In Google we trust

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Abstract

This paper investigates the incentives of a monopoly search engine that offers organic and sponsored search (or position auctions). We identify novel incentives to distort each type of search. First, the engine may distort organic searches towards those websites least effective for display advertising, since display advertising alternatives lower merchants’ willingness to bid in sponsored search auctions. Second, its optimal sponsored search auctions may cash in on merchants’ willingness to pay by underweighting their consumer relevance. We analyze the interplay of these effects by explicitly modeling the markets for both sponsored and display advertising. We also analyze how vertical integration by a monopolistic engine into ad-intermediation affects biases and welfare. When third-party websites are symmetric, the search engine may monopolize the entire ad intermediation market and fully internalize the vertical externality, raising consumer surplus and the quality of organic search. Partial monopolization of the ad-intermediation market, however, motivates the engine to divert traffic towards websites that use its ad intermediation services (“affiliate publishers”). This distortion may outweigh the internalization effect and result in lower consumer and producer surplus.

Keywords: Search engine bias, internet economics, vertical integration, two-sided markets, antitrust. JEL Classifications: L13, L41, L44, L86.

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1 Introduction

Accounting for over 90% of the market share in online search for most European countries and over 60% for the Americas, Google unquestionably dominates the global market for online search (State-of-search, 2012). This could reflect widespread trust in Google’s motives (“do no evil”) and ability (developed through early innovations like PageRank and ongoing investments in vast server farms) to deliver reliable search results, but some (not least, Microsoft) argue against trusting in Google’s promise to deliver unbiased results. While they point to evidence of specific organic search biases that raise the ranking of Google’s own (non-search) services and content (see e.g., Edelman and Lockwood, 2011), proving more general bias is very difficult (especially search bias among websites not owned by Google). So we need to better understand Google’s incentives to distort, both for sponsored and organic search. In addition, a rigorous model of the underlying mechanisms and their welfare implications may serve to guide the ongoing regulatory investigations of Google’s search algorithms and vertical integrations (such as YouTube, DoubleClick and Zagat).

This paper develops a micro-founded economic model involving consumers, merchants (who sell offline products), publishers (who offer online content or services), advertising intermediaries (who mediate between merchants and publishers) and a monopoly search engine, called Google[1]. We model both organic search (where rankings are not paid for) and sponsored search (where an auction determines the position of links) and characterize Google’s incentives to distort each type of search result. The quality of search has implications for welfare through

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[1] Of course, alternative search engines do exist, but the small cost of locating and clicking onto a competing search engine may not ensure effective competition. Argenton and Prufer (2012) demonstrate how “indirect network externalities” (searchers indirectly learn from each other’s past experiences) can lead to market tipping and subsequent monopolization. In addition, searchers may have difficulty observing the relative quality of search results and they may face switching costs (e.g., in thinking which search engine to use for each small search query). As a result, many searchers develop the habit of using one search engine and rarely consider whether to try an alternative mode of search.
four channels: (a) matching consumers with products they may wish to buy and (b) matching consumers with content they may wish to consume online, thereby (c) determining the effectiveness of display advertising and (d) influencing surplus appropriation which can affect investment incentives of the five different groups of actors. We derive two novel types of bias - one affecting organic search and one affecting sponsored search - that operate independently but also interact. We then investigate the impact of integration by Google into advertising intermediation (between merchants and publishers), as well as integration of Google into publishing. We refer to both forms of integration as vertical integration.

In our model consumers conduct two types of searches. A “product search” for (offline) goods and a “content search” for online goods. Most people conduct both types of search through the same search engine. So we model consumers’ participation in Google as a single decision that depends on the reliability of sponsored and organic results (implying a joint constraint for Google). The key feature distinguishing content from products is that content is readily tied with advertising, whereas products are not. In other words, the publishers who offer content on their websites can readily expose their consumers (web visitors) to advertisements from third-party merchants. In addition, publishers typically offer their content free of charge and make money from advertising alone (owing to paywall problems and/or asymmetries in the two-sided market), while merchants naturally charge positive prices for their products.

In principle, consumers could reach publishers via sponsored or organic search results and the same is true for merchants, but in our simple model, we derive the following simplifying split: consumers conduct their content searches by looking at organic results alone and conduct their product searches only through sponsored results. Greenspan (2004) and Jansen (2007) offer empirical support for the prod-

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2 The integration is vertical in the sense that Google’s search engine, publishers and ad intermediaries are three steps in a chain that can produce valuable matches between consumers and merchants (those that end up in productive trades). To spell this out, Google’s (organic) search engine sends visitors to publishers who then expose those visitors to advertisements served by ad intermediaries and paid for by the merchants who make the ads.
uct search side of this approximate split: they find that people use sponsored links more often than organic results when conducting product searches ("e-commerce search queries"). The explanation is straightforward. For product search queries, Google has an incentive to distort its organic results so heavily as to make them useless, because giving any (free) organic link (direct) to merchants selling relevant products reduces those merchants’ willingness to pay in sponsored search auctions[3]. The restriction of content searches to organic results is less easily explained, but follows from publishers’ typical inability to anticipate the myriad of keywords that may indicate a good match between their content and a searching consumer’s interest.

There are two channels for advertising by merchants: display and search advertising. Search advertising is typically more effective, because people declare what they are looking for and merchants can bid for the search queries that let them target the specific consumers to whom their products are relevant. By contrast, display advertising is an imperfect substitute. The main reason is that merchants cannot target their specific, relevant consumers (in the baseline model, display advertising has no way to enable contextual or behavioral targeting). In addition, publishers have limited space on which to expose people to ads (invasive ads can backfire as nuisance costs dissuade reduce publisher traffic).

The effectiveness of display advertising for merchants depends indirectly on organic search quality. This can happen in a number of ways. In our model it is because consumers spend more time on publisher webs if Google’s organic search engine sends them to sites with content that is more relevant to their interests (see Ellman and Germano, 2009, and Wilbur, 2008, for alternative motivations and relationships between content and advertising effectiveness).

Search advertising is fully effective for a merchant whenever winning the top

[3]In our model, this disruption of the organic search channel has no cost for consumers, because sponsored results are free to consumers and Google can make these as reliable (in equilibrium) as it can make its organic results. See also the empirical work of Yang and Ghose (2010) and the theory of Chen et al (2012) who suggest that behavioral factors may sometimes create complementarities between organic and sponsored results.
position on Google’s auction of sponsored links for search queries relevant to that merchant’s products. So merchants bid for relevant queries, but their winning probability for a given (pay per click) bid depends on the nature of Google’s sponsored search. Google claims to allocate its sponsored search positions using a scoring auction that weights each merchant’s bid by that merchant’s quality or relevance from the perspective of the searching consumer (see Varian, 2007). In essence, Google must discriminate against the merchants that consumers value less, raising the bid they need to win a top position. Google faces a tradeoff between serving the merchants with highest willingness to pay on one hand, and serving consumer interests by discriminating against merchants with low quality or consumer relevance despite higher willingness to pay (say because they have higher margins).

A profit-maximizing search engine generally distorts search results with respect to the allocation rules that maximize consumer surplus. When Google is purely a search engine, not integrated into advertising or publishing, our analysis finds three main results on search distortion: (1) Google has an incentive to distort organic search to impoverish display advertising which is an (imperfect) substitute for Google’s search advertising\(^4\); (2) Google does have an incentive to use a scoring auction that discounts “less relevant” merchants, i.e., merchants whose products provide lower net value to consumers; however, a profit-maximizing Google does not weigh relevance sufficiently to ensure that the most relevant merchants always wins, and so the model only partially supports Google’s claims about the position auctions it uses for sponsored search\(^5\); (3) Google’s incentives to distort both organic and sponsored search results are naturally tempered by the need to attract consumers to use Google for their searches, as opposed to abandoning search or using alternative search methods. In fact, distortions in both types of search results

\(^4\)The intuition is simple: consumers spend less time on the publishers’ websites which are less attractive and so the publishers have less time to expose them to advertising by merchants.

\(^5\)We suggest that the simple conflict of interest between merchants and consumers underlying this new result is a good explanation of Hotchkiss et al’s (2005) survey evidence that people report more trust in organic than sponsored results.
interact through various channels, including consumers’ decisions to use the search engine. As a result, at least under some conditions, there may be a substitution effect between distortions: strong motives for biasing organic search lead to less bias of sponsored search and vice versa.

Regarding the impact of vertical integration into ad intermediation, we present three main findings (similar results hold for integration into publishing): (1) Vertical integration into display ad intermediation leads to full monopolization of that market if publishers are symmetric; Google’s control over organic search gives Google the gatekeeper power to restrict the flow of traffic onto any publisher website that does not become a Google advertising affiliate (trading instead with a non-Google ad intermediary or negotiating with merchants directly); (2) vertical integration then has the positive effect of reducing the incentive to distort organic search, because Google internalizes a share of the profits from display advertising; (3) on the other hand, vertical integration that only involves partial monopolization of ad intermediation (as can arise when publishers are heterogeneous or as a result of regulation) introduces the risk of a serious bias where Google’s organic search favors publishers that deal with Google as ad intermediary against those that do not. Concretely, we identify conditions under which non-integration generates higher total welfare and higher consumer surplus than does vertical integration with partial monopolizations. Similarly, vertical integration with full monopolization can have negative consequences for organic search when publishers are asymmetric in that certain types of publisher are more effective than others as platforms for display advertising.

In addition to identifying how integration can affect the interests of consumers and of society as a whole (the social welfare), our analysis characterizes the sharing of surplus among consumers and each class of business actor involved in “producing” web-mediated “trades”, namely the search engine, the publishers, the merchants and the ad intermediaries. These shares are important because advertising

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6Search distortions are not only detrimental to consumer surplus but also to total surplus.
revenues are fundamental to the business models of most web-based publishers, not just to Google. For example, online news content is mostly free to consumers and the publishers (news companies) make most of their money from advertising (ongoing attempts to set up pay walls and charge for access have met only limited success - see e.g., Evans, 2008, and The Economist, 2011). We show that vertical integration may reduce the share of advertising surplus that publishers can appropriate. In an extended model where publishers must invest in quality to attract consumers, a lower equilibrium share of advertising surplus implies weaker incentives to invest in quality. The dynamic consequences for social surplus may be significant. Similarly, merchants have less incentive to invest in creating good products if their share of advertising surplus is low.

Internet trade and search engine incentives is a very active research field in economics. Hahn and Singer (2008) provide a thoughtful law and economics analysis of the Google-DoubleClick merger of 2007 and their survey provides empirical evidence suggesting that advertisers do perceive search and display advertising as imperfect substitutes, as captured in our formal model. White (2009) also discusses this merger and the interaction between organic and sponsored search, but lacks a micro-founded model of search (merchant products and prices are identical). Most works in the literature have studied sponsored search. Chen and He (2011) and Athey and Ellison (2011) study position auctions in a context with asymmetric information. Their main result is that auctions induce a positive self-selection effect: merchants with better matches bid higher, because their links are more likely to lead to a trade. In this respect, their baseline settings rule out any conflict of interest between merchants (and Google) and consumers. Borgers et al. (2007), consider a more general setting, but their goal is to empirically estimate how much merchants value different positions; they do not characterize or analyze

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7 Auction theorists generalized first and second price auctions to allow for bidding on multiple rankings (see e.g., early papers including Lehaie, 2006, Edelman and Ostrovsky, 2007, Edelman, Ostrovsky and Schwarz, 2007 and Varian, 2007) and care was taken over equilibrium selection issues (see e.g., Borgers et al., 2007, Varian, 2007, and Edelman and Ostrovsky, 2010). Equilibrium selection is not a problem in our environment.
the conflict of interest between consumers and merchants.

Borgers et al. (2007), Liu, Chen and Whinston (2010) and Athey and Ellison (2012) develop useful models of scoring auctions, but they focus on click-through-rates (CTRs) that differ among merchants. Again the conflict of interest is limited. Google discounts merchants with low CTRs and this benefits social surplus and consumer surplus. Athey and Ellison (2012) do show that in the presence of private consumer search costs, Google raises the reserve price on its search auctions to increase the minimal CTR in equilibrium, but insufficiently from the perspective of consumers who face private search costs. This is reminiscent of Hagiu and Jullien’s (2011) result (in a non-auction setting) that distorting search from the consumer’s ideal can be socially advantageous owing to the positive externality of their searches on merchants and intermediaries, such as Google. Our conflict of interest is quite different: merchants care about their margins, while consumers care about net benefit. Our paper is the first to capture this stark and simple cause of bias.

The paper proceeds as follows. In section 2, we present the baseline model and characterize the first-best and second-best where the only constraint is the incentive compatibility of consumer participation in search. In Section 3, we analyze the equilibrium of the game, where Google, merchants, publishers, and ad intermediaries are all vertically separate profit-maximizers. Here we begin by deriving the equilibria for a class of scoring auctions compatible with Google’s description of its sponsored search auctions (Lemma 1). Then we characterize the distortions in each type of search; first organic alone, then sponsored alone, and then their interactions. In section 4, we repeat this equilibrium analysis in the case with vertical integration into ad intermediation (demonstrating the logic of full monopolization). We use our results to make welfare and consumer surplus comparisons between the two cases. In section 5, we extend the analysis to treat the case of

\footnote{A few papers in this literature model the determination of merchant values by modeling price competition in the product market (see e.g., Chen and He, 2011, Xu, Chen, and Whinston, 2010 and 2011).}
vertical integration with only partial monopolization and again compare surplus implications with those of no integration. We gather the proofs in the Appendix.

2 The baseline model

There are five types of agent: consumers, merchants, publishers, intermediaries (between publishers and merchants) in the display advertising market, and a search engine, $G$.

Merchants and publishers. Each merchant owns and maintains a website on which consumers can read about and purchase (or learn how to purchase) a product that interests them. Similarly, each publisher owns and maintains a website that offers online “content” such as news, information, blogs, music and video, that has direct consumption value for some consumers. In addition, publishers offer “space” where merchants can insert their advertising messages (in textual, graphic or, more recently, audiovisual format). Consumers who wish to consume a publisher’s content must visit the publisher’s website to access its content and they thereby become accessible, during their visit, to any advertising the publisher chooses to expose. In other words, the publisher can expose its visitors to the third-party ads it displays.

This channel for third-party advertising is the critical feature that distinguishes publishers from merchants. Merchants can advertise on publisher sites, but not conversely, because publisher content is readily bundled with third-party advertising, while merchant products and websites are not. One practical reason is that publisher content is typically consumed online (which greatly facilitates bundling with advertising) while merchant products are typically consumed offline (and merchant websites are only attractive to potential consumers to whom that merchant is unwilling to expose competing merchants’ ads)\textsuperscript{9}

\textsuperscript{9}Online content consumption is necessary for ads to offer immediate links to merchant websites (or to vary with real-time auctions on an ad exchange). In special cases where a publisher’s main attraction is free electronic downloads of content (or software) for offline consumption, the publisher tends to impose relatively invasive ads on the download page (or to bundle the
Publishers do not charge consumers for access to their content. Instead they make their money from the advertising tied to their content. Publishers compete as platforms in a two-sided market featuring advertisers and consumers; zero pricing on the consumer side arises endogenously when a relatively high potential advertiser surplus drives publishers to compete intensely for consumers. Exogenous frictions in charging consumers (such as internet piracy and hacker access) may also force a zero price (see e.g., Evans, 2008, or The Economist, 2011). We focus on the zero price case since, empirically, most online content is free to consumers, but our main points continue to hold when publishers charge for content as well as advertising.

**Products and content.** We can now describe the details of the model more precisely. A mass one of consumers, indexed by $i$, value specific varieties of off-line and on-line goods. There are $2J$ off-line goods (“products”), produced by $J$ different merchants, indexed by $j$. Merchants are symmetric. Each merchant produces 2 varieties, one of type 1 and one of type 2; product $(j, k)$ is merchant $j$’s type $k$ product ($j = 1,..., J; \ k = 1, 2$). Consumers are very choosy. Each consumer is characterized by only two products $(j, k)$ and $(j', k')$ that give positive net utility. We suppose $k \neq k'$ and $j \neq j'$ so that each consumer values one product of each type and no single merchant produces both products that interest any given consumer. Consumer preferences are uniformly distributed over all such pairs. Each consumer’s net utility (net of price) is $v > 0$, from one unit of her preferred type 1 product, and $\lambda v$, $0 < \lambda < 1$, from one unit of her preferred type two product. That is, each consumer’s best match is a type one product and a second best match is always a type two product (and all other matches of consumers and downloaded content with ads that automatically appear during consumption).

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10This two variety set-up simplifies the analysis, because the $2J$ merchant model where each sells one product has merchant asymmetries (two types of merchant).

11Excluding $k = k'$ maintains consumer symmetry. Excluding $j = j'$ ensures competition for the top position of any given keyword auction. With fully independent probabilities across merchants for each consumer, there is the slight $\frac{1}{J}$ probability complication of no competition in the keyword auction of a consumer’s product query (in any case, the probability is negligible if $J$ is large).
products generate no value).

Merchants have the opposite ranking over trading their type 1 and 2 products: denoting their margins from selling one unit of type 1 and type 2 products by \( m_1 \) and \( m_2 \), respectively, we assume \( m_1 < m_2 \). This is a convenient special case of a more general formulation discussed below.

There are \( N \) on-line goods, which we call “content” (but software is included). Content is available for free consumption on websites owned and produced by publishers. Each publisher produces exactly one (distinct) content website, so we label publishers and online goods (content websites) by the same index \( n \). Each consumer has a favorite website (or “best match”) which generates direct net utility \( u, u > 0 \), while all other websites generate zero net utility. Publishers are symmetric: each publisher is the best match for a fraction \( \frac{1}{N} \) of consumers. Publishers also generate indirect benefits, because their content websites provide a channel for merchants to advertise to consumers. This is called “(third-party) display advertising” (the merchant is the third-party).

Concretely, each publisher has an exogenously fixed quantity of perfectly divisible ad space, which we normalize to \( J \), and can supply this space at zero marginal cost (to merchants). So each publisher \( n \) earns profits \( \sum_{j=1}^{J} q_{jn} \) by renting quantities of advertising \( a_{jn} \) (this can be interpreted in terms of time, probability, size and salience) to merchants \( j \in J \) for displaying \( j \)’s ads on \( n \)’s website at a monetary charge \( q_{jn} \). Each publisher \( n \) makes a take-it-or-leave-it offer \((a_{jn}, q_{jn})\) to each merchant \( j \). The equilibrium quantities must satisfy the space constraint \( \sum_{j=1}^{J} a_{jn} \leq J \) and the merchant participation constraint (i.e., \( j \) pays nothing - \( q_{jn} = 0 \) - if \( j \) buys no ads on \( n \)). Notice that we allow nonlinear pricing and third degree price discrimination, but the latter plays no role in the baseline model since merchants are symmetric given the information available to publishers. For simplicity, we assume that merchant \( j \) can advertise both its products \( k = 1, 2 \) at the same time with no efficiency costs in ad-effectiveness.\(^{12} \) The only costs

\(^{12}\) I.e., a consumer visiting a publisher website that impresses \( j \)’s ad for both products is as
for publishers may come from payments for ad intermediation, but are zero in the baseline model since ad intermediation is perfectly competitive and has zero cost. To analyze the competitive outcome, we need to specify the effectiveness of advertising.

When merchant \( j \) buys ad “space” \( a_{jn} \) on website \( n \), consumer \( i \)’s visit to website \( n \) leads \( i \) to clicks through to visit \( j \)’s website with probability \( \alpha(a_{jn}) \) if \( n \) is \( i \)’s best-match content site and with probability \( \beta \alpha(a_{jn}) \) if \( n \) is any other publisher site where \( \alpha(\cdot) \) is a concave function and \( \beta \in (0,1) \). These probabilities or “click-through rates” (CTR) capture how “advertising effectiveness” varies with ad intensity \( a \) and varies with publisher-searcher match quality. We make three assumptions. (i) The CTR scales down by the factor \( \beta \) when \( n \) is a poor match for \( i \)’s content preferences. We motivate this reduced advertising efficiency by the idea that \( i \) spends longer on webs with more interesting content (that of better match publishers) and is therefore more likely to notice ads (and click on them). (ii) For a given match quality, the CTR reaches its upper bound at a finite ad intensity \( \bar{a} \). We assume that increasing advertising intensity \( a \) (initially) raises click-through at a decreasing rate, becoming useless or counterproductive (as when consumers become annoyed by overly obtrusive ads) beyond a finite level \( \bar{a} \). Formally, the CTR \( \alpha(a) \) satisfies \( \alpha'(a) > 0 \) and \( \alpha''(a) \leq 0 \) for all \( a < \bar{a} \) and \( \alpha(a) \leq \alpha(\bar{a}) \) for responsive to each of the two products as when \( j \)’s ad instead features that product alone. Since at most one product is relevant, we are simply assuming that the consumer can immediately focus on the relevant product in a multi-product ad by any single merchant. This simplifying assumption avoids the complication of differential display advertising by product type. Note that we do not consider publisher-imposed restrictions on how merchants use their ad space. This is because, even if feasible, such restrictions offer no benefit to publishers in our symmetric merchant setup.

To be explicit, we assume intermediation between publishers and merchants has constant (zero) marginal cost and free entry. So in the baseline model where Google is not vertically integrated into ad intermediation, this is a perfectly competitive industry and it is immediate that ad intermediation has zero prices. We can therefore neglect ad intermediation in the remainder of this section.

For alternative motivations and more general settings, we refer to Ellman and Germano (2009); in particular, Wilbur (2008) provides related empirical evidence that specific types of video content (in offline TV) are more conducive to effective advertising and is relevant to our extensions with asymmetric publishers.
all $a > \bar{a}$. (iii) Moreover, we focus on the case where publishers can offer this maximal amount of advertising $\bar{a}$ to all merchants; that is we assume $\bar{a} = 1$. We denote the maximized ad effectiveness or CTR by $\bar{\alpha} \equiv \alpha(\bar{a}) = \alpha(1)$. Assumption (i) is the root cause of organic bias. Assumptions (ii) and (iii) ensure that display ad inventory $J$ is not a binding constraint in equilibrium. Assumption (ii) places an upper bound on the demand for display advertising of $J\bar{a}$ and assumption (iii) ensures that publishers can satisfy this maximal demand.

The search engine. Consumers use $G$ to search for products and then for content. To conduct each type of search, they type in a query (consisting of a set of keywords) and $G$ responds by providing a set of results which consist of links to merchants’ or publishers’ websites. These results are separated into two groups: a list of “sponsored results” (typically appearing on the right and at the top of the screen with a yellow background) where website owners can sponsor a link to their site (in that they pay for a well-placed link) and a list of nonsponsored or “organic results” where $G$ commits against adjusting results as a function of any such sponsorship. Sponsorship takes the form of a position auction (also called pay-for-placement auctions) which we describe below.

Consumers do not know which merchants produce their best and second-best matches for offline products. However, $G$ is sometimes able to interpret their queries and locate the identity of these two merchants. There are as many queries as pairs of products and when consumer $i$ types her query, with probability $\phi$, $G$ is able to identify both her best and second best match products, and the merchants that sell those commodities. With probability $1 - \phi$, $G$ is unable to interpret the query and can do nothing better than deliver an entirely random list of results. The parameter $\phi$ measures the degree of substitutability between sponsored search advertising and display advertising, with a higher value of $\phi$ indicating a higher degree of substitutability. For technical reasons that will become clear below, we focus on the case $\phi \leq \frac{1}{2}$. When $G$ is able to identify the best and the second best matches for the consumer, then $G$ observes the outcome of the position auction
corresponding to such query. The winner of this auction will appear in the list of sponsored results, that contains only one entry. A possible justification is the cost of a second click for a consumer is sufficiently large. The looser and all other merchants will be randomly placed in the organic results. The rules of the auction are as follows. If we denote by $b_1$, $b_2$ the bids placed by the merchants offering the first and second best products, respectively, then $G$ will take into account the merchant’s “relevance” and will discount the bid of the type 2 merchant by a factor $\mu$, $\mu < 1$.\footnote{We assume that, in all cases, any merchant other than those selling the consumer’s favorite pair have their (never profitable) bids discounted down to zero, so we do not have to worry about irrelevant participation.} Bids are the merchants’ declared willingness to pay per click (PPC). Then, every time a consumer clicks on the sponsored result the merchant will pay $G$ the PPC determined in the corresponding auction. The winner of the auction is determined by the straight comparison between $b_1$ and $\mu b_2$. The PPC rate is set equal to the second highest “effective” bid: the lowest bid that would have allowed the winner to win the auction. Thus, if $b_1 > \mu b_2$ then merchant type 1 wins the auction and the PPC rate is set equal to $\mu b_2$. If $b_1 < \mu b_2$ then merchant type 2 wins the auction and the PPC rate is set equal to $\frac{b_1}{\mu}$. In case of tie, then with probability $r^S$ the type 1 merchant wins and the PPC is equal to $\mu b_2$ and with probability $1 - r^S$ type 2 merchant wins and the PPC rate is equal to $\frac{b_1}{\mu}$. Both $\mu$ and $r^S$ are set by $G$.

The mechanism for determining PPCs aims at capturing in a simplified framework the mechanism that Google has declared to use in real situations. In other words, the outcome of the auction is not solely dependent on merchants’ bids but also on the quality score of different ads. In doing this, Google recognizes that there may be in fact a conflict of interests between consumers and merchants. In our model, such conflict of interests arises from the fact that consumers and merchants rank the two products differently. In other words, consumers would like $G$ to show the producer of type 1 good at the top, but the producer of the type 2 good has a higher willingness to pay for the single position in the list of...
sponsored results. The choice of μ and r^S indicates G’s compromise between these two objectives.

Similarly, consumers ignore the identity of the publisher that provides their favorite content, but G is also able to interpret their queries and locate the identity of the optimal publisher. However, we also allow G the possibility of distorting its recommendations. In other words, as a response to queries about content on publisher websites, G displays a list of organic results. G announces that with probability r^O (the degree of “reliability”) it will respond to i’s search query by ranking the consumer’s best match at the top of the list of organic results, and with probability 1 − r^O G will ignore its information and the entire ranking will be completely random (in particular, the first position will be taken by a random publisher.)

Consumers have a cost c_i of using the search engine. This is the joint cost of both queries (for off-line and online goods). We assume that c_i is an independent draw from a differentiable density function f(c) on [0, c_H] and cumulative distribution function F(c). We assume that the hazard rate is decreasing. That is, if we let H(c) = \frac{f(c)}{F(c)} then H'(c) < 0. For simplicity, we assume that consumers do not consider the indirect benefits of visiting a publisher’s website (the possibility of being attracted by display advertising and ending up purchasing offline products). We also assume that c_H > \phi u + v, which guarantees that in equilibrium consumer participation in online search has always an interior solution.

The timing of the game is as follows. In the first stage, G announces μ, r^S and r^O as well as the rules of the position auctions. In the second stage, publishers announce their offers (a_{jn}, q_{jn}). In the third stage, merchants choose their bidding strategies in the position auctions and their (accept or reject) responses to publishers’ offers. In the fourth stage, consumers decide whether or not to use the search engine. If they do search on Google, they type in their query for offline products and, if G successfully interprets their query, they visit one merchant’s website. If the website turns out to produce their first or their second best product
then they purchase the good. Next, they type in the queries for online content and visit one publisher’s website. If they are attracted by display advertisement of the merchants producing their first- or second-best product, then they make a purchase, provided they have not bought the good before.

Merchants, publishers and ad intermediaries can observe the actions chosen in the previous stages. However, for simplicity, consumers can only observe $G$’s choices in the first stage but can only conjecture merchants’ and publishers’ choices.

In the main text we assume that functions and parameter values satisfy the restrictions necessary to guarantee the concavity of $G$’s optimization problems. In the Appendix, we specify the exact restrictions on the primitives of the model (or at least the sufficient conditions) that we need, in addition to the ones we have already laid out.

2.1 Discussion

We could have started out with a more general description of consumer preferences on off-line goods. Let $p^F(j, k)$ be the probability that product $(j, k)$, $j = 1, ..., J$, $k = 1, 2$, is the best match for an arbitrary consumer. Also, let $p^S(j', k' \mid j, k)$ be the probability that product $(j', k')$ is the second best match conditional on $(j, k)$ being the best. For the moment let us assume that, with probability $\phi$, the query contains all the relevant information to identify any ordered pair. This formulation opens the door to uninteresting situations in position auctions. First, some queries involve two type 1 or two type 2 products. In those cases, merchants would place the same bid and hence $G$’s optimal policy would consist of recommending consumers’ optimal product (no conflict of interest). Second, some queries involve a type 2 product as the best match and a type 1 product as the second best. Once again, there is no conflict of interest since consumers and merchants rank the two products in the same order. The existence of these segments of the market do not add any insight and have only a quantitative, not qualitative, effect on $G$’s incentives to distort its recommendations. Hence, in the baseline model we have
assumed that $p^P(j, 2) = 0$ for all $(j, 2)$, and $p^S(j', 1 | j, 1) = 0$ for all $(j, 1), (j', 1)$.

We may also assume that queries do not provide enough information to identify consumers’ ranking of the two products with probability 1. For instance, suppose that a query allows $G$ to identify a pair $\{(j, 1), (j', 2)\}$, but consumers may disagree on the ranking of these two products. In particular, suppose that a fraction $\rho$ have product $(j, 1)$ as their best match while product $(j', 2)$ is the best match for the rest, but $G$ is unable to identify to which group the consumer typing in the query belongs to. Clearly, only if $\rho > \frac{1}{2}$ then there is a conflict of interest and the value of $\rho$ would parameterize the intensity of such conflict. Fixing $\rho = 1$, as we did in the description of the baseline model, does not alter the qualitative results.

In our model, aside from consumer preferences, off-line goods (types) differ only in the merchant’s margin. This may be due to differences in access to alternative channels for attracting consumers. For instance, suppose that merchants realize the same margin per sale, $m$, in both types of goods. However, for type 1 goods, a fraction $\tau$ of consumers whose best match is $(j, 1)$ and who did not purchase good $(j, 1)$, neither through sponsored results nor as a result of being attracted by display advertisement in publishers’ websites, will still purchase the good through alternative channels, perhaps offline. It is straightforward to show that in the equilibrium of this alternative formulation, the effective internet margin of types 1 and 2 products are $(1 - \tau) m$ and $m$, respectively. In other words, merchant’s willingness to pay in position auctions and display advertisement are given by the same formulas of the baseline model just letting $m_1 = (1 - \tau) m$ and $m_2 = m$.

### 2.2 The social planner’s problem

Suppose that the social planner maximizes total surplus (the sum of the surpluses obtained by all agents)\textsuperscript{16} and can only control not only the search engine’s behavior $(r^O, r^S)$, but not consumer participation, $X$, which is chosen by individual

\textsuperscript{16}We will mention below how results change if the social planner only cares about consumer surplus.
consumers in order to maximize their utility. Note that it will be optimal allocate
space $a = 1$ to each merchant in each publisher’s website. Also, since the cost
of a second click is sufficiently large, the social planner will not alter consumers’
clicking behavior.

Consumer surplus can be written as\textsuperscript{17}

$$CS = \int_0^\infty \left\{ \phi \left[ r^S v + (1 - r^S) \lambda v \right] + r^O u - c \right\} f(c) dc,$$

(1)

where $\overline{c} = \phi \left[ r^S v + (1 - r^S) \lambda v \right] + r^O u$. The first two terms in the integrand represent the expected surplus associated with finding the first and second best matches thanks to sponsored search results; the third term is the surplus associated with finding the right publisher’s content; and the fourth term is the cost of using $G$. Note that $CS$ increases with both $r^O$ and $r^S$. The rest of the surplus generated in this economy, the producer surplus, is generated by merchants, publishers and the search agent. This producer surplus can be written as:

$$PS = \left\{ m_1 \left[ \phi r^S + (1 - \phi r^S) \overline{e} \left( r^O \right) \right] + m_2 \left[ \phi (1 - r^S) + [1 - \phi (1 - r^S)] \overline{e} \left( r^O \right) \right] \right\} F(\overline{c}),$$

(2)

where

$$e \left( r^O \right) = r^O + (1 - r^O) \beta,$$

(3)

is the effective precision of organic search, from the point of view of advertisers. Note that $PS$ increases with $r^O$ but it may increase or decrease with $r^S$.

The social planner chooses $(r^O, r^S)$ in order to maximize $CS + PS$. In the optimal plan $r^O = 1$, since both consumer and producer surplus increase with $r^O$. However, the social planner faces a tradeoff when choosing $r^S$: consumer surplus increases, but producer surplus decreases, with $r^S$. The optimal value depends on

\textsuperscript{17}Notice that we use here the consumer surplus as perceived by consumers ex ante which neglects the possible unsought benefits from being distracted by adverts during the content search stage and clicking through and consuming offline products. There are arguments to take an alternative view but they do not change any of our main effects.
the relative strength of these two effects. In particular, $r^S$ should be set lower than 1 if the following condition holds:

$$H (\phi v + u) \frac{w (\phi m_1 + m_2 \overline{\pi})}{\phi (m_2 - m_1) (1 - \overline{\pi}) - w} < 1$$

(4)

**Remark 1** If the goal is to maximize total surplus (and provided consumer participation is endogenously determined) then the search engine’s optimal policy consists of $r^O = 1$, and $r^S$ is equal to 1 if and only if condition (4) fails.

If on top of $(r^O, r^S)$ the social planner can also choose consumer participation, $c$, then $c > \phi \left[r^S v + (1 - r^S) \lambda v\right] + r^O u$ and it is more likely that the optimal $r^S$ is lower than 1. In other words, in the previous formulation of the social optimum consumers do not internalize the positive externality their participation decision generates on producer surplus and as a result consumer participation is inefficiently low. Also, the social planner had an additional incentive to set a higher $r^S$: increase consumer participation.

Finally, if the social planner maximizes consumer surplus then the optimal policy consists of $r^O = r^S = 1$.

### 3 Equilibrium analysis under vertical separation

We restrict attention to symmetric equilibria. In particular, all publishers make the same offer to all merchants, and the latter take exactly the same decision regarding both display advertising and position auctions. More specifically, all merchants use the same bidding strategy, which only depends on the type of product. That is, all merchants selling type 1 product will bid $b_1$, and those selling type 2 products will bid $b_2$. Such a symmetric bidding behavior also requires that merchants have the same willingness to pay to appear in the list of sponsored results. Finally, in a symmetric equilibrium all participating consumers also make the same choices and obtain the same expected utility (gross of search costs).
Let us begin by studying the optimal design of sponsored search auctions. Let \( R_1, R_2 \) denote merchants’ willingness to pay at a position auction if she holds the type 1 and the type 2 offline good, respectively. Without loss of generality, suppose \( R_1 \leq R_2 \). Given the competing merchant’s bid, \( b_2 \), the merchant offering the relevant type 1 good wins the auction with probability one if her bid is strictly above \( \mu b_2 \) or with probability \( r^S \) if it is equal to \( \mu b_2 \). In either case, she will pay \( \mu b_2 \). Similarly, given the rival bid \( b_1 \), the merchant offering the relevant type 2 good wins the auction with probability one if she bids strictly above \( \frac{b_1}{\mu} \), and with probability \( (1 - r^S) \) if the bid is equal to \( \frac{b_1}{\mu} \). In either case she will pay \( \frac{b_1}{\mu} \). Consequently, each merchant has a unique weakly dominant strategy: \( b_k = R_k \).

**Lemma 1** In any auction, the strategy profile \((b_1, b_2)\), where \( b_k = R_k \), is the unique equilibrium in weakly dominant strategies.

We focus on such type of equilibrium. In such equilibrium, consumer participation will depend only on whether \( \mu \) is above, below, or equal to \( \frac{R_1}{R_2} \), and only in the later case, on the value of \( r^S \). It is then straightforward that \( \mu = \frac{R_1}{R_2} \) maximizes \( G \)'s expected profit. Indeed, if \( G \) prefers that merchant 1 always wins, then setting that value of \( \mu \) and \( r^S = 1 \) not only attains that goal but also maximizes \( G \)'s revenues conditional on it. Similarly, if \( G \) prefers merchant 2 to always win then that value of \( \mu \) and \( r^S = 0 \) is its best choice. Finally, the only way to randomize is also by setting that value of \( \mu \) and set an interior value for \( r^S \).

Note that for \( \mu = \frac{R_1}{R_2} \) the two effective bids are equal, and hence by choosing \( r^S \) \( G \) can determine the probability that the PPC equals \( R_1 \) and \( R_2 \). That is, this second price auction is equivalent to a mechanism consisting of making a take-it-or-leave-it offer to merchant 1 with probability \( r^S \), and to merchant 2 with probability \( 1 - r^S \). Summarizing:

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18Note that there are a continuum of pure strategy equilibria where, for instance, merchant 2’s bid when competing with merchant 1 is \( \mu R_2' > b_1 > R_1 \). Bidding above her reservation prices do not change the identity of the winner but it does change the price the rival pays. For the standard reasons, we select the equilibrium in dominant strategies.
Remark 2 G’s optimal auction involves \( \mu = \frac{R_1}{R_2} \), which implies that with probability \( r^S \) the merchant selling the type one product wins the auction (and pays the price \( R_1 \)), and with probability \( 1 - r^S \) the merchant selling the type 2 product wins the auction (and pays the price \( R_2 \)).

In a symmetric equilibrium consumer participation, \( X \), is given by

\[
X = F[\bar{c}],
\]

where, given \( \mu = \frac{R_1}{R_2} \),

\[
\bar{c} = \phi \left[ r^S v + (1 - r^S)\lambda v \right] + r^O u. \tag{5}
\]

Suppose that in stage 2 all publishers have made identical offers to merchants: \((a, q)\). We will argue below that in the unique symmetric equilibrium, in stage 3, all merchants accept these offers and make the same bids in all position auctions, \( b_k = R_k, k = 1, 2 \). We are now ready to compute these willingness to pay for sponsored results.

The profits for merchant \( j \), \( \Pi_j \), can be written as:

\[
\Pi_j = \frac{\phi X}{J} \left[ r^S (m_1 - R_1) + (1 - r^S) e \left( r^O \right) \alpha(a) m_1 + (1 - r^S)(m_2 - R_2) + r^S e \left( r^O \right) \alpha(a) m_2 \right] + \frac{(1 - \phi) X}{J} e \left( r^O \right) \alpha(a) (m_1 + m_2) - qN. \tag{6}
\]

That is, a fraction \( \phi \) of participating consumers will purchase one unit of the offline good by clicking at the top sponsored result. Within this group, a fraction \( r^S \) purchase a type one good, and a fraction \( 1 - r^S \) a type two good. These consumers next search for online content and they still have a probability \( e \left( r^O \right) \alpha(a) \) of being attracted by display ads and purchasing the second unit. Also, a fraction \( 1 - \phi \) of participating consumers arrive at publishers’ website with a demand of two units and they will purchase one unit of each with probability \( e \left( r^O \right) \alpha(a) \).

Suppose instead that merchant \( j \) deviates, bids \( b_1 < R_1 \), and looses all position auctions for its type one products. Then for each consumer that successfully uses \( G \)’s sponsored results, instead of making profits equal to \( r^S (m_1 - \frac{R_1}{R_2} n_1 - \frac{R_1}{R_2} n_2) \),
\[ R_1 + (1 - r^S) e (r^O) \alpha(a)m_1 \text{ then the merchant makes } e (r^O) \alpha(a)m_1. \] By equating these two amounts then we can compute the highest willingness to pay: \[ R_1 = [1 - \alpha(a) e (r^O)] m_1. \] The same argument applies to type two products: \[ R_2 = [1 - \alpha(a) e (r^O)] m_2. \] In this case merchant \( j \)'s profits are:

\[
\Pi_j = \frac{X \phi}{J} e (r^O) \alpha(a)(m_1 + m_2) + \frac{X}{J} (1 - \phi) e (r^O) \alpha(a)(m_1 + m_2) - qN
\]

Next we compute the maximum willingness to pay for display advertising. Suppose that merchant \( j \) rejects all publishers’ offers. In this case its willingness to pay to appear in the list of sponsored results becomes \( R_k = m_k \). Thus, merchant \( j \) would win all auctions and make profits:

\[
\Pi_j = \frac{X \phi}{J} e (r^O) \alpha(a)(m_1 + m_2)
\]

Comparing these two last expression we obtain merchants’ willingness to pay for display advertising:

\[
q = \frac{X}{JN} (1 - \phi) e (r^O) \alpha(a)(m_1 + m_2)
\]

Thus, given the substitutability between sponsored and display advertising for a fraction \( \phi \) of participating consumers, merchants are not willing to pay for the potential rents obtains from these consumers through display advertising. In fact, they are only willing to pay for the rents they obtain from the fraction of consumers that can only be reached through display advertising.

Finally note that in stage 2 publishers will maximize their expected profits by offering the efficient level of space, \( a = 1 \), and charging the merchants’ reservation price, \( q = \frac{X(1 - \phi)}{JN} e (r^O) \bar{\alpha}(m_1 + m_2) \). In the Appendix we show that this is part of the unique symmetric equilibrium of the game:

**Proposition 1** Given \( r^S, r^O, \mu = \frac{m_1}{m_2} \), in the unique pure strategy, symmetric equilibrium of the continuation game, publishers set \( a = 1, q = \frac{X(1 - \phi)}{JN} e (r^O) \bar{\alpha}(m_1 + m_2) \), and merchants accept these offers and bid in position auctions according to \( b_k = [1 - \bar{\alpha} e (r^O)] m_k \).
We can now write $G$’s profits:

$$\Pi^G = \phi X \left[ r^S R_1 + (1 - r^S) R_2 \right] = \phi X \left[ 1 - e \left( r^O \right) \bar{\alpha} \right] M \left( r^S \right) .$$

where

$$M \left( r^S \right) = r^S m_1 + (1 - r^S) m_2$$

is the average margin of all transactions materialized through sponsored search advertising.

We are now ready to discuss $G$’s incentives for traffic management. The first order condition with respect to $r^O$ is:

$$\frac{\partial \Pi^G}{\partial r^O} = \frac{\partial X \Pi^G}{\partial r^O} X + X \left( r^S \frac{\partial R_1}{\partial r^O} + (1 - r^S) \frac{\partial R_2}{\partial r^O} \right) = 0. \quad (7)$$

Again, the first term is positive ($\frac{\partial X}{\partial r^O} = f(\bar{\pi}) u$) and represents the increase in demand associated with a larger probability of finding the content the consumer is looking for. The second term is negative. A more accurate organic search increases the value of ads in the publishers’ sites, and so reduces the opportunity cost of not advertising through sponsored search. That reduces the prices that merchants bid in the auction for position ($\frac{\partial R_k}{\partial r^O} = -\bar{\alpha} m_k (1 - \beta)$).

The first order condition with respect to $r^S$ is:

$$\frac{\partial \Pi^G}{\partial r^S} = \frac{\partial X \Pi^G}{\partial r^S} X - \phi X \left( R_2 - R_1 \right) = 0. \quad (8)$$

The first term is positive: a larger value of $r^S$ attracts more demand ($\frac{\partial X}{\partial r^S} = f(\bar{\pi}) w$), since it results in a larger probability of finding the right merchant when looking for offline goods. On the other hand, a larger $r^S$ reduces the profitability of each visit (consumer), since the price that $G$ obtains when the type 2 merchant wins the auction, $R_2$, is larger than when the type 1 merchant wins, $R_1$. It might be enlightening to consider first the incentives to distort each type of search separately. The first order condition for an interior solution with respect to $r^O$ (7) can be rewritten as:

$$H \left( \bar{\pi} \right) \frac{u \left[ 1 - e \left( r^O \right) \bar{\alpha} \right]}{\bar{\alpha} \left( 1 - \beta \right)} = 1. \quad (9)$$
Thus, a higher value of $\alpha$ and a lower value of $\beta$ both push the optimal value of $r^O$ downwards. In other words, the incentives to distort organic search are exacerbated as the effectiveness of display advertising, $\alpha$, and the effectiveness of distorting organic search, $1 - \beta$, increase. In case of no distortion of sponsored search, $r^S = 1$, $G$ will find it optimal to distort organic search, $r^O < 1$, if and only if:

$$H (\phi v + u) \frac{u(1 - \alpha)}{\alpha(1 - \beta)} < 1.$$ \hspace{1cm} (10)

Therefore,

**Remark 3** In equilibrium conditional on $r^S = 1$, if condition (10) fails then $r^O = 1$ as in the social benchmark. If condition (10) holds then $r^O$ is inefficiently low, with respect to the social benchmark.

The reason behind the inefficiency is straightforward: $G$ does not take into account the reduction in producer surplus associated to a value of $r^O < 1$.

Then, the first order condition for an interior solution with respect to $r^S$ (8) can be written as:

$$H (\bar{\tau}) \frac{wM (r^S)}{m_2 - m_1} = 1.$$ \hspace{1cm} (11)

Thus, a higher value of $m_1$ and a lower value of $m_2$ both push the optimal value of $r^S$ upwards. In other words, the incentives to distort (with respect to consumers’ optimal benchmark) sponsored search are exacerbated as the opportunity cost of pursuing consumers’ interests, $m_2 - m_1$, increases. In case of no distortion of organic search, $r^O = 1$, $G$ will find it optimal to distort sponsored search, $r^S < 1$, if and only if:

$$H (\phi v + u) \frac{um_1}{m_2 - m_1} < 1.$$ \hspace{1cm} (12)

If we compare conditions (4) and (12) then it is clear that the set of parameters for which in the conditional equilibrium $r^S = 1$ is smaller than the set that prescribes $r^S = 1$ in the social benchmark. Moreover, even when in the social benchmark $r^S < 1$, the equilibrium value of $r^S$ is inefficiently low. Summarizing:
Remark 4 In equilibrium conditional on $r^O = 1$, if condition (12) fails then $r^S = 1$ as in the social benchmark. If condition (12) holds then $r^S$ is inefficiently low with respect to the social benchmark.

The reason why in equilibrium $r^S$ is inefficiently low is that $G$ does not take into account the positive effect of $r^S$ on consumer surplus. Moreover, $G$ only captures a fraction of producer surplus and hence has lower incentives to foster consumer participation and expand producer surplus.

Summarizing, our model predicts that $G$ has incentives to distort sponsored search as well as organic search (with respect to consumers’ optimal benchmark), even when considered the two instruments in isolation. With respect to total surplus maximization, incentives to distort both organic and sponsored search also tend to be excessive.

Now we can look at the interactions between these two instruments. From equation (9) we can see that the only effect of $r^S$ on the optimal value of $r^O$ is through consumer participation, $\ell$. Since $H$ is a decreasing function, then it is immediate that a higher value of $r^S$ implies a lower value of $r^O$. Similarly, from equation (11) we conclude that a higher value of $r^O$ implies a lower optimal value of $r^S$. In other words, these two instruments are imperfect substitutes from $G$’s point of view.

4 Equilibrium analysis under vertical integration

Now, assume that $G$ merges with one particular ad intermediary. In the absence of regulatory supervision, $G$ could set different values of $r^O$ for publisher websites whose advertising business is run by $G$’s ad intermediary and for the rest. In fact, $G$ may “threaten” websites with $r^O = 0$ in case they deal with a different intermediary. Suppose that ad intermediaries offer their services to publishers in exchange for a tariff $T$, and next publishers make their offers to merchants, $(a_{jn}, q_{jn})$. Then, if $G$ has no constraint on the number of publishers that its ad
intermediary can handle, it can capture the entire surplus from display advertising. Indeed, in equilibrium all other intermediaries will offer $T = 0$, $G$ will set $r^O = 0$ for publishers dealing with any other intermediary, and $G$’s intermediary will set $T^G = qJ$. The price $q$ will be set by publishers according to equation:

$$q = \frac{X (1 - \phi)}{JN} e \left( r^O \right) \bar{\pi} (m_1 + m_2) .$$

Thus, $G$’s profits will now be

$$\Pi^G = X \left\{ \phi \left[ 1 - \bar{\pi} e \left( r^O \right) \right] M \left( r^S \right) + (1 - \phi) e \left( r^O \right) \bar{\pi} (m_1 + m_2) \right\} .$$

Once again, let us first examine $G$’s optimal behavior in allocating traffic, considering sponsored search and organic search in isolation. The effect of $r^O$ on $G$’s profits is given now by:

$$\frac{\partial \Pi^G}{\partial r^O} = \frac{\partial X \Pi^G}{\partial r^O} X + X \left( 1 - \beta \right) \bar{\pi} \left( -\phi M \left( r^S \right) + (1 - \phi) (m_1 + m_2) \right) .$$

Two new effects appear relative to the case of no integration (condition (7)). First, the per-consumer rents, i.e., $\frac{\Pi^G}{X}$, that $G$ gains from consumer participation, are higher. Second, a higher $r^O$ increases the rents that $G$ extracts from publishers, $X \left( 1 - \phi \right) \left( 1 - \beta \right) \bar{\pi} (m_1 + m_2)$. Both effects are positive. In fact, note that the only negative effect of a higher value of $r^O$, through lower bids in position auctions, is more than compensated by the positive effect through the higher rents extracted from publishers. That is, $\frac{\partial \Pi^G}{\partial r^O} > 0$.

**Remark 5** Under vertical integration $G$ sets $r^O = 1$, which contributes to increase total surplus.

The first order condition with respect to $r^S$ is:

$$\frac{\partial \Pi^G}{\partial r^S} = \frac{\partial X \Pi^G}{\partial r^S} X - \phi X \left[ 1 - \bar{\pi} e \left( r^O \right) \right] (m_2 - m_1) = 0 .$$

Once again, with respect to the case of no integration (condition (8)) we have an additional effect. Namely, the gains from consumer participation apply to a
larger base. Hence, $G$ faces stronger incentives to set a higher value of $r^S$ with respect to the case of no integration. However, the comparison with respect to the social benchmark is somewhat more complicated. With respect to the social planner, $G$: (i) ignores the positive effect of $r^S$ on consumer surplus, (ii) does not fully internalize the positive effect on producer surplus through higher consumer participation, and (iii) undervalues the negative effect of $r^S$ on per capita producer surplus. The first two effects induce $G$ to set lower values, but the third effect induces $G$ to set a higher value of $r^S$. It is easy to find examples of parameter values for which the equilibrium value of $r^S$ is lower than in the social benchmark, and other examples where the opposite is true.

**Remark 6** Under vertical integration $G$ tends to set a higher value of $r^S$ than in the case of no integration. As a result, $r^S$ may still be lower than, and closer, to the total surplus maximizing level, but it may also overshoot.

So far we have discussed $G$'s incentives to set $r^O$ and $r^S$ in isolation. In contrast to the case of no integration the interaction effects can now have any sign; that is, $r^S$ and $r^O$ may be substitutes or complements. Therefore, the overall analysis of the effects of vertical integration are complex. However, the next proposition shows that the in fact one of the effects always dominates.

**Proposition 2** Integration reduces organic search distortion, which raises participation and both consumer and producer surplus. Formally, the equilibrium values of $r^O$, $X$, $CS$ and $PS$ are higher under integration than under separation.

The proof can be found in the Appendix.

Under integration $G$ monopolizes the display advertising market and as a result incentives to allocate traffic in both types of queries are altered. Now $G$ sets $r^O = 1$ because the negative effect on its profits from position auctions are more than compensated by the rents appropriated in the display advertising market. Such an improvement in the reliability of organic results will also attract new consumers.
The incentives with respect to $r^S$ are still mixed. With respect to the case of no integration, now $G$ has additional incentives to raise $r^S$ (attracting more consumers generate rents not only in position auctions but also in display advertising) but, on the other hand, participation is also higher due to a higher value of $r^O$, which aggravates the conflict of interest in position auctions: it increases the incentives to disregard relevance and take advantage of the higher willingness to pay of the least popular merchants.

Overall, however, we have found that vertical integration with ad intermediaries makes $G$ internalize a vertical externality, and that works in the interest of consumers. This is similar to the case of upstream-downstream integration in markets mediated by prices. When $G$ integrates with ad intermediaries, and then is able to appropriate the rents of publishers, it acquires incentives to increase these rents. Participation and organic search precision both have this effect, and also both are positive for consumers.

In our model the quality of online goods, available to consumers for free, is exogenous. In fact, we have ignored many of the distributional consequences of vertical integration and, in particular, the potential effect on the quality of on-line content. In a richer model with endogenous content quality, the type of integration we have considered in this section is likely to have an additional effect on consumers’ welfare, since publishers’ incentives to invest in quality could be significantly reduced.

Another potential drawback of integration has to do with $G$’s incentives to discriminate against publishers not dealing with $G$’s intermediaries. This effect is absent in the above, extreme case of full monopolization, since all publishers deal with $G$ in equilibrium. In the next section we illustrate how this effect comes to play in a less extreme market structure.
5 Integration with partial monopolization

The assumptions in our base model, in particular publishers’ symmetry and constant returns to scale in the ad intermediation technology, result in full monopolization when $G$ enters the ad intermediation market. This, in turn, meant that discrimination against publishers not dealing with $G$’s ad intermediary, although part of the equilibrium strategy, had no impact on $G$’s incentives or consumer surplus. In a more realistic setting, publishers would be heterogeneous and $G$’s integration with one ad intermediary would result in partial monopolization. In this section, we make a step in this direction in order to explain the consequences of partial monopolization of the display advertising market.

Suppose that for exogenous reasons $G$’s ad intermediary can at most handle the advertising business of a fraction $\gamma$ of publishers.\textsuperscript{19} As in the previous section we let $G$ treat these two types of websites differently. It offers their potential customers the possibility or receiving more traffic in exchange for a tariff, $T_G$. We assume that $G$ must offer the same deal to all websites ($G$ cannot price discriminate). In particular, $G$ announces $r^O_G$ and $r^O_{NG}$, the probabilities that a customer looking for content offered by a publisher is sent to that publisher when it is affiliated with $G$ and when it is not, respectively. Clearly, $G$ will find it optimal to send the diverted traffic to a different publisher in the $G$ system.\textsuperscript{20}

In equilibrium $T_G$ will be such that all websites are indifferent between accepting and not accepting the deal offered by $G$, but it has to be the case that at least a fraction $\gamma$ of websites will accept the offer and $G$ makes a deal with exactly a fraction $\gamma$. We let $e_G$ and $e_{NG}$ be the fraction of “effective” visits received by

\textsuperscript{19}One possible interpretation of such exogenous constraint is that an increase of $G$’s market share above $\gamma$ might trigger an unwanted investigation by the regulatory agency

\textsuperscript{20}The effect on customer participation of distorting search for online goods is the same whether the destination is a publisher affiliated or not with $G$. Also, the effect on the willingness to pay of merchants for sponsored ads is also independent of the destination of diverted traffic, but only depends on whether traffic is indeed diverted. However, the amount of traffic that a publisher’s site receives affects the willingness to pay of merchants for ads in that site. The latter effect is positive for $G$ when the destination of diverted traffic is a site affiliated with $G$ and inexistent otherwise.
publishers in the G system or outside, respectively. That is:
\[ e_G (r_G^O, r_{NG}^O) = r_G^O + (1 - r_G^O) \beta + \frac{1 - \gamma}{\gamma} (1 - r_{NG}^O) \beta, \]
and
\[ e_{NG} (r_{NG}^O) = r_{NG}^O. \]

Conditional on \( a = 1 \), the price of display advertisement in these two types of websites will be given by:
\[ q_d = \frac{X (1 - \phi)}{JN} e_d \bar{\alpha} (m_1 + m_2), \tag{13} \]
where subscript \( d \) denotes whether the website is or is not affiliated with G; i.e., \( d = G, NG \). Similarly, merchants’ bids in position auctions will be:
\[ b_k = \{1 - \bar{\alpha} E (r_G^O, r_{NG}^O)\} m_k, \]
where \( k = 1, 2 \), and \( E (r_G^O, r_{NG}^O) \) is the average number of effective visits received by publishers. That is,
\[ E (r_G^O, r_{NG}^O) \equiv \gamma e_G + (1 - \gamma) e_{NG} = \beta + (1 - \beta) [\gamma r_G^O + (1 - \gamma) r_{NG}^O]. \tag{14} \]

Finally, \( T_G \) will be equal to the maximum willingness to pay to be part of the G system:
\[ T_G = \frac{X (1 - \phi)}{N} \alpha (e_G - e_{NG}) (m_1 + m_2). \]

Consumer participation is determined by the equation analogous to (5):
\[ \bar{\sigma} = \phi \left[ r^S v + (1 - r^S) \lambda v \right] + [\gamma r_G^O + (1 - \gamma) r_{NG}^O] u. \tag{15} \]

Then, \( G \)'s profits can be written as:
\[ \Pi^G = X \{ \phi \left[ 1 - \bar{\sigma} E (r_G^O, r_{NG}^O) \right] M (r^S) + \gamma (1 - \phi) (e_G - e_{NG}) \bar{\sigma} (m_1 + m_2) \}, \]

The effect of \( r_G^O, r_{NG}^O \) on \( \Pi^G \) are given by
\[ \frac{\partial \Pi^G}{\partial r_G^O} = f (\bar{\sigma}) \gamma u \frac{\Pi^G}{X} + F (\bar{\sigma}) (1 - \beta) \gamma \bar{\alpha} \left[ -\phi M (r^S) + (1 - \phi) (m_1 + m_2) \right], \tag{16} \]
\[
\frac{\partial \Pi^G}{\partial r_{NG}^O} = f(\bar{\pi})(1-\gamma)u \frac{\Pi^G}{X} - F(\bar{\pi}) \left\{ \phi(1-\beta)(1-\gamma)\bar{\alpha}M(r^S) + [\gamma + (1-\gamma)\beta](1-\phi)\bar{\alpha}(m_1 + m_2) \right\}
\]

(17)

The effect of \( r_{NG}^O \) is analogous to the one in the baseline model, only weighted by the proportion of publishers dealing with \( G, \gamma \). The main distinctive feature of the current scenario is reflected in the first order condition with respect to \( r_{NG}^O \). In this case, since \( \gamma < 1 \), \( r_{NG}^O \) positively influences consumer participation (the first term of (17)). Thus, in contrast with the baseline model, \( G \)'s incentives to set a low \( r_{NG}^O \) are moderated and it is no longer true that the optimal \( r_{NG}^O \) is always equal to zero. However, in case that \( G \) chooses to set a low value of \( r_{NG}^O \) then affect consumer and producer surpluses will be negatively affected.

As \( \gamma \) increases the effect of \( r_{NG}^O \) on consumer participation and merchants’ bids in sponsored search auctions (first and second term of (17), respectively) shrink, and go to zero as \( \gamma \) goes to one, while the effect on \( G \)'s rents in the ad intermediation market converges to a negative value. As a result, if \( \gamma \) is sufficiently large then the optimal value of \( r_{NG}^O \) is lower than 1. In fact, if \( \gamma \) is sufficiently close to 1 \( r_{NG}^O = 0 \).

In the baseline model integration (with full monopolization) reduces the incentives to distort traffic: \( r^O \) is higher and as a result consumers are better off. The reason is that \( G \) internalizes a higher fraction of the producer rents generated by each participating consumer, and hence \( G \) has additional incentives to attract new consumers, which requires more reliable search results. In the current set up there is a new effect that works in the opposite direction: \( G \) tends to distort more heavily the traffic to websites that remain outside of \( G \)'s system. If this latter effect becomes relatively more important, then consumers would end up being worse off under vertical integration.

It is straightforward to demonstrate such a possibility. For instance, suppose that in the case of no integration \( G \) finds it optimal to set \( r^S = r^O = 1 \). This occurs when conditions (10) and (12) both fail, which can occur, for instance, if \( \bar{\pi} \) and
(m_2 - m_1) are sufficiently small. Also, as shown above, under vertical integration with partial monopolization, if \( \gamma \) is sufficiently close to 1 then \( r_{NG}^O < 1 \) hence, independently of the values of \( r^S \) and \( r_G^O \) chosen by \( G \), consumers are worse off and participation declines.

In this particular example, the social planner also chooses \( r^S = r^O = 1 \). Consequently, total surplus under vertical integration with partial monopolization is lower than with no integration. Summarizing:

**Proposition 3** There exists a region of parameter values for which vertical integration with partial monopolization reduces both consumer and total surplus.

6 Concluding remarks

In this paper, we have constructed a model that explicitly describes the workings of markets for both search and display advertising. These two modes of advertising are imperfect substitutes for merchants. As a result, a monopoly search engine may have incentives to distort, not only sponsored search (in favor of merchants with higher willingness to pay but lower consumer relevance), but also organic search in order to reduce the effectiveness of display advertising and raise the value of sponsored search.

We have also shown how, in a scenario with symmetric publishers, if the monopoly search engine owns an intermediary in the display advertising market then it may be able to monopolize the entire market. In this context, the incentives to distort organic search disappear and consumer participation as well as total surplus increase. However, if there exist any obstacles to full monopolization then the search engine finds it optimal to distort organic search towards its affiliate publishers. In fact, we argue that the latter effect may dominate so that integration between the search engine and an ad intermediary results in lower consumer surplus as well as lower total welfare.

Several important further issues are missing from the current draft of the paper.
First, the search engine may own some publishers. It may then be able to directly manipulate the prices of display advertising. Second, integration between the search engine and an ad intermediary or a publisher may facilitate “behavioral targeting”, where publishers learn the type of consumers visiting their websites and can show different display ads contingent on consumer type. Third, in general environments, contextual, semantic and behavioral targeting can all increase the effectiveness of display advertising, with possible consequences for the tradeoffs that we have depicted. Fourth, asymmetries in the advertising effectiveness of different types of publisher generate additional distortions that can be more severe under (monopolizing) integration. Fifth, implications for publishers’ incentives to invest in quality are already visible, but merit further investigation.
Proof of Proposition 1

Existence: We first show that the proposed outcomes are part of an equilibrium. Given \( a \) and \( q \), and given that all other merchants accept the publishers’ offers and bid according to \( b_k = [1 - \alpha e (r^O)] m_k \), consider merchant \( j \)’s best response. If \( j \) rejects some of the publishers’ offers, say a proportion \( \sigma \), then its willingness to pay for sponsored advertising is higher than any other merchant’s, and so its best bidding will guarantee all wins in position auctions. On the other hand, of the \( X (1 - \phi) J \) consumers who would have demand for its goods and could not be directed by \( G, j \) will only meet a proportion \( (1 - \sigma) \). Thus, \( j \)’s profits will be

\[
\Pi_j(\sigma) = \frac{X \phi}{J} e (r^O) \bar{\pi} (m_1 + m_2) - (1 - \sigma) \left[ \frac{X (1 - \phi)}{J} e (r^O) \bar{\pi} (m_1 + m_2) - qN \right],
\]

which for \( q = \frac{X (1 - \phi)}{JN} e (r^O) \bar{\pi} (m_1 + m_2) \) is independent of \( \sigma \). Thus, \( \sigma = 0 \) and consequently \( b_k = [1 - \alpha e (r^O)] m_k \) are best responses. Now assume that a publisher deviates. It may offer all merchants a \((a, q)\) with \( a \neq 1 \) and/or \( q \) different to the value in the text of the proposition. A lower price cannot be a profitable deviation, since demand is already maximal at the proposed equilibrium outcome. Likewise, \( a \neq 1 \) cannot be a profitable deviation, since it can only lower the price that can be charged to merchants, never increase it. Thus, let one publisher offer each merchant a contract \((a = 1, q')\) with \( q' > \frac{X (1 - \phi)}{JN} e (r^O) \bar{\pi} (m_1 + m_2) \). We need computing the equilibrium reaction by merchants. We propose that only \( J_1 \) merchants will accept this offer, whereas all merchants still accept all other offers. If one of these \( J_1 \) merchants rejects the offer, then it will tie with \( J - J_1 \) other merchants in paid for placement auctions, and bid higher than \( J_1 - 1 \) other merchants. Indeed, \( J_1 - 1 \) merchants have lower willingness to pay, \((1 - e (r^O) \bar{\pi}) m_k\), whereas the rest have the same willingness to pay, \((1 - \frac{N-1}{N} e (r^O) \bar{\pi}) m_k\), for a position.
Then it gets profits

\[
\frac{X\phi}{J} \left[ \frac{J_1 - 1}{J - 1} e^{(r^O) \frac{\alpha}{\bar{\alpha}}} + \left( 1 - \frac{J_1 - 1}{J - 1} \right) \frac{N - 1}{N} e^{(r^O) \frac{\alpha}{\bar{\alpha}}} \right] (m_1 + m_2),
\]

(18)

where we are using the fact that \( q = \frac{X(1 - \phi)}{JN} e^{(r^O) \frac{\alpha}{\bar{\alpha}}} (m_1 + m_2) \) for all offers accepted, and also the fact that when tying the auction the merchant obtains the same profits winning and loosing the auction. If the merchant rejects more offers, the profits would still be given by (18): it would win all auctions, and the profits in a fraction \( \frac{J_1 - 1}{J - 1} \) of them would be \( e^{(r^O) \frac{\alpha}{\bar{\alpha}}} \) and in the rest they would be \( \frac{N - 1}{N} e^{(r^O) \frac{\alpha}{\bar{\alpha}}} \). If the merchant accepts all offers, then its profits are

\[
\frac{X\phi}{J} e^{(r^O) \frac{\alpha}{\bar{\alpha}}} (m_1 + m_2) + \frac{X (1 - \phi)}{JN} e^{(r^O) \frac{\alpha}{\bar{\alpha}}} (m_1 + m_2) - q'.
\]

(19)

Again, we use the value of \( q \) to get this expression, and the fact that the merchant will either tie or loose in all auctions. For the merchant not to prefer rejecting the offer, we then need

\[
\frac{X}{JN} (m_1 + m_2) e^{(r^O) \frac{\alpha}{\bar{\alpha}}} \left( 1 - \phi \frac{J_1 - 1}{J - 1} \right) \geq q'.
\]

Now if a merchant that is buying only from the inexpensive merchants also buys from the expensive one, then its profits will be (19), again, instead of

\[
\frac{X\phi}{J} \left[ \frac{J_1 - 1}{J - 1} e^{(r^O) \frac{\alpha}{\bar{\alpha}}} + \left( 1 - \frac{J_1 - 1}{J - 1} \right) \frac{N - 1}{N} e^{(r^O) \frac{\alpha}{\bar{\alpha}}} \right] (m_1 + m_2).
\]

Thus, for the merchant not to have incentives to deviate, it should be satisfied that

\[
\frac{X}{JN} (m_1 + m_2) e^{(r^O) \frac{\alpha}{\bar{\alpha}}} \left( 1 - \phi \frac{J_1 - 1}{J - 1} \right) \leq q'.
\]

Thus, if we define \( J_1 \) as the integer that satisfies both expressions, and treating this as a real number, the profits of a deviating publishers are bounded above by

\[
\frac{X}{JN} (m_1 + m_2) e^{(r^O) \frac{\alpha}{\bar{\alpha}}} \left( 1 - \phi \frac{J_1 - 1}{J - 1} \right) J_1,
\]

whose derivative with respect to \( J_1 \) is proportional to

\[
J - (1 - \phi) - 2\phi J_1,
\]
positive for all values of \( J_1 \), since \( \phi \leq \frac{1}{2} \). We should still consider other deviations by publishers, like offering \((a', q')\) to less than \( J \) publishers. Note that there is no reason to offer \( a' \neq 1 \), even in this case. Also, offering \( q' < 1 \) could not be a profitable deviation. Finally, if \((1, q')\) is offered to more than \( J_1 \) merchants, where \( J_1 \) is obtained as above, then there would be still equilibrium if only \( J_1 \) accept that offer. If the offer is made to less than \( J_1 \), then the profits for the publisher could be no higher than when the offer is made to \( J_1 \), and so could not be profitable either. Finally, assume the publisher makes different offers to different merchants, \((a_j', q'_j)\). Order from lower to higher all prices \( q'_j \) and for each of them, compute the value \( J_1(q'_j) \) obtained above. There is a price \( q' \) among the \( q'_j \)'s such that no more than \( J_1(q') \) received an offer with price below that one. Then, along the lines of the proof above, an equilibrium exists where all merchants receiving an offer with a price lower or equal to \( q' \) accept and all the rest reject. The profits for the publisher are lower than \( J_1(q')q' \), and so the deviation is not profitable either.

**Uniqueness:** Assume there is another symmetric, pure strategy equilibrium where all publishers offer a contract \((a', q')\). The contract must then be accepted. Otherwise, the profits of all trivial deviations would give positive profits for deviants. But if all offers are accepted, then the willingness to pay of each of the merchants is

\[
q(a') = \frac{X_{rO}}{\phi} \alpha(a') (m_1 + m_2). \tag{1}
\]

(Note that \( q(a') \) attains a maximum at \( a' = 1 \).) Thus, the equilibrium could have \( q' \) no higher than that. Thus, we need only rule out equilibrium offers \((a', q')\) with \( a' \neq 1 \) and/or price \( q' < q(a') \).

Consider such strategy by all publishers, and assume that a publisher deviates and offers \((1, q(1) - \varepsilon)\), for arbitrary small \( \varepsilon \). We show that all merchants accept. Indeed, assume a merchant rejects the offer (with some probability). If this merchant deviates, bids for positions exactly as before, but accepts the offer (with probability 1), then it will win the auctions with the same probability as before, pay the same prices, and face the same traffic in all publishers except the deviant. The profits obtained through traffic at this publisher will now be positive, since
they will be, at least,

\[
\frac{X (1 - \phi)}{J} e (r^O) \alpha (1) (m_1 + m_2) - (q(1) - \varepsilon) > 0.
\]

Thus, the offer will be accepted by all merchants, and so the deviation is profitable for the publisher, since there always exists an \(\varepsilon\) such that \(q(1) - \varepsilon > q(a')\) whenever \(q(a') < q(1)\). ■

Proof of Proposition 2.

Consider the first order condition for optimal \(r^S\) under vertical integration,

\[
H(\bar{c}) w \left\{ \phi \left[ 1 - \alpha e (r^O) \right] M (r^S) + (1 - \phi) e (r^O) \alpha (m_1 + m_2) \right\} - \phi \left[ 1 - \alpha e (r^O) \right] (m_2 - m_1) = 0.
\]

We can write this condition as

\[
H(\bar{c}) w \frac{M (r^S)}{m_2 - m_1} + \frac{(1 - \phi) \alpha e (r^O) (m_1 + m_2)}{\phi \left[ 1 - \alpha e (r^O) \right] (m_2 - m_1)} = 1. \tag{20}
\]

Under vertical separation the first order condition with respect to \(r^S\) is

\[
H(\bar{c}) w \phi \left[ 1 - e (r^O) \alpha \right] M (r^S) - \phi \left[ 1 - \alpha e (r^O) \right] (m_2 - m_1) = 0,
\]

which can be written as

\[
H(\bar{c}) w \frac{M (r^S)}{m_2 - m_1} = 1. \tag{21}
\]

Now we show that \(\bar{c}\) in (20) cannot be smaller than \(\bar{c}\) in (21). Assume otherwise. Then, on the one hand, since \(r^O = 1\) in (20), this requires that \(r^S\) is larger in (21) as well, and so \(M (r^S)\) to be smaller in (21). On the other hand, since \(H(\bar{c})\) is decreasing, \(H(\bar{c}) w\) would be also smaller larger in (21). Thus, since the extra term in (20) is positive, we would have that \(H(\bar{c}) w \frac{M (r^S)}{m_2 - m_1} < 1\) in (21). This implies that there could be no interior (in \(r^S\)) solution under vertical separation with more consumer participation and surplus than under vertical integration. Corner solutions with this property could only occur when \(r^S = 1\) under vertical separation or \(r^S = 0\) under vertical integration. In the first case, the left hand
side in (21) is (weakly) larger than 1, and by a similar argument so would be the left hand side in (20), which implies that $r^S = 1$ also under vertical integration. In the second case, the left hand side in (20) would have to be smaller than 1 at $r^S = 0$, and again by a similar argument so would be the left hand side in (21). Since the objective function is continuous in both cases and the optimization is over a compact set, a solution does exist, and so it has to involve (weakly) larger participation under vertical integration.

We can write producer surplus as:

$$PS = X \left\{ \phi M \left( r^S \right) + \alpha e \left( r^O \right) \left( m_1 + m_2 \right) \right\}$$

Let us now consider $(r^O, r^S, X)$ as independent variables. Note that $PS$ increases with $r^O$ and $X$ and decreases with $r^S$. Therefore, if integration involves a lower value of $r^S$ then $PS$ would be larger.

Alternatively, suppose that $r^S$ is higher after integration. Consider the values of $(r^O, r^S)$ that maximize producer surplus. First, $\frac{\partial PS}{\partial r^O} > 0$, and hence the optimal value of $r^O$ is equal to 1. Second, the first order condition with respect to $r^S$ can be written as:

$$H \left( \tau \right) w \left\{ \frac{\phi M \left( r^S \right) + \alpha e \left( r^O \right) \left( m_1 + m_2 \right)}{\phi \left[ 1 - \alpha e \left( r^O \right) \right] \left( m_2 - m_1 \right)} \right\} = 1 \ (22)$$

Since $\frac{\partial^2 PS}{\partial r^S \partial r^O} < 0$, if we compare (20) and (22), evaluated at $r^O = 1$, we conclude that the equilibrium value of $r^S$ under vertical integration is below the level that maximizes $PS$. Moreover, any increase in the value of $r^S$ starting at a level lower than the one that satisfies condition (22) and for $r^O = 1$, results in higher $PS$. Hence, if vertical integration implies a higher value of both $r^O$ and $r^S$ then this should also increase $PS$. First, starting from the equilibrium values of $(r^O, r^S)$ in the case of separation, if we raise $r^O$ to 1 then, since $\frac{\partial PS}{\partial r^O} > 0$, $PS$ increases. Second, starting at $r^O = 1$ and the equilibrium value of $r^S$ under separation, if we increase $r^S$ to the equilibrium level under integration, then $PS$ increases again. ■

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Proof of Proposition 3.

For this to be an equilibrium, merchants should have no incentive to deviate. The proof that this is the case follows exactly the same argument as in the proof of Proposition 1. Also, for an argument as before, and given acceptance of G offers by all publishers offered, no publisher has an incentive to deviate. Indeed, only \( (a = 1, q') \) deviations, with \( q' > q_d \) for the corresponding index \( d \), have a chance to be profitable. The same merchants’ behavior, with only substituting \( E (r_G^O, r^O_{NG}) \) or \( e_d (r_d^O) \) for \( e (r^O) \) (and \( E (r_G^O, r^O_{NG}) - \frac{1}{N} e_d (r_d^O) \) for \( \frac{N-1}{N} e (r^O) \) in the derivations), constitutes an equilibrium in the continuation game after such deviation. Then, the profits from such deviation will be bounded above by

\[
\frac{X}{JN} (m_1 + m_2) e_d (r_d^O) \alpha \left( 1 - \phi \frac{J_1 - 1}{J - 1} \right) J_1,
\]

and as before, for \( \phi \frac{J_1 - 1}{J - 1} > \frac{1}{2} \) such deviation is not profitable. Thus, we only need considering deviations where a publisher changes intermediary and/or the offer it makes. Only publishers who are offered the choice to join G can deviate in this sense. Assume it rejects G’s offer. If it offers price \( q_{NG} \) and \( a = 1 \), then it is easy to see that the same behavior, where every merchant accepts all offers and bids accordingly, is still an equilibrium behavior, but the profits of the deviant do not change, and so the deviation is not profitable. Likewise, any deviation with \( a \neq 1 \) and \( q < q_{NG} \) could not be profitable. Thus, assume that the publisher offers \( a = 1 \) and \( q > q_{NG} \). We can replicate the same derivation of an equilibrium for the continuation game, as above, except that now we substitute

\[
\hat{E} (r_G^O, r^O_{NG}) \equiv \left( \gamma - \frac{1}{N} \right) e_G + \left( 1 - \gamma + \frac{1}{N} \right) e_{NG}
\]

for \( E (r_G^O, r^O_{NG}) \). That defines the equilibrium number of acceptants of the publisher’s offer, and shows that the optimal price upon rejection of G’s offer is still \( q_{NG} \) and therefore the deviation is not profitable. ■


8 References


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