and $\theta^* > \underline{\theta}$. (In this case $s_F(\underline{\theta}) < s_T(\underline{\theta})$.) In summary,

there is a critical $\bar{h}_0 \in (0, 1)$ such that for $h(\underline{\theta}) < \bar{h}_0$ we have that $\theta^* > \underline{\theta}$ and for

$h(\underline{\theta}) \geq \bar{h}_0$ there is always one equilibrium with $\theta^* = \underline{\theta}$.

Let $h(\underline{\theta}) < \bar{h}_0$, then equations (1) $\Phi(\sqrt{\tau_\varepsilon}(s^* - \theta^*)) = h(\theta^*)$ and (2)

$\Phi\left(\sqrt{\tau_\theta + \tau_\varepsilon}\left(\theta^* - \frac{\tau_\theta \mu_\theta + \tau_\varepsilon s^*}{\tau_\theta + \tau_\varepsilon}\right)\right) = \gamma$ combine into equation

$$\varphi(\theta^*) \equiv \tau_\theta(\theta^* - \mu_\theta) - \sqrt{\tau_\varepsilon}\Phi^{-1}(h(\theta^*)) - \sqrt{\tau_\theta + \tau_\varepsilon}\Phi^{-1}(\gamma) = 0,$$

by substituting the value of s^* from (1) into (2). This equation may have multiple

solutions in θ^* . As $\theta \rightarrow \tilde{\theta}$ we have that $\Phi^{-1}(h(\theta))$ tends to $+\infty$ and $\varphi \rightarrow -\infty$; as

$\theta \rightarrow \underline{\theta}$ we have that $h(\theta) \rightarrow h(\underline{\theta})$ and $\varphi \rightarrow \varphi(\underline{\theta}) > 0$ whenever $h(\underline{\theta}) < \bar{h}_0$. There

is at least a solution $\theta^* \in [\underline{\theta}, \tilde{\theta}]$. The solution will be unique if $\varphi' < 0$; there will be multiple solutions if $\varphi'(\theta) > 0$ for a potential solution $\varphi(\theta) = 0$ (three in fact when $h(\cdot)$ is linear, and two when there is a θ such that $\varphi(\theta) = 0$ and $\varphi'(\theta) = 0$). As γ tends to 0 (1) we have that $\Phi^{-1}(\gamma)$ tends to $-\infty$ ($+\infty$), s^* tends to (minus) infinity and θ^* tends to $\tilde{\theta}$ ($\underline{\theta}$). This follows from equation (2) and the fact that $\theta^* \in [\underline{\theta}, \tilde{\theta}]$. There is a unique solution if $\tau_\theta/\sqrt{\tau_\varepsilon} \leq h_1\sqrt{2\pi}$. Indeed, $\varphi' = 1 - (\sqrt{\tau_\varepsilon}/\tau_\theta)h'[\phi(\Phi^{-1}(h(\theta)))]^{-1}$, where ϕ is the density of the standard normal. Since ϕ is bounded above by $1/\sqrt{2\pi}$ and h_1 is the smallest slope of $h(\cdot)$, it follows that φ' is bounded above: $\varphi' \leq 1 - h_1\sqrt{2\pi\tau_\varepsilon}/\tau_\theta$ (with strict inequality, except possibly when $h(\theta) = 1/2$ because then $\Phi^{-1}(1/2) = 0$ and ϕ attains its maximum: $\phi(0) = 1/\sqrt{2\pi}$). Therefore, if $\tau_\theta/\sqrt{\tau_\varepsilon} \leq h_1\sqrt{2\pi}$ then $\varphi' \leq 0$. In this case the equilibrium is unique and the game is dominance solvable because then $\bar{s} = \underline{s}$. Furthermore, it should be clear that the critical thresholds θ^* and s^* move together.

Suppose $h(\cdot)$ is linear, then if $\tau_\theta/\sqrt{\tau_\varepsilon} > h_1\sqrt{2\pi}$ there is a range of γ for which there are multiple equilibria. Indeed, choose γ such that $h(\theta^*) = 1/2$, then $\varphi'(\theta^*) > 0$ and there must be three equilibria, and by continuity there is a neighborhood of such γ with multiple equilibria. Note that for γ small we will have a unique equilibrium (high) and for γ high a unique equilibrium (low) since as γ tends to 0 (1) we have that $\Phi^{-1}(\gamma)$ tends to $-\infty$ ($+\infty$) and φ to $+\infty$ ($-\infty$) for any given $\theta^* \in (\underline{\theta}, \tilde{\theta})$. Given the shape of φ (decreasing-increasing-decreasing) for γ small we obtain an equilibrium in the third decreasing portion and for γ high an equilibrium in the first decreasing portion. ■

Proof of Proposition 2: When $h(\underline{\theta}) < \bar{h}_0$, the equation

$$\varphi(\theta^*) \equiv \tau_\theta(\theta^* - \mu_\theta) - \sqrt{\tau_\varepsilon}\Phi^{-1}(h(\theta^*)) - \sqrt{\tau_\theta + \tau_\varepsilon}\Phi^{-1}(\gamma) = 0$$

determines θ^* . We obtain the results by looking at how parameter changes impinge on $\varphi(\cdot)$. When $\tau_\theta/\sqrt{\tau_\varepsilon} \leq h_1\sqrt{2\pi}$, we have that $\varphi' < 0$ and there is a unique

equilibrium. The usual comparative static analysis applies. When $\tau_\theta/\sqrt{\tau_\epsilon} > h_1\sqrt{2\pi}$ there may be multiple equilibria and the results will apply to the extremal ones. With adaptive dynamics the results apply in general.

(i) The result for θ^* follows since φ is decreasing in γ and μ_θ , and increasing in α since $\partial h/\partial\alpha < 0$. The threshold s^* moves with θ^* .⁴⁶ The result for $Pr(\theta < \theta^*)$ is immediate for γ and α , and for μ_θ also since increases in μ_θ move the distribution of θ to the right. (ii) Suppose the equilibrium is unique. The equilibrium signal threshold is determined by $r(s^*; \mu_\theta) - s^* = 0$. From which it follows that for a marginal change in μ_θ

$$\left| \frac{ds^*}{d\mu_\theta} \right| = \frac{|\partial r/\partial\mu_\theta|}{1-r'} > \left| \frac{\partial r}{\partial\mu_\theta} \right|$$

whenever $r' < 1$ is met and $r' > 0$. In consequence, an increase in μ_θ will have a larger effect on the equilibrium threshold s^* than the direct impact on the best response of a player $\partial r/\partial\mu_\theta = -\tau_\theta/\tau_\epsilon$. The same is true for discrete changes even with multiple equilibria if we restrict attention to extremal equilibria or in general with adaptive dynamics. This multiplier effect is largest when r' is close to 1 at equilibrium, that is, when strategic complementarities are strong, and we approach the region of multiplicity of equilibria. This is so when τ_θ is large since r' is increasing in τ_θ . (iii) The first part follows since $\partial\varphi/\partial\tau_\theta = \theta^* - \mu_\theta - (\tau_\theta + \tau_\epsilon)^{-1/2} \Phi^{-1}(\gamma)/2$ and therefore $\partial\varphi/\partial\tau_\theta > 0$ since $\theta^* > \underline{\theta} \geq \mu_\theta$ and $\Phi^{-1}(\gamma) < 0$ for $\gamma < 1/2$. Note that if $\theta^* > \mu_\theta$ when τ_θ increases $Pr(\theta < \theta^*) = \Phi(\sqrt{\tau_\theta}(\theta^* - \mu_\theta))$ also increases. For the second part, using the equation $\varphi(\theta^*) = 0$ in $\partial\varphi/\partial\tau_\epsilon$ we obtain $\partial\varphi/\partial\tau_\epsilon = -(\theta^* - \mu_\theta - (\tau_\theta + \tau_\epsilon)^{-1/2} \Phi^{-1}(\gamma))\tau_\theta/2\tau_\epsilon$ and $\partial\varphi/\partial\tau_\epsilon < 0$ under the assumptions. If $\mu_\theta \geq \tilde{\theta} - (\tau_\theta + \tau_\epsilon)^{-1/2} \Phi^{-1}(\gamma)$ we have $\theta^* < \mu_\theta$ and the results are

⁴⁶ The result for μ_θ follows also from a general argument in monotone supermodular games. We know that extremal equilibria of monotone supermodular games are increasing in the posteriors of the players (Van Zandt and Vives (2007)). A sufficient statistic for the posterior of a player under normality is the conditional expectation $E[\theta | s] = (\tau_\theta\mu_\theta + \tau_\epsilon s)/(\tau_\theta + \tau_\epsilon)$, which is increasing in μ_θ . It follows then that extremal equilibrium thresholds $(-\theta^*, -s^*)$ increase with μ_θ .

reversed since $\theta^* < \tilde{\theta} \leq \mu_\theta$ (note that since $\Phi^{-1}(\gamma) < 0$ we have that $-\Phi^{-1}(\gamma) > -\Phi^{-1}(\gamma)/2$). ■

Claim 2. Comparative static results in the bank run model.

Proof: From the proof of Proposition 1 it follows that the critical ratio \bar{m} fulfils

$$m = \Phi\left(\frac{\tau_\theta}{\sqrt{\tau_\varepsilon}}(\underline{\theta} - \mu_\theta) - \sqrt{1 + \frac{\tau_\theta}{\tau_\varepsilon}}\Phi^{-1}(\gamma)\right) \text{ where } \underline{\theta} \equiv (1-m)/(\ell^{-1} + d^{-1} - m). \text{ Note}$$

that the right hand side is decreasing in m if $\partial\underline{\theta}/\partial m < 0$, which obtains according to our maintained assumption $1 - \ell^{-1} - d^{-1} < 0$. We have also that $\partial\underline{\theta}/\partial d > 0$, $\partial\underline{\theta}/\partial \ell > 0$ and $\underline{\theta}$ is independent of λ and γ . We know also that $\partial h/\partial m > 0$ (under the maintained assumption), $\partial h/\partial \ell^{-1} > 0$ and $\partial h/\partial d^{-1} > 0$. The other results follow from Proposition 2/Remark 1 or by direct inspection of the equilibrium condition $\varphi(\theta) = 0$. ■

Proof of Proposition 3:

(i) We have that $Pr(\theta < \underline{\theta}) = \Phi(\sqrt{\tau_\theta}(\underline{\theta} - \mu_\theta))$ and therefore the solvency constraint (S) $Pr(\theta < \underline{\theta}) \leq q$ holds if and only if $\underline{\theta} \leq \underline{\theta}_q \equiv \mu_\theta + \Phi^{-1}(q)/\sqrt{\tau_\theta}$ where $\underline{\theta} \equiv (1-m)/(\ell^{-1} + d^{-1} - m)$. The constraint S follows immediately from $\underline{\theta} \leq \underline{\theta}_q \equiv \mu_\theta + \Phi^{-1}(q)/\sqrt{\tau_\theta}$ yielding $\ell^{-1} \geq (\underline{\theta}_q)^{-1} - d^{-1} - ((\underline{\theta}_q)^{-1} - 1)m$. Note that S is independent of γ and λ . Furthermore, $Pr(\theta < \theta^*) = \Phi(\sqrt{\tau_\theta}(\theta^* - \mu_\theta))$ therefore $Pr(\theta < \theta^*) \leq p$ if and only if $\theta^* \leq \theta_p^* \equiv \mu_\theta + \Phi^{-1}(p)/\sqrt{\tau_\theta}$. From Corollary 1 we know that the probability of insolvency is decreasing in m , ℓ^{-1} and d^{-1} . The cutoff θ^* and the probability of a crisis are decreasing in m , ℓ^{-1} (and also in γ , d^{-1} and λ^{-1}).

The constraint L follows from $\theta^* \leq \theta_p^* \equiv \mu_\theta + \Phi^{-1}(p)/\sqrt{\tau_\theta}$ noting that at the boundary $h(\theta^*) = h(\theta_p^*) = m + \frac{\ell^{-1} + d^{-1} - m}{\lambda}\theta_p^* - \frac{1-m}{\lambda}$ and we trade off ℓ^{-1} and m in a linear way to keep $h(\theta_p^*)$ constant at level k . It follows that L is linear

$\ell^{-1} \geq \frac{1+k\lambda}{\theta_p^*} - d^{-1} - \left(\frac{1+\lambda}{\theta_p^*} - 1 \right) m$ where, from $\varphi(\theta_p^*) = 0$, we obtain

$$k = h(\theta_p^*) = \Phi \left(\frac{\tau_\theta}{\sqrt{\tau_\varepsilon}} (\theta_p^* - \mu_\theta) - \sqrt{1 + \frac{\tau_\theta}{\tau_\varepsilon}} \Phi^{-1}(\gamma) \right) = \Phi \left(\sqrt{\frac{\tau_\theta}{\tau_\varepsilon}} \Phi^{-1}(p) - \sqrt{1 + \frac{\tau_\theta}{\tau_\varepsilon}} \Phi^{-1}(\gamma) \right)$$

since $\theta_p^* - \mu_\theta = \Phi^{-1}(p) / \sqrt{\tau_\theta}$. We have that $k \in (0,1)$ is increasing in p , decreasing in γ , and with ambiguous dependence on $\tau_\theta/\tau_\varepsilon$. When $\tau_\varepsilon \rightarrow \infty$, $k = 1 - \gamma$. When $p > \gamma$ and τ_ε is small, k is increasing in $\tau_\theta/\tau_\varepsilon$. The latter follows since

$$\text{sgn} \{ \partial k / \partial \tau_\theta \} = \text{sgn} \left\{ \left(\frac{\tau_\theta}{\tau_\varepsilon} \right)^{-1/2} \Phi^{-1}(p) - \left(1 + \frac{\tau_\theta}{\tau_\varepsilon} \right)^{-1/2} \Phi^{-1}(\gamma) \right\}$$
 is positive when

$$1/2 > p > \gamma \text{ since then } \Phi^{-1}(\gamma) < \Phi^{-1}(p) < 0 \text{ and } \left(\frac{\tau_\theta}{\tau_\varepsilon} \right)^{-1/2} / \left(1 + \frac{\tau_\theta}{\tau_\varepsilon} \right)^{-1/2} = \sqrt{1 + \frac{\tau_\varepsilon}{\tau_\theta}}.$$

When $p \leq \gamma$ then $\partial k / \partial \tau_\theta < 0$.

Note that when $\underline{\theta}_q < 1$ both constraints S and L are downward sloping since then we have $(\underline{\theta}_q)^{-1} - d^{-1} > (\underline{\theta}_q)^{-1} - 1 > 0$ and $(1+\lambda)(\theta_p^*)^{-1} > (\underline{\theta}_q)^{-1} > 1$.

(ii) The constraints S and L become tighter when d increases. L becomes tighter when λ increases (since $k - m > 0$ for interior solutions for m) or γ decreases (since k is decreasing in γ). Both constraints are more relaxed with higher μ_θ (for S since $\underline{\theta}_q$ is increasing in μ_θ and $m < 1$, and for L since $\partial k / \partial \mu_\theta = 0$ and θ_p^* is increasing in μ_θ , $m < 1$ and $k - m > 0$). Increases in τ_θ increase $\underline{\theta}_q$ and θ_p^* provided that $1/2 > p$ since then $\Phi^{-1}(q) < \Phi^{-1}(p) < 0$. This implies that increases in τ_θ relax S but have an ambiguous impact on L since k may increase or decrease with τ_θ . A sufficient condition for k to be increasing in τ_θ is that $p \leq \gamma < 1/2$. For $p > q$, $\theta_p^* / \underline{\theta}_q$ is decreasing in μ_θ and therefore L is relaxed relatively less than S.

(iii) The minimal $(\hat{m}, \hat{\ell}^{-1})$ ratios will be given by the intersection of the boundaries of the solvency

$$\ell^{-1} = (\underline{\theta}_q)^{-1} - d^{-1} - \left((\underline{\theta}_q)^{-1} - 1 \right) m$$

and the liquidity

$$\ell^{-1} = \frac{1+k\lambda}{\theta_p^*} - d^{-1} - \left(\frac{1+\lambda}{\theta_p^*} - 1 \right) m$$

constraints (note that $\frac{1+k\lambda}{\theta_p^*} - (\theta_q)^{-1} > 0$ if $\lambda k > \left(\frac{\theta_p^*}{\theta_q} - 1 \right)$). We obtain for positive solutions

$$\hat{m} = 1 - \frac{1-k}{1 - \lambda^{-1} \left(\frac{\theta_p^*}{\theta_q} - 1 \right)} \text{ and}$$

$$\hat{\ell}^{-1} = (\theta_q)^{-1} - d^{-1} - \left((\theta_q)^{-1} - 1 \right) \hat{m}.$$

It follows that

$$\hat{m} = \max \left\{ 1 - (1-k) \left(1 - \lambda^{-1} \left(\frac{\theta_p^*}{\theta_q} - 1 \right) \right)^{-1}, 0 \right\} \text{ and if } p = q \text{ then } \hat{m} = k. \text{ Note also}$$

that for $p > q$ if $\lambda k > \left(\frac{\theta_p^*}{\theta_q} - 1 \right)$ then $k > \hat{m} > 0$. The comparative statics of the

regulatory ratios given in Table 2 follow: $\partial \hat{m} / \partial \lambda > 0$, $\partial \hat{m} / \partial \gamma < 0$, $\partial \hat{m} / \partial d = 0$,

$\partial \hat{\ell}^{-1} / \partial \lambda < 0$, $\partial \hat{\ell}^{-1} / \partial \gamma > 0$, and $\partial \hat{\ell}^{-1} / \partial d > 0$. We have also that $\partial \hat{m} / \partial \mu_\theta > 0$ since

k is independent of μ_θ , $\partial(\theta_p^* / \theta_q) / \partial \mu_\theta < 0$ (for $p > q$), and \hat{m} is decreasing in

θ_p^* / θ_q . Correspondingly, and since $1 - \hat{m} > 0$, we obtain that $\partial \hat{\ell}^{-1} / \partial \mu_\theta < 0$ from

$\partial(\theta_q)^{-1} / \partial \mu_\theta < 0$ and $\partial \hat{m} / \partial \mu_\theta > 0$. The same results hold for an increase in τ_θ in the

case $\tau_\varepsilon \rightarrow \infty$ (for which $k = 1 - \gamma$) since $\partial(\theta_p^* / \theta_q) / \partial \tau_\theta < 0$ and $\partial(\theta_q)^{-1} / \partial \tau_\theta < 0$

when $q < 1/2$. The same applies also if $p > \gamma$ and τ_ε is small. Then k increases with

τ_θ and $\partial \hat{m} / \partial \tau_\theta > 0$. It follows then also that $\partial \hat{\ell}^{-1} / \partial \tau_\theta < 0$ since

$\partial \hat{\ell}^{-1} / \partial \tau_\theta = - \left((\theta_q)^{-1} - 1 \right) \partial \hat{m} / \partial \tau_\theta + (1 - \hat{m}) \partial(\theta_q)^{-1} / \partial \tau_\theta < 0$ as $\partial(\theta_q)^{-1} / \partial \tau_\theta < 0$ for

$1/2 > p$. ■

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