Interconnection and Competition among Asymmetric Networks in the Internet Backbone Market∗

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Abstract

We examine the interrelation between interconnection and competition in the Internet backbone market. Networks that are asymmetric in size choose among different interconnection regimes and compete for end-users. We show that a direct interconnection regime, peering, softens competition as compared to indirect interconnection since asymmetries become less influential when networks peer. If interconnection fees are paid, the smaller network pays the larger one. Sufficiently symmetric networks enter a Peering agreement while others use an intermediary network for exchanging traffic. This is in line with considerations of a non-U.S. policy maker. In contrast, U.S. policy makers prefer that relatively asymmetric networks peer.

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1 Introduction

The rapid development of e-commerce industries and the emergence of Voice over IP and Video-on-Demand services, which all rely on the Internet Protocol (IP) standard, have increased the importance of the Internet as a global medium of data exchange. Being a communications industry, the Internet is subject to network externalities. These externalities have forced Internet Backbone Providers (IBPs) to interconnect with each other in order to provide their customers with “worldwide connectivity”, hence increasing consumers’ benefits and willingness-to-pay for Internet access. From an economic perspective there are several ways to interconnect with other networks. The specific type of interconnection influences competition for end-users, and vice versa.

This paper aims to provide a general analysis of the industrial organization of an unregulated Internet backbone market, i.e. the market for interconnection among IP-networks, which also sell Internet access to end-users. We endogenize both the networks’ interconnection and their competition decisions whilst explicitly accounting for asymmetric network sizes, which are widely observed in practice. We study the following questions: What determines networks’ choices of interconnection? How do different types of interconnection affect competition for end-users? Who pays whom for interconnecting networks? Are networks’ decisions in line with welfare considerations?

We consider a new interconnection regime, Paid Peering, and find that networks that are sufficiently symmetric in size prefer it (together with the better known Bill-and-Keep Peering regime) over using an intermediary network to exchange data. For medium ranges of network asymmetry, Paid Peering dominates both alternative interconnection regimes. By choosing Paid Peering networks can raise profits in comparison to a situation where they are restricted to choosing between Bill-and-Keep Peering and IP-Transit. Only for large asymmetries do they buy IP-Transit from an intermediary network in equilibrium. Our model suggests that this interconnection behavior is not always desirable from a welfare point of view. Finally, taking into account the fact that the market for IP-Transit is dominated by U.S. carriers, a non-U.S. trade policy oriented regulator, who does not value profits of the largest IBPs, finds that there is too much Peering and seeks to restrict Peering of networks which are sufficiently asymmetric in size.
Our model has the following timing: First, two networks, which are ex ante connected via an intermediary backbone, negotiate their interconnection regime. In case of Paid Peering, they also bargain for a settlement-fee (interconnection fee or access price) that could flow in either direction. In stage two, they compete in prices for consumers with heterogeneous preferences in a Hotelling model. Finally, consumers choose the network that maximizes their net benefits.

Our results show that the initial level of asymmetry in network sizes affects equilibrium outcomes: the larger the ex ante asymmetry is, the larger the profit differences between the networks when using an intermediary backbone, which in turn serve as threat points in the Nash bargaining game. As a consequence, both the settlement-fee resulting from the bargaining process and the interconnection decision reached in equilibrium depend on the degree of network asymmetry.

We obtain these results without assuming direct network externalities in the utility function of consumers. If Internet-users valued direct connection to a large network over a small network, our results would be even more pronounced.

There is a large body of literature on interconnection and two-way access pricing in telecommunications, which one might think of as being related to the Internet backbone market. Armstrong (1998) and Laffont et al. (1998) constitute two fundamental works, while Vogelsang (2003) provides a comprehensive survey of this literature. However, there are two crucial differences that make an adoption of the analysis on the telecommunications market to the Internet backbone highly problematic: First, interconnection in the Internet backbone is not subject to regulation. Cash flows associated with interconnection on the Internet do not depend on the direction of traffic but may be negotiated freely in the market.\(^1\) Second, destination based price discrimination is usual in telecommunications, while it is practically impossible on the Internet.\(^2\)

There is also a more recent theoretical literature on telecommunications relaxing these industry specific restrictions: Carter and Wright (2003), Armstrong (2004),

\(^1\)In the telecommunications industry there exist various regulatory schemes around the globe, which rule network interconnection. Moreover, policy makers often require termination charges or “access charges” to be set reciprocally.

\(^2\)It is standard for consumers to pay more for long-distance or international phone calls than for local calls. To imitate such price discrimination on the Internet, a consumer would have to be asked before each click on a Web link whether she would be willing to pay a specific price depending on the network distance to a specific target Web site’s location.
Gilo and Spiegel (2004) and Peitz (2005) study competitively chosen asymmetric access prices, asymmetric networks or IP-Transit as an outside option when negotiating the terms of interconnection. Our paper is the first to unify all three issues in one model.

Focusing on the Internet, Laffont et al. (2003) study the strategic behavior of backbone operators in an environment of reciprocal access pricing in two-sided markets. Mendelson and Shneorson (2003) extend this framework to consumer delay costs and capacity decisions. Contrarily, because of already existing world-wide connectivity we abstract from network externalities in consumers’ utility functions. Because of the unregulated nature of the Internet backbone, we let networks negotiate access prices freely.

Using a model of price competition, Giovannetti (2002) shows that the introduction of competition for Transit services may lead to fiercer competition in original areas of the Internet and thereby lower access and retail prices. Crémer et al. (2000) analyze in a Cournot model (thus endogenizing capacity) whether dominant network operators have incentives to lower the interconnection quality to rival networks. By extending the Katz and Shapiro (1985) network competition model they show that a network with a large installed base of customers is likely to degrade its interconnection quality with smaller networks.3 However, nowadays there is excess capacity all over the backbone market,4 and the marginal costs of data transmission are virtually zero.5 Instead of modelling competition based on capacities we take excess capacity as given and focus on price competition with differentiated products in the retail market against the background of (exogenous) competition in the Transit market. Instead of competition based on quality of interconnection we assume perfect transmission quality, which is due to the existing world-wide connectivity and the absence

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3 Foros and Hansen (2001) also study interconnection quality and competition between IBPs but derive opposing results concerning the development of market shares. Roson (2002) provides a more thorough discussion of Crémer et al. (2000) and that article. Foros et al. (2005) analyze interconnection in a two-stage game where networks first decide about interconnection quality and compete in quantities thereafter.

4 Telegeography, a consultancy, notes: “Despite significant and consistent growth in data traffic flows across the world’s communications networks, a huge portion of potential network capacity remains unused. [...] only three percent of the maximum possible intercity bandwidth in Europe and the U.S. has been ’lit’ for service provision.” (http://www.telegeography.com/press/releases/2005-04-20.php)

or bottlenecks, and let networks choose amongst several interconnection regimes.\textsuperscript{6}

The papers connected closest to our’s are Baake and Wichmann (1999) and Besen et al. (2001) in the sense that they also endogenize the choice of IBPs’ interconnection regime. The former studies the Transit vs. Peering decision in the context of quality differentials, while the latter provides a bargaining process of Peering partners (implicitly introducing the option for Paid Peering). Neither considers the effects on competition for end-users. To the best of our knowledge, our paper is the first to attempt to endogenize both networks’ interconnection and competition decisions among asymmetric networks whilst taking into account the economic differences between the Internet backbone and telecommunications markets.

The paper is organized as follows. Section 2 describes the most widely used interconnection regimes in more detail. Section 3 introduces the model. Section 4 analyzes networks’ equilibrium prices, market shares and profits in the retail market while distinguishing Intermediary and Bill-and-Keep Peering regimes. Section 5 examines the incentives to peer and finds interconnection equilibria. Section 6 takes a welfare perspective. Section 7 discusses the robustness of our results to relaxing some of the central assumptions of this model, while section 8 concludes.

2 Interconnection Practice in the Internet Backbone Market

The United Nations Conference on Trade and Development (UNCTAD) mentions in its Information Economy Report 2005 (p.93) that over 300 operators were providing commercial backbone services at the end of 2004. According to the report, the “broader network services industry sales are estimated at about $1.3 trillion worldwide. [...] Of the 300 backbone networks mentioned before, the top 50 carry nearly 95 per cent of all IP traffic, and only five of them can be considered to have a truly global presence.” These numbers suggest that IP-networks are heterogenous in terms of size, i.e. in terms of traffic volume or subscribers, which could have an impact on interconnection practice.

How does traffic get from consumer 1 to consumer 2? Suppose 1 and 2 are

\textsuperscript{6}In the Internet backbone, excess capacity leads to virtually perfect quality of interconnection.
communicating via the Internet and consumer 1 (2) is connected to network A (B). The networks have two main options to exchange traffic, Transit and Peering.

**IP-Transit/Intermediary:** If a direct connection is not feasible or desirable, two networks can buy so-called Transit services from a third network. Under such an arrangement each network pays a variable charge per unit of traffic to the intermediary network which commits to deliver the traffic to any specified destination and from a given origination. For being able to fulfill this obligation, networks offering Transit mostly have a large physical network and are connected to many other networks via Peering or further Transit sales. The IP-Transit market is dominated by so-called Tier-1 networks which are mainly U.S. based.\(^7\)

**Peering:**\(^8\) *Bill-and-Keep Peering*, also called settlement-free Peering, has evolved as the regular type of direct interconnection regime between two networks since privatization of the Internet. Networks exchange traffic without charging any fees to each other. However, under such a Peering agreement no participating network has the obligation to terminate traffic to or from a third party. Each network must only process traffic from the Peering partner to its own customers (and the customers of their customers and so on), but not to the remainder of the Internet. This constitutes a major difference between IP-Transit and Peering. In our example, consumers 1 and 2 can exchange traffic without causing any interconnection costs to the networks they have subscribed to if those networks are peering.

A *Paid Peering* regime between two networks implies the same rights concerning their exchange of traffic. In contrast to Bill-and-Keep, one network charges the other for exchanging traffic. We may emphasize that Paid Peering is a relatively new type of interconnection regime and has only recently begun to be employed.\(^9\) In our example, suppose network A agreed to pay for traffic exchange with network B, thereby forming a Paid Peering interconnection regime. In this environment it has no impact on the stream of money whether 1 sends an e-mail to 2, or vice versa: in

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\(^7\)A network is regarded to have Tier-1 status if it is connected to the whole Internet while never paying for interconnection itself.

\(^8\)In the industry, there is a difference between “Private Peering”, where exactly two networks build or lease lines to interconnect, and “Public Peering” where several networks interconnect their lines in a node, a so-called Internet Exchange Point. As economic differences are not very significant and more and more networks use Private Peering, we only consider this type in our model. See Kende (2000) for more details.

\(^9\)A Paid Peering settlement could appear in several different forms of payment, either fixed amount payments or a variable charge per unit of traffic (or a combination of both).
both cases network A will pay B. However, since Paid Peering is no Transit contract, network B will not proceed traffic from A to a third party that is not a customer of B.\footnote{For more details on Internet traffic, see Giovanetti and Ristuccia (2005) or Kende (2000).}

\section{The Model}

There are two networks $i \in \{A, B\}$ each having a fixed installed base of $\alpha_i$ customers that is not subject to competition.\footnote{Internet Service Providers selling Internet access to those consumers are vertically integrated.} Without loss of generality we assume $\alpha_A \geq \alpha_B$. On top, $\bar{\alpha}$ consumers are situated in a \textit{battlezone}, where networks A and B compete in prices.\footnote{The most intuitive explanation is geographic: the locked customers of network $i$ can only be directly connected to network $j$ for prohibitively high costs, e.g. because they live in a rural area. The battlezone, however, consists of consumers living in large cities where both networks have a point of presence (POP). Another interpretation is that A and B compete in new services, e.g. Voice over IP, in the battlezone but also have legacy customers who are not interested in such services.} We assume excess capacity on the part of the networks so they can serve battlezone consumers without extra investments. Ex ante both networks are connected to the remainder of the Internet by using an Intermediary, thereby offering their customers world-wide connectivity.\footnote{In line with this, we model no quality differentials among Peering and Transit, unlike Crémon et al. (2000) or Baake and Wichmann (1998), since, according to industry representatives, there is no clear relationship between interconnection quality and regimes. Consequently, demand-side network effects do not play a role in the model since customers enjoy world-wide connectivity on a constant quality level regardless of the networks’ interconnection decision or competition.} As there is Bertrand price competition in the market for IP-Transit, we assume the Intermediary to be the cheapest Tier-1 network available, by definition offering access to all remaining consumers connected to the Internet, $\kappa$.\footnote{In our model we do not cover competition where one of the two networks has Tier-1 status. Therefore, we do not endogenize the Intermediary’s price of IP-Transit. See Prüfer and Jahn (2007) for a discussion of the influence of Bertrand competition on the Internet backbone industry’s outlook and market structure.} It is not relevant whether the Intermediary directly serves the $\kappa$ consumers as ISP or connects other networks’ consumers via its backbone to networks A and B. There is a continuum of consumers, of mass 1, so $\alpha_A + \alpha_B + \bar{\alpha} + \kappa = 1$. Figure 1 shows the competition set-up.

Networks’ cost structure:

- Networks face an exogenous market price for upstream Transit, $t$, per unit of
Figure 1: Network interconnection via an Intermediary

- Technical marginal costs of sending data are zero.\(^\text{15}\)
- Costs of connecting customers to a network within the battlezone are symmetric and, for simplicity, normalized to zero.
- In case of a Peering arrangement, each network bears fixed cost \(F > 0\).\(^\text{16}\)

Since top-level backbones do not charge different fees for upstream and downstream traffic, we assume that each consumer sends one unit of data to each other consumer and receives one unit of data from each other consumer, thereby not taking into account which network the other consumer is connected to (balanced calling pattern). This yields every consumer a gross benefit, \(v\). Finally, we assume that prices \(p_i^L\) in the locked areas are not affected by competition in the battlezone, where both networks charge every customer a price \(p_i\).

The timing of the game is as follows:

1. Networks A and B decide non-cooperatively about the interconnection regime between them, Intermediary, Bill-and-Keep Peering or Paid Peering. Without

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\(^{15}\)Refer to the literature mentioned in footnote 5. We discuss this assumption in section 7.

\(^{16}\)\(F\) encompasses all fixed-step costs for setting up a physical interconnection, buying routers, etc. and organizational costs for managing a Peering agreement.
agreement both are forced to use the Intermediary. In case of Paid Peering, networks bargain for a fixed settlement which may flow in either direction.

2. Networks A and B set prices $p_i$ for consumers in the battlezone and compete in a Hotelling-like environment.\textsuperscript{17}

3. Consumers in the battlezone choose the network that maximizes their net benefits.

We begin by characterizing the equilibrium retail profits under Bill-and Keep Peering (BK) and Intermediary. We then distinguish between BK and Paid Peering (PP). Finally we compare the profits under the three regimes to reveal the incentives for choosing one over the other.

4 Retail Competition Analysis

4.1 The Intermediary Regime

Consider a standard Hotelling (1929) model. Consumers are indexed by $x$ and uniformly distributed on the interval $[0, 1]$ with increasing preference for network B. The network differentiation parameter (transportation cost parameter) is $\tau > 0$, so that a consumer’s utility function is given by

$$U = \begin{cases} 
  v - \tau x - p_A & \text{if buying from network A} \\
  v - \tau (1 - x) - p_B & \text{if buying from network B} \\
  0 & \text{otherwise.}
\end{cases}$$

(1)

We assume that $v$ is sufficiently large such that the market is covered. The marginal consumer, who is indifferent between A and B, is denoted by:

$$\hat{x} = \frac{1}{2} + \frac{p_B - p_A}{2\tau}.$$  \hspace{1cm} (2)

\textsuperscript{17}Consumer heterogeneity could depend on different complementary services offered by the networks, e.g. specific Web content or software applications certain consumers are already used to. Note that heterogeneity refers to the retail market of Internet access, while data exchange between networks is a homogenous good.
Note that $\hat{x}$ also specifies A’s market share within the battlezone, while $(1 - \hat{x})$ is B’s battlezone market share. Profit functions under the Intermediary regime are given by:

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$$
\Pi^I_A = \hat{x}\bar{\alpha}(p_A - 2\kappa t) + \alpha_A(p_A^L - 2\kappa t) - 2t(\hat{x}\bar{\alpha} + \alpha_A)((1 - \hat{x})\bar{\alpha} + \alpha_B),
$$

$$
\Pi^I_B = (1 - \hat{x})\bar{\alpha}(p_B - 2\kappa t) + \alpha_B(p_B^L - 2\kappa t) - 2t(\hat{x}\bar{\alpha} + \alpha_A)((1 - \hat{x})\bar{\alpha} + \alpha_B).
$$

The first term of each function describes a network’s direct profits from customers in the battlezone net of Transit costs which stem from sending data to or receiving data from customers of the other network. The second term denotes the same for its locked customers, while the third term adjusts for the traffic that is exchanged between A and B. This term has to be paid to the Intermediary by each network, is of equal size for both firms and becomes a driver of the main results of this model. Note that traffic has to be paid twice for each consumer since we have assumed that all consumers both send data to and receive data from all other consumers.

Second-order-conditions are satisfied and the slopes of the reaction functions are between zero and one for $\tau > 2\bar{\alpha}t$. We assume henceforth that this condition holds. Equilibrium prices are given by:

$$
p^*_A = \tau(1 - z\Delta) + 2\kappa t
$$

$$
p^*_B = \tau(1 + z\Delta) + 2\kappa t,
$$

where $\Delta \equiv \alpha_A - \alpha_B \geq 0$ and $z \equiv \frac{2t}{(3\tau - 4t\bar{\alpha})} > 0$. Hence, A’s equilibrium market share is:

$$
\hat{x} = \frac{1}{2} + z\Delta.
$$

Equation (7) implies that, to receive interior solutions for $\hat{x}$ such that $\hat{x} \in [0, 1]$, it is necessary that $\Delta \leq \frac{1}{2z} \equiv \Delta_{\text{max}}$. If $\Delta > \Delta_{\text{max}}$, the larger network’s low pricing drives out the smaller network from the battlezone market. A and B would consequently keep using the Intermediary for exchanging traffic. Henceforth, we restrict our analysis to $\Delta \in [0, \Delta_{\text{max}}]$.

\[18\]

We assume that networks are able to discriminate prices between locked consumers and the battlezone. If that was not possible, as $\alpha_A \geq \alpha_B$, there would be no price Nash equilibrium in pure strategies. Because of this and actual usage of price discrimination based on the sender’s—not the receiver’s—location in the Internet, we restrict our analysis to this case.
To facilitate further analysis, let us define \( w \) as the amount of data A and B exchange if they split the battlezone equally, i.e. if \( \hat{x} = \frac{1}{2} \):

\[
w \equiv 2(\alpha_A + \frac{\bar{\alpha}}{2})(\alpha_B + \frac{\bar{\alpha}}{2}). \tag{8}\]

We can express equilibrium profits under the Intermediary regime as:

\[
\Pi^I_A = \frac{1}{2} \tau \bar{\alpha}(1 + z\Delta - 2z^2\Delta^2) + z^2\Delta^2\bar{\alpha}(3\tau - 2t\bar{\alpha}) - tw + \alpha_A(p^L_A - 2\kappa t), \tag{9}
\]

\[
\Pi^I_B = \frac{1}{2} \tau \bar{\alpha}(1 - z\Delta - 2z^2\Delta^2) + z^2\Delta^2\bar{\alpha}(3\tau - 2t\bar{\alpha}) - tw + \alpha_B(p^L_B - 2\kappa t). \tag{10}
\]

It is obvious that A’s direct profits from the battlezone, \( \frac{1}{2} \tau \bar{\alpha}(1 + z\Delta - 2z^2\Delta^2) \), increase while B’s direct profits decrease with growing asymmetry \( \Delta \). Furthermore, total Transit costs of each network, \( tw + \alpha_i2\kappa t - z^2\Delta^2\bar{\alpha}(3\tau - 2t\bar{\alpha}) \), are maximized for symmetry \((\Delta = 0)\). We find:

**Lemma 1** Under the Intermediary regime, network A prices more aggressively, and obtains a higher market share and larger profits in the battlezone than B.

The key to understanding this Lemma is that Transit payments of A and B to the Intermediary decrease with growing network asymmetry. Thus, the larger network A has a greater incentive to increase its market share than the smaller one: if A sold to the marginal consumer, its income would increase and its Transit costs would decrease. B faces an extra trade-off: when acquiring a marginal customer within the battlezone, its income would increase, but the corresponding Transit costs would increase as well. Therefore, A’s marginal profit from acquiring another customer is larger than B’s, making A more aggressive. Similarly, A’s ex post profits increase with growing ex ante asymmetry, which also minimizes both networks’ Transit payments since more traffic is exchanged ”on-net”, i.e., if sender and receiver are customers of the same network.

### 4.2 Bill-and-Keep Peering

If networks peer with each other, their profit functions show two differences in relation to the case without Peering: Peering’s upside is that networks do not have to
pay the Intermediary for traffic that is exchanged solely between the two networks involved. Its downside is that the Peering partners have to set up direct lines, buy new equipment such as routers and have to bear Peering management costs. All these types of costs are compiled in the variable $F$, which is not, according to various industry talks, correlated with either the network size or the amount of traffic transmitted.

This leads to the following profit functions under Peering:

$$
\Pi^P_A = \hat{x}\bar{\alpha}(p_A - 2\kappa t) + \alpha_A(p^L_A - 2\kappa t) - F, \quad (11)
$$

$$
\Pi^P_B = (1 - \hat{x})\bar{\alpha}(p_B - 2\kappa t) + \alpha_B(p^L_B - 2\kappa t) - F. \quad (12)
$$

Equilibrium prices, market shares, and profits can be characterized as:

$$
p^*_A = \tau + 2\kappa t = p^*_B \quad (13)
$$

$$
\hat{x} = \frac{1}{2} \quad (14)
$$

$$
\Pi^P_A = \frac{1}{2}\tau\bar{\alpha} + \alpha_A(p^L_A - 2\kappa t) - F \quad (15)
$$

$$
\Pi^P_B = \frac{1}{2}\tau\bar{\alpha} + \alpha_B(p^L_B - 2\kappa t) - F \quad (16)
$$

**Lemma 2** Under the Peering regime of interconnection, (i) regardless of asymmetries in installed bases, networks’ pricing behavior is symmetric. (ii) Market shares in the battlezone are symmetric. (iii) Ignoring profits from the installed bases, profits from competition in the battlezone are symmetric. (iv) If installed bases were symmetric ($\Delta = 0$), equilibrium prices and battlezone market shares would be the same under Intermediary and Peering regimes.

The intuition for (i) through (iii) is that, since under a Peering regime Transit costs for traffic between the two parties are waived, the larger network has no extra incentive to undercut the smaller one. Therefore, incentive structures, behavior and profits are symmetric. This intuition is confirmed by (iv) stating that symmetric networks always behave in the same way regardless of the interconnection regime.
5 Equilibrium Interconnection Regimes

5.1 Comparing Transit and Peering

Being aware of Nash equilibria in retail prices, we proceed to analyze incentives in the first stage of the game: When do networks wish to peer with a specific competitor? What form of Peering would prevail? We will first distinguish between Intermediary and Peering equilibria—equating “Peering” with its most flexible form, Paid Peering—and then distinguish Bill-and-Keep from Paid Peering outcomes.

To facilitate further analysis, let us define $\Pi_I$ as networks’ aggregate profits under Intermediary:

$$\Pi_I \equiv \Pi^I_A + \Pi^I_B = \tau \bar{\alpha}(1 - 2z^2\Delta^2) + 2z^2\Delta^2\bar{\alpha}(3\tau - 2t\bar{\alpha})$$

$$- 2tw + \alpha_A(p^L_A - 2\kappa t) + \alpha_B(p^L_B - 2\kappa t). \quad (17)$$

If we assume that, in line with agreeing to Paid Peering, networks bargain about a lump-sum settlement-fee before they enter retail competition, then this fee will have no allocative, only distributive effects. Hence, aggregate profits from BK will equal aggregate profits from PP and we can define both as:

$$\Pi_P \equiv \Pi^P_A + \Pi^P_B = \tau \bar{\alpha} + \alpha_A(p^L_A - 2\kappa t) + \alpha_B(p^L_B - 2\kappa t) - 2F. \quad (18)$$

Aggregate Intermediary profits are larger than aggregate Peering profits if:

$$\Pi^I - \Pi^P = 2F - 2tw + 4\bar{\alpha}z^2\Delta^2(\tau - \bar{\alpha}t) > 0. \quad (19)$$

Define $\Delta_P$ as the threshold level, where $\Pi^I = \Pi^P$ and above which $\Pi^I > \Pi^P$. We have to resubstitute $w$, as it depends on $\alpha_A$ and $\alpha_B$, in (19), which yields after rearranging:\n
$$\Delta_P \equiv \sqrt{(4\bar{\alpha}t - 3\tau)^2(-2F + (\kappa - 1)^2t) \over t(8\bar{\alpha}t + 9\tau)}. \quad (20)$$

Lemma 3 (i) Intermediary constitutes a Nash equilibrium for any level of $\Delta$. (ii) For $\Delta > \Delta_P$, Intermediary constitutes the unique Nash equilibrium. (iii) For $\Delta \leq \Delta_P$, Paid Peering constitutes a Nash equilibrium.

\footnote{See Equation (A.3) and its explanations in Appendix A.2 for more information.}
Proof: refer to the appendix.

Figure 2: LEFT: Contour plot of constant \((\Pi^I - \Pi^P)\) - levels; lighter-colored regions refer to higher levels; RIGHT: \(\Delta_P\) - curve.

Part \((i)\) of the Lemma depends on our assumption that it is only possible to deviate from the Intermediary strategy jointly. If only one network chooses the Intermediary strategy in the first stage, the other is forced to pay Transit fees too.\(^{20}\) Figure 2 supports the intuition of Lemma 3, parts \((ii)\) and \((iii)\).\(^{21}\) The LEFT panel shows that \((\Pi^I - \Pi^P)\) is increasing both in \(\Delta\) and in \(F\), which is intuitive: if \(F\), the fixed cost of the Peering regimes, grows larger, Peering gets less attractive as compared to Intermediary. Similarly, as explained below Lemma 1, if the asymmetry between networks A and B increases, i.e. if \(\Delta\) grows, the total cost to be paid to the Intermediary decreases and the Intermediary regime becomes more attractive. The RIGHT panel depicts the one curve of the left panel where \((\Pi^I - \Pi^P) = 0\), i.e., it shows \(\Delta_P\). Above this curve, the Intermediary regime is more attractive than any Peering regime—and, consequently, a unique Nash equilibrium. Below this curve,

\(^{20}\)This implies that we could use a stronger equilibrium concept, *equilibrium in weakly dominant strategies*, to rule out (Intermediary, Intermediary) as an equilibrium strategy for all \(\Delta < \Delta_P\). For the sake of consistency with stage 2 of the game, however, we stay with (subgame-perfect) Nash equilibrium as our solution concept.

\(^{21}\)For details of the numerical example used see appendix A.2.
Peering is more attractive than Intermediary given that excess profits can be split between A and B via side-payments without frictions, which we assume in the Paid Peering regime.

Note that, to yield the results presented here, it is not necessary to specify the Paid Peering settlement-fee. It is only helpful to assume that this fee—or access charge—unlike in most papers on interconnection in telecommunications, is of a lump-sum type, not a per unit of data fee.\textsuperscript{22} Given this we can expect that individual profits from the Intermediary case serve as threat points in the bargaining process while only “excess” profits \( (\Pi_P - \Pi_I) \) are shared according to some bargaining rule.

5.2 Comparing Bill-and-Keep with Paid Peering

Assume \( \Delta \leq \Delta_P \). We showed that Paid Peering constitutes a Nash equilibrium in this range. What about Bill-and-Keep? It facilitates further analysis if we first characterize the networks’ relative individual incentives to accept Bill-and-Keep Peering.

**Lemma 4** The smaller network has stronger incentives to reach a Peering agreement relative to using the Intermediary than the larger network, i.e., \( \Pi_P^A - \Pi_I^A < \Pi_P^B - \Pi_I^B \) \( \forall \Delta > 0 \).

Proof: See the appendix.

Consequently, in any Paid Peering regime network B pays a settlement-fee to network A, not vice versa.\textsuperscript{23}

If \( \Delta \leq \Delta_P \), we necessarily have \( \Pi_P^B - \Pi_I^B > 0 \). Therefore, when comparing BK and Intermediary, in this range the smaller network always agrees to BK. Since, by definition, no side-payments can be exchanged under the BK regime, BK will constitute a Nash equilibrium iff \( \Pi_P^A - \Pi_I^A \geq 0 \), too.

Define \( \Delta_{BK} \) as the threshold level, where \( \Pi_P^A = \Pi_I^A \) and below which \( \Pi_P^A > \Pi_I^A \).

By using (15) and (10) and rearranging we get:

\textsuperscript{22}See section 7 and appendix A.7 for a comparative analysis of the per-unit case. Our assumption relates to Besen et al. (2001), whose approach is based on the Nash bargaining model of Binmore et al. (1986).

\textsuperscript{23}It is noteworthy that we obtain this finding even without assuming network externalities in the utility functions of consumers. If we assumed such externalities, consumers would ex ante prefer the larger network A over the smaller network B, which would increase A’s bargaining power and the settlement-fee paid from B to A even more.
\[
\Delta_{BK} \equiv \frac{(\tilde{\alpha}z\tau + \sqrt{\tilde{\alpha}^2 z^2 \tau^2 - 4(2F - (\kappa - 1)^2 t)(t - 4\tilde{\alpha}^2 tz^2 + 4\tilde{\alpha}z^2 \tau))}}{t(8\tilde{\alpha}^2 z^2 - 2) - 8\tilde{\alpha}z^2 \tau}.
\] 

(21)

Lemma 5  (i) \(\Delta_{BK} < \Delta_P\).  (ii) For \(\Delta \leq \Delta_{BK}\), Bill-and-Keep constitutes a Nash equilibrium.

Proof: Refer to the appendix.

Lemma 5.(ii) expresses that, if two networks are sufficiently symmetric, each one gains individually by switching from Intermediary to BK, even without a lump-sum being paid by the other one. However, there is an upper bound, \(\Delta_{BK}\), for the range where this occurs in equilibrium. Lemma 5.(i) states that, as long as networks are asymmetric in size, this upper bound is strictly below the upper bound of Paid Peering, \(\Delta_P\). Hence, there exists an intermediate \(\Delta\)-range, where \(\Delta_{BK} < \Delta \leq \Delta_P\). In this range, a subsidy is required in order for network A to be willing to peer. But since \(\Pi^P_B \geq \Pi^I_B\), network B is willing to pay it via a Paid Peering settlement-fee; and since \(\Pi^P \geq \Pi^I\), there is sufficient surplus to compensate network A.

Based on Lemmas 3 and 5, we summarize our key insights in:

Proposition 1  (i) For \(\Delta \in [0, \Delta_{BK}]\), Intermediary, Bill-and-Keep, and Paid Peering constitute Nash equilibria.  (b) For \(\Delta \in (\Delta_{BK}, \Delta_P]\), Intermediary and Paid Peering constitute Nash equilibria.  (iii) For \(\Delta \in (\Delta_P, \Delta_{max}]\), Intermediary constitutes a unique Nash equilibrium.

Figure 3 illustrates Proposition 1:24 It plots \(\Delta_P\) and \(\Delta_{BK}\) and shows, where all three regimes (region \(X\)), Intermediary and Paid Peering (region \(Y\)), and uniquely Intermediary (region \(Z\)) constitute Nash equilibria.

As we explained below Lemma 3, Intermediary is a Nash equilibrium for all levels of asymmetry. This might explain why we observe usage of IP-Transit among both symmetric and asymmetric networks in practice. Below \(\Delta_P\), however, at least one Peering regime is preferred by the networks over buying IP-Transit. Moreover, here they can deviate from playing an Intermediary strategy without risk because the fallback outcome, if the other network does not pick the same Peering strategy, is

\[\text{For details of the numerical example used see appendix A.2.}\]
Figure 3: Network interconnection in equilibrium

Intermediary. This suggests an intuition why, according to anecdotal evidence, Bill-and-Keep Peering has been the dominant Peering regime in practice. If networks are sufficiently symmetric (region X) and the smaller network can credibly announce that it will not bargain over a settlement-fee (which is not captured by our model), the larger network is better off by accepting BK instead of paying the Intermediary.\footnote{One reason for the smaller network’s resistance to bargain at all in practice could be explained by the fact that the bargaining process associated with Paid Peering may involve extra transaction costs in comparison to BK. Another explanation could be legacy which is, however, questionable from a purely rational point of view. The argument claims that, at the beginning of the commercial Internet era, networks did not focus on the strategic aspects of interconnection but strived for reaching world-wide connectivity fast. Nowadays, they found themselves in the resource consuming process of reviewing their existing Peering policies.}

If networks’ asymmetry is intermediate (region Y), the smaller network knows that the larger would never accept BK because, as an outside option, Intermediary is more attractive. Then, the smaller network is better off paying some of its gains from Peering via a settlement-fee thereby compensating the larger one for its losses. Reflecting on these two arguments indicates that in practice—and outside of our model—the sequence of moves is crucial.

Reflecting on these two arguments indicates that in practice—and outside of our model—the sequence of moves is crucial.
6 Welfare

6.1 Consumer Surplus

We restrict the analysis to the $\tilde{\alpha}$ consumers residing in the battlezone since consumer surplus within the locked regions is neither a function of the networks’ interconnection regime nor of their battlezone prices. Hence aggregate consumer surplus is the integral over individual net benefit, equation (1). As under (Paid) Peering, equilibrium prices of networks A and B are equal and each one gets a market share of 0.5, we can calculate consumer surplus as:

$$CS^P = 2\tilde{\alpha} \int_0^{0.5} (v - \tau x - p_A)dx = \tilde{\alpha}(v - \frac{5}{4}\tau - 2\kappa t).$$  \hspace{1cm} (22)

In contrast, consumer surplus under Intermediary is denoted by:

$$CS^I = \tilde{\alpha} \left( \int_0^{\hat{x}} (v - \tau x - p_A)dx + \int_{\hat{x}}^1 (v - \tau (1 - x) - p_B)dx \right)$$

$$= \tilde{\alpha}(v - \frac{5}{4}\tau - 2\kappa t) + \tilde{\alpha}\tau z^2\Delta^2 = CS^P + \tilde{\alpha}\tau z^2\Delta^2. $$  \hspace{1cm} (23)

Analogously to Lemma 2.(iv), we have $CS^P = CS^I$ if networks are symmetric ($\Delta = 0$). But for all $\Delta > 0$ consumer surplus is larger under the Intermediary regime. This is intuitive since under Intermediary the larger network competes more aggressively in prices than under Peering and it also obtains a higher market share within the battlezone. Hence a majority of consumers enjoy extra surplus which is not offset completely by higher prices that are paid by the fewer customers of the smaller network. It is straightforward to observe from (23) that consumer surplus under Intermediary relative to Peering increases even further with growing network asymmetry.

6.2 Total Welfare

What is the maximum level of asymmetry up to which networks should peer from a social perspective? We define the threshold level $\Delta^S_{soc}$, at which a social planner who includes both consumer surplus and producer surplus (profits of networks A
and B and the Intermediary network) in his objective function is indifferent between Peering and Intermediary. It satisfies the following:

\[ CS^P + \Pi^P + \Pi^P_{\text{Int}} = CS^I + \Pi^I + \Pi^I_{\text{Int}}, \]  

(24)

where profits of the Intermediary network are denoted by \( \Pi^P_{\text{Int}} = 2\kappa t (\alpha_A + \alpha_B + \bar{\alpha}) \) when A and B peer and by \( \Pi^I_{\text{Int}} = \Pi^P_{\text{Int}} + 2tu \omega - 2z^2\Delta^2\bar{\alpha}(3\tau - 2t\bar{\alpha}) \) when A and B do not peer. Employing these equations and (22), (18), (23), and (17) yields that from a social planner’s perspective networks should peer iff:

\[ \Delta \geq \frac{(3\tau - 4\bar{\alpha}t)\sqrt{F}}{t\sqrt{2\bar{\alpha}\tau}} \equiv \Delta_{\text{Soc}}^P. \]  

(25)

However, since currently all major Intermediary backbones are U.S. based firms,\(^{26}\) we are also interested in the ranges of asymmetry where a non-U.S. policy maker would like networks to peer, i.e. without taking into account the profits of the Intermediary network. Therefore, we set:

\[ CS^P + \Pi^P = CS^I + \Pi^I \]  

(26)

and find that in this “trade policy” case a regulator would want networks to peer as long as the following holds:

\[ \Delta \leq \frac{\sqrt{(4\bar{\alpha}t - 3\tau)^2(-2F + (\kappa - 1)^2t)} \sqrt{t\tau(9\tau - 4\bar{\alpha}t)}}{t\tau(9\tau - 4\bar{\alpha}t)} \equiv \Delta_{\text{TP}}^P. \]  

(27)

These insights establish the following:

**Proposition 2**  
(i) A social planner prefers Peering among relatively asymmetric networks (for \( \Delta \leq \Delta_{\text{Soc}}^P \)).  
(ii) “Trade policy” regulators prefer Peering among relatively symmetric networks (for \( \Delta \leq \Delta_{\text{TP}}^P \)).  
(iii) Networks peer excessively: \( \Delta_P > \Delta_{\text{TP}}^P \).

Proof: See the appendix.

It might be startling that both a trade policy regulator and the profit maximizing networks prefer Peering for a lesser degree of asymmetry, while a social planner

prefers Peering for larger asymmetry. To understand the intuition of Proposition 2, parts (i) and (ii), recall that the respective optimizers include different parameters in their calculi.

*Networks* trade-off Peering costs $F$ versus Transit costs depending on $t$. If $\Delta$ increases, $F$ remains constant whilst joint Transit costs decrease. Therefore, above a certain level of asymmetry, $\Delta_P$, networks prefer the Intermediary regime.

A "trade policy" regulator faces the same trade-off and hence prefers Peering for low levels of asymmetry. But in addition he takes into account consumer surplus, which grows with $\Delta$ under Intermediary due to fiercer network competition but remains constant under Peering. Therefore, trade policy makers wish to have the Intermediary regime implemented for lower levels of asymmetry than the networks themselves.

A social planner, in contrast, does not observe the effect of decreasing Transit costs for larger asymmetry as this money flows to the Intermediary backbone, which is included in his optimization calculus. Therefore, for low levels of asymmetry he only considers Peering costs $F$—and prefers Intermediary regimes. With rising $\Delta$, under Intermediary the social planner observes distortions due to networks’ fiercer competition, which depend on the transportation cost $\tau$ in the model. As a consequence, above a threshold, $\Delta_{SP}^S$, he prefers interconnection via Peering regimes.

When comparing $\Delta_P$, $\Delta_{SP}^S$, and $\Delta_{TP}^S$, the only general statement we can make is Proposition 2.(iii). Only if A and B are completely symmetric ($\Delta = 0$), networks peer when a “trade policy” regulator wants them to peer. In all other circumstances, they peer excessively from this perspective.

Figure 4 displays the three $\Delta$-thresholds using the same numerical example as before (see Appendix A.2). A social planner prefers Peering to the right of $\Delta_{SP}^S$ (region X), while a “trade policy” regulator prefers Peering to the left of $\Delta_{TP}^S$ (region Y). Networks peer to the left of $\Delta_P$ (regions Y and Z). Notice that unlike in previous figures we plotted the curves on the entire support of $\Delta$ (which is $[0, 0.4]$ in our numerical example). In equilibrium, however, battlezone competition does not exist to the right of $\Delta_{max}$. Hence, there networks always use the Intermediary to exchange traffic.

We plotted the full support of $\Delta$ to show that $\Delta_{SP}^S$ can intersect with $\Delta_P$ and $\Delta_{TP}^S$ (depending on parameter values, the intersection can lie to the left of $\Delta_{max}$).
Therefore, we can make no general statement on the relative positions of the curves. However, we can show that the following holds:

**Proposition 3** If \( \kappa < 1 \) and \( \Delta = 0 \), \( F(\Delta_{P}^{Soc}) < F(\Delta_{P}^{TP}) = F(\Delta_{P}) \).

**Proof:** See the appendix.

Proposition 3 implies that, given that there exists a battlezone or at least one network has a positive installed base (notably the starting point of this paper), \( \Delta_{P}^{Soc} \) serves as an upper bound with respect to \( F \) and as a lower bound with respect to \( \Delta \) for a region, in which both the social planner and the trade policy regulator agree with networks’ Peering decisions. For other \((\Delta, F)\)-combinations, the regulatory bodies would like to intervene—not necessarily in the same direction.

### 7 Discussion

**Paid Peering using a per-unit access charge:** Hitherto we assumed the transfer payment or *access charge* between networks to be a lump-sum, not a per-unit of
data fee (henceforth: \textit{variable fee}). The two are structurally similar as long as the variable fee does not influence pricing behavior in the retail market. Given this, the lump-sum could be interpreted as the sum of all per-unit fees in a given period. In contrast, a variable fee does indeed have an influence on the networks’ retail pricing: they tacitly collude even more than under lump-sum Paid Peering by splitting the battlezone 50:50 and symmetrically increasing retail prices. Thus, some consumer surplus is shifted to the networks. Qualitatively our results, namely Propositions 1 to 3, remain unchanged, though. For a more detailed analysis of variable Paid Peering refer to appendix A.7.

Positive marginal costs of sending data: In our analysis, building on established institutional literature and interviews with industry representatives, we assumed the marginal costs of sending data to be zero.\footnote{Refer to the literature mentioned in footnote 5.} If those costs were positive, they would influence retail prices as a mark-up in all interconnection regimes symmetrically\footnote{This is a standard effect in Hotelling models.} as long as there would be no differences in costs for sending on-net or off-net traffic, which there is no technical reason for. Atkinson and Barnekov (2004, p.3) support our view by pointing out that the operating costs of a telecommunications network can be estimated well by the number of end-users connected to that network. They reject the idea that traffic volume is a major determinant for networks’ operating costs.

Non-covered market: Let $s_i$ be the market share of network $i$ in the battlezone. If we lift our restriction, that the market be covered, we have $s_A + s_B \leq 1$. Under the Intermediary regime, the costs that a marginal consumer residing in the battlezone creates when buying from network A by exchanging data with consumers connected to network B are $(\alpha_B + \bar{\alpha}s_B)2t$. If he buys from network B, these marginal costs are $(\alpha_A + \bar{\alpha}s_A)2t$. Assuming that both networks start with an equal battlezone market share, say $s_A = s_B = 0$, and using $\alpha_A > \alpha_B$, it is obvious that the marginal costs of connecting an additional consumer are lower for network A than for network B. Because of the uniform distribution of consumers along the $[0, 1]$-interval, marginal revenues are equal for both networks. Consequently, network A’s marginal profit for connecting another consumer in the battlezone is larger than B’s. This makes A more aggressive and leads to $p_A < p_B$, just as in Lemma 1. Under the Peering
regime, when assuming equal battlezone market shares $s_A = s_B$ as a starting point, ceteris paribus the marginal incentives to attract another consumer are equal for both networks, just as in Lemma 2. Depending on the shape of the demand curve, it is possible, however, that elastic demand has an effect on our welfare conclusions, Propositions 2 and 3.

8 Conclusion

In this paper we have suggested a model of the Internet backbone market, which explicitly takes into account differences in the size of networks. We have analyzed the consequences of those asymmetries for the optimal interconnection decisions of IBPs, which are strategically linked to retail competition for end-users. In line with that, we have studied the role of unregulated access charges as a means of tacit collusion when networks choose a Paid Peering regime. The main practical implications of our model both for networks, consumers, and policy makers inside and outside the U.S. are the following:

1. If, besides Intermediary and Bill-and-Keep, networks also consider Paid Peering as a possible type of interconnection, we expect to observe more Paid Peering in the future. This would translate to more Peering agreements in general which, in turn, would lead to higher profits of IBPs.

2. This development would harm consumer surplus.

3. Since the emergence of Paid Peering also lowers demand for IP-Transit, top level backbones can be expected to lose revenues.

4. As all top level backbones are U.S.-based, non-U.S. policy makers do not include profits from IP-Transit in their calculus. Instead of considering punishing large networks who refuse (Bill-and-Keep) Peering to smaller ones, these policy makers should consider restricting Peering because networks do not care about the fact that fiercer competition under Intermediary benefits consumers, and peer excessively instead. In contrast, since U.S.-based policy makers do account for profits from IP-Transit, they should favor Peering among networks sufficiently asymmetric in size. Hence, they should seek to discourage large networks from refusing to peer with smaller ones.
These implications could also be applied to a telecommunications market which was both unregulated in terms of inter-carrier compensation fees and not subject to price discrimination regarding the destinations of calls.

\section*{A Appendix}

\subsection*{A.1 Proof of Lemma 3}

(i): This follows from our assumption that the agreement of both networks is needed to deviate from the Intermediary regime.

(ii): $\Delta > \Delta_P \iff \Pi^P < \Pi^I$. There average Peering profits are smaller than average Intermediary profits. Hence, at least one network has an incentive to deviate from a Peering strategy and switch to Intermediary. The other network cannot gain enough profits from the Peering regime to offer a side-payment (via a settlement-fee) that induces the deviating network not to deviate. Formally:

$$\Pi_i^{PP} > \Pi_i^I \quad \text{but} \quad \Pi_j^{PP} < \Pi_j^I \quad \forall \ i, j \in \{A, B\}. \quad (A.1)$$

(iii): $\Delta \leq \Delta_P \iff \Pi^P \geq \Pi^I$. There exists a settlement-fee $S \in \mathbb{N}$ that makes both networks better off under Paid Peering than under Intermediary such that:

$$\Pi_i^{PP} = \Pi_i^P + S \geq \Pi_i^I \quad \text{and} \quad \Pi_j^{PP} = \Pi_j^P - S \geq \Pi_j^I \quad \forall \ i, j \in \{A, B\}. \quad \square \quad (A.2)$$

\subsection*{A.2 Numerical example}

Using $\alpha_A = \Delta + \alpha_B$ and $\alpha_B = 1 - \kappa - \alpha_A - \bar{\alpha}$ yields:

$$w = 2(\bar{\alpha} + \frac{1 - \kappa - \bar{\alpha} - \Delta}{2})(\bar{\alpha} + \Delta + \frac{1 - \kappa - \bar{\alpha} - \Delta}{2}). \quad (A.3)$$

Hence, any increase in $\Delta$ reflects an increase in $\alpha_A$ and a decrease in $\alpha_B$.

In the numerical example of Figure 2 we used $\kappa = 0.4$, $\bar{\alpha} = 0.2$. Hence $\alpha_A + \alpha_B = 0.4 \Rightarrow \alpha_A \in [0.2, 0.4] \Rightarrow \Delta \in [0, 0.4]$. Furthermore, we used $t = 1$ and $\tau = 0.5 > 2\bar{\alpha}t = 0.4$, by assumption. $z = \frac{2\bar{t}}{3\tau - 4\bar{\alpha}t} = 2.857$. Substituting these values
we get:

$$\Pi^I - \Pi^P = 2F - 0.36 + 2.959\Delta^2 \quad \text{(A.4)}$$

The LEFT panel of Figure 2 displays curves of constant $\Pi^I - \Pi^P$ levels while the RIGHT panel picks the one curve where $\Pi^I - \Pi^P = 0$. Both panels only cover $\Delta \in [0, 0.175]$ because $\Delta_{\max} = \frac{1}{2z} = 0.175$.

A.3 Proof of Lemma 4

Network A’s incentives to BK are smaller than B’s, if $\Pi^P_A - \Pi^I_A < \Pi^P_B - \Pi^I_B$, or:

$$\frac{1}{2}\tau\bar{\alpha} + \alpha_A(p^L_A - 2\kappa t) - F - \frac{1}{2}\tau\bar{\alpha}(1 + z\Delta - 2z^2\Delta^2) - z^2\Delta^2\bar{\alpha}(3\tau - 2t\bar{\alpha}) + tw - \alpha_A(p^L_A - 2\kappa t) < \frac{1}{2}\tau\bar{\alpha} + \alpha_B(p^L_B - 2\kappa t) - F - \frac{1}{2}\tau\bar{\alpha}(1 - z\Delta - 2z^2\Delta^2) - z^2\Delta^2\bar{\alpha}(3\tau - 2t\bar{\alpha}) + tw - \alpha_B(p^L_B - 2\kappa t),$$

which can be rearranged as:

$$-F - \frac{1}{2}\tau\bar{\alpha}(z\Delta - 2z^2\Delta^2) - z^2\Delta^2\bar{\alpha}(3\tau - 2t\bar{\alpha}) + tw < -F - \frac{1}{2}\tau\bar{\alpha}(-z\Delta - 2z^2\Delta^2) - z^2\Delta^2\bar{\alpha}(3\tau - 2t\bar{\alpha}) + tw,$$

and reduces further to:

$$\frac{1}{2}\tau\bar{\alpha}z\Delta < -\frac{1}{2}\tau\bar{\alpha}(-z\Delta),$$

$$\tau\bar{\alpha}z\Delta > 0.$$

This is true as long as $\Delta > 0$ because $\tau, \bar{\alpha}, z > 0$, by definition. \Box

A.4 Proof of Lemma 5

(i): For $\Delta = \Delta_{BK}$, by definition we have:

$$\pi^P_A = \pi^I_A, \quad \text{(A.5)}$$
and, by Lemma 4 and assuming $\Delta > 0$, there we have:

$$\pi_B^P > \pi_B^I. \quad (A.6)$$

We shall distinguish among three cases:

1. Assume $\Delta_{BK} = \Delta_P$. Then, $\Delta_P$ requires $\pi_A^P + \pi_B^P = \pi_A^I + \pi_B^I$. Substituting (A.5) in this condition yields $\pi_B^P = \pi_B^I$, which is in contradiction to (A.6).

2. Assume $\Delta_{BK} > \Delta_P$. Then, $\Delta_P$ requires $\pi_A^P + \pi_B^P < \pi_A^I + \pi_B^I$. Substituting (A.5) in this condition yields $\pi_B^P < \pi_B^I$, which is in contradiction to (A.6).

3. Assume $\Delta_{BK} < \Delta_P$. Then, $\Delta_P$ requires $\pi_A^P + \pi_B^P > \pi_A^I + \pi_B^I$. Substituting (A.5) in this condition yields $\pi_B^P > \pi_B^I$, which is in line with (A.6).

(ii): If one network plays BK and the other does not, they end up using the Intermediary. If $\Delta \leq \Delta_{BK}$, by definition, $\Pi_i^P \geq \Pi_i^I \forall i$. Hence, there is no incentive to deviate from playing BK. Therefore, for $\Delta \in [0, \Delta_{BK}]$ both BK and PP are equilibria, while for $\Delta \in (\Delta_{BK}, \Delta_P]$ only PP is a Peering equilibrium. \( \square \)

A.5 Proof of Proposition 2

(i): This follows from equation (25).

(ii): This follows from equation (27).

(iii): Assume: $\Delta_P > \Delta_{TP}^P$. This equals:

$$\frac{\sqrt{(4\alpha t - 3\tau)^2(-2F + (\kappa - 1)^2t)}}{\sqrt{t\tau(-8\alpha t + 9\tau)}} > \frac{\sqrt{(4\alpha t - 3\tau)^2(-2F + (\kappa - 1)^2t)}}{\sqrt{t\tau(-4\alpha t + 9\tau)}}. \quad (A.7)$$

Both numerators are equal. If $F < \frac{(\kappa - 1)^2t}{2}$, the numerators are positive and (A.7) equals:

$$\frac{1}{\sqrt{9\tau - 8\alpha t}} > \frac{1}{\sqrt{9\tau - 4\alpha t}}. \quad (A.8)$$

25
Due to our assumptions, namely $\tau > 2\hat{\alpha}t$, both expressions are positive. The LHS’s denominator is smaller than the RHS’s. Consequently:

$$\Delta_P > \Delta_P^{TP} \quad \forall \quad F < \frac{(\kappa - 1)^2t}{2}. \quad \text{(A.9)}$$

For the supremum, $F \neq \frac{(\kappa - 1)^2t}{2}$, we have $\Delta_P = \Delta_P^{TP} = 0$. Hence, if $\Delta > 0$, $F(\Delta_P), F(\Delta_P^{TP}) < \frac{(\kappa - 1)^2t}{2}$ and we have: ²⁹

$$\Delta_P > \Delta_P^{TP} \quad \forall \quad \Delta > 0. \quad \square \quad \text{(A.10)}$$

### A.6 Proof of Proposition 3

We have proven $F(\Delta_P|\Delta = 0) = F(\Delta_P^{TP}|\Delta = 0)$ in Proof of Proposition 2.(iii). $F(\Delta_P^{TP}) > F(\Delta_P^{SPC})$ equals:

$$\frac{t}{2}(1 + (\kappa - 2)\kappa + \frac{\Delta^2(4\hat{\alpha}t - 9\tau)\tau}{(4\hat{\alpha}t - 3\tau)^2}) > \frac{2\hat{\alpha}\Delta^2t^2\tau}{(4\hat{\alpha}t - 3\tau)^2}. \quad \text{(A.11)}$$

For $\Delta = 0$, this reduces to:

$$\frac{t}{2}(1 + (\kappa - 2)\kappa) > 0, \quad \text{(A.12)}$$

which is true $\forall \quad \kappa < 1. \quad \square$

### A.7 Analysis of Per-Unit Access Fees/Variable Paid Peering

Assume that $a$ is a fee that network B has to pay network A for every unit of data exchanged between the two networks under a Paid Peering regime. Because of Lemma 4, $a > 0 \quad \forall \quad \Delta > 0$. Profit functions in the retail market change to:

$$\Pi_A^{PP} = \hat{x}\alpha(p_A - 2\kappa t) + \alpha_A(p_A^L - 2\kappa t) + 2a(\hat{x}\alpha + \alpha_A)((1 - \hat{x})\alpha + \alpha_B) - F$$

$$\Pi_B^{PP} = (1 - \hat{x})\alpha(p_B - 2\kappa t) + \alpha_B(p_B^L - 2\kappa t) - 2a(\hat{x}\alpha + \alpha_A)((1 - \hat{x})\alpha + \alpha_B) - F.$$ ²⁹This can be seen—for the numerical example—in Figure 4.
Equilibrium prices are derived as \( p^{vPP}_A = \tau + 2\kappa t + 2a\Delta = p^{vPP}_B \), leading to an equilibrium market share for A (and for B, respectively) of \( \hat{x} = \frac{1}{2} \). Consequently, equilibrium retail profits under variable Paid Peering are denoted by:

\[
\begin{align*}
\Pi^{vPP}_A &= \frac{1}{2} \tilde{a}(\tau + 2a\Delta) + \alpha_A(p^L_A - 2\kappa t) + aw - F \\
\Pi^{vPP}_B &= \frac{1}{2} \tilde{a}(\tau + 2a\Delta) + \alpha_B(p^L_B - 2\kappa t) - aw - F.
\end{align*}
\]

Because of \( p^{vPP}_i = p^P_i + 2a\Delta \), we have \( \Pi^{vPP}_A + \Pi^{vPP}_B > \Pi^P \). Hence, variable Paid Peering is sustainable for the networks for even larger asymmetries and we find:

\[
\Delta^{vPP}_P > \Delta_P \quad \forall \quad \Delta > 0. \tag{A.13}
\]

However, since the other firm-level results remain unchanged, Proposition 1 does not change. Due to the fact that, under variable Paid Peering, the reduction of consumer surplus is completely redistributed to networks, \( \Delta^{SOC}_P \) and \( \Delta^{TP}_P \) are not altered, too. Consequently, Propositions 2 and 3 hold.

**Interpretation:** Using a variable Paid Peering fee lets networks not only tacitly collude in the battlezone (and share that market 50:50) but it lets them increase prices even more than under lump-sum Paid Peering. A variable access charge is used to increase the other network’s perceived marginal cost (even if the access charge is received, not paid!). Consequently, joint profits are larger and consumer surplus is smaller when a variable fee is used.

**References**


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30 There exists a large body of literature studying the potential usage of two-way access charges as an instrument of tacit collusion in telecommunications. See Armstrong (1998) or Laffont et al. (1998) for more details.


