Robust Optimization in Simulation: Taguchi and Krige Combined

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Simulation-optimization aims to identify the setting of the input parameters (input variables, factors) of the simulated system that leads to the optimal system performance. In practice, however, some of these parameters cannot be perfectly controlled due to measurement errors or other implementation issues, and the inherent uncertainty caused by fluctuations in the environment (e.g., demand). Consequently, the classic optimal solution may turn out to be either sub-optimal or infeasible.

The solution is Robust Optimization (RO), which derives solutions that are relatively insensitive to perturbations caused by the so-called noise factors. Our research proposal focuses on achieving RO control and noise factors — denoted by $d_i$, $j = 1, \ldots, m$ — respectively; the outputs of interests are denoted by $f_1, \ldots, f_k$.

**Figure 1** shows a black-box representation of a simulation model of a system, whose inputs are both control and noise factors — denoted by $d_i$, $i = 1, \ldots, n$, and $\xi_j$, $j = 1, \ldots, m$ respectively; the outputs of interests are denoted by $f_1, \ldots, f_k$.

**Taguchi RO** simple and useful approach to solve robust simulation-optimization problems, which does not require any strong assumptions about the uncertain parameters (see, for example, Beyer and Sendhoff [1]).

**Kriging** metamodelling technique which may be applicable in RO context, providing good approximations of highly nonlinear functions.

At least two Kriging metamodels are considered, namely one for the expected (mean) main performance function and one for its variance caused by the noise factors. During the iterative RO process, we need to consider the updating and validation of these metamodels. Our simplest RO solves a constrained stochastic optimization problem, namely minimizing (or maximizing) the expected main objective function, such that the variance does not exceed a user-defined threshold. Due to the discrete nature of the decision variables, our RO applies Nonlinear Integer Mathematical Programming to the Kriging metamodels, see Kleijnen et al. [5] and also Biles et al. [2].

**Figure 2** sketches the main steps of the proposed approach.

**Method**

1. Initial experimental design for control and noise factors
2. Simulation model evaluation on the design
3. Data set for metamodels construction
4a. Kriging Metamodel (MM)
   4b. Validation of the main MM
   5a. Validation of the variance MM
   6. Adding new design points to improve MM validity
5. Kriging Metamodel (MM)
   6. Validation of the variance MM
6. Kriging Metamodel (MM)
   of the variance $\sigma^2$
   of the main performance function $\mu$
7. Validation of the main MM
8. Validation of the variance MM
9. Kriging Metamodel (MM)
   of the main performance function $\mu$
10. Kriging Metamodel (MM)
11. Validation of the variance MM
12. RO optimization
13. Validation of the main MM
14. Validation of the variance MM
15. RO optimization
16. New optimum point
17. Validation of the main MM
18. Validation of the variance MM
19. RO optimization
20. Same optimum
21. RO optimization
22. New optimum?
23. Add new optimum points to design
24. Find new optimum
25. Stop

The robust simulation-optimization approach is widely applicable in the context of inventory management. One of the simplest examples in the $(s,S)$ inventory model, which could be sketched as follows.

- **Goal**: minimize the expected steady-state total costs, given by the sum of the order setup, ordering, and holding costs
- **Control variables**: $s$ ( reorder level), $S$ ( order up-to level)
- **Noise factors**: e.g., demand
- **Box constraints**: fixing lower and upper bounds on the decision variables
- **Deterministic constraints**: establishing relationships between the decision variables
- **Stochastic constraints**: related to other outputs of interests, such as the expected steady-state fill rate (i.e. the fraction of demand directly met from stock on hand)

Looking for a robust solution of this problem means finding an $(s,S)$ policy that can minimize the expected total costs, guaranteeing at the same time a low degree of sensitiveness to the variation of parameters such as the mean demand and the various cost parameters.

**References**

