

## Entry Requirements MSc QFAS

In order to be able to enter the MSc QFAS, ***the topics and the corresponding skills related to mathematical proofs and mathematical reasoning*** covered in the following nine courses offered by the BSc Econometrics and Operations Research (EOR) at Tilburg University is required:

Year 1 (30 ECTS)					
#	Semester	Unit	Course name	Course number	ECTS
1	1	1	Linear Algebra	35B203	6
2	1	1 & 2	Introduction Analysis and Probability Theory	35B113	6
3	1	2	Mathematical Analysis 1	35B105	6
4	2	3	Mathematical Analysis 2	35B107	6
5	2	3 & 4	Probability and Statistics	35B402	6
Year 2 (24 ECTS)					
#	Semester	Unit	Course name	Course number	ECTS
6	1	1 & 2	Statistics for Econometrics	35B501	6
7	1	1 & 2	Advanced Linear Algebra	35B801	6
8	2	3 & 4	Differentiation and Integration Theory	35B601	6
9	2	3 & 4	Econometrics	35B206	6

The content of these nine courses is available below.

Please note that if you have no knowledge of Quantitative Finance, you may also benefit from following the BSc EOR course Quantitative Finance before starting the MSc QFAS. This, however, is not mandatory.

## Linear Algebra

*Linear Algebra and Its Applications – David C. Lay*

1. Linear Equations in Linear Algebra
  - 1.1 Systems of Linear Equations
  - 1.2 Row Reduction and Echelon Forms
  - 1.3. Vector Equations
  - 1.4. The Matrix Equation  $Ax=b$
  - 1.5. Solution Sets of Linear Systems
  - 1.6. Applications of Linear Systems
  - 1.7. Linear Independence
  - 1.8. Introduction to Linear Transformations
  - 1.9. The Matrix of a Linear Transformation
  - 1.10. Linear Models in Business, Science and Engineering
2. Matrix Algebra
  - 2.1. Matrix Operations
  - 2.2. The Inverse of a Matrix
  - 2.3. Characterizations of Invertible Matrices
  - 2.4. Partitioned Matrices
  - 2.5. Matrix Factorizations
  - 2.8. Subspaces of  $\mathbb{R}^n$
  - 2.9. Dimension and Rank
3. Determinants
  - 3.1. Introduction to Determinants
  - 3.2. Properties of Determinants
  - 3.3. Cramer's Rule, Volume, and Linear Transformations
4. Vector Spaces
  - 4.1. Vector Spaces and Subspaces
  - 4.2. Null Spaces, Column Spaces, and Linear Transformations
  - 4.3. Linearly Independent Sets; Bases
  - 4.5. The Dimension of a Vector Space
  - 4.6. Rank
  - 4.7. Change of Basis
6. Orthogonality and Least Squares
  - 6.1. Inner Product, Length, and Orthogonality
  - 6.2. Orthogonal Sets
  - 6.3. Orthogonal Projections

## Introduction Analysis and Probability Theory

### *Introduction Analysis (reader)*

1. Equations, inequalities and absolute value
  - 1.1. Number sets
  - 1.2. Solving equations
  - 1.3. Solving inequalities
  - 1.4. Absolute value
  - 1.5. Mixed exercises
2. Logic reasoning and the set of real numbers
  - 2.1. Logic reasoning
  - 2.2. The Algebraic Axioms
  - 2.3. The Ordering Axioms
  - 2.4. The Axiom of Completeness
  - 2.5. Mixed exercises
3. The principle of induction
  - 3.1. The summation and product sign
  - 3.2. The principle of induction
  - 3.3. Newton's binomium
  - 3.4. Mixed exercises
4. Geometry
  - 4.1. Plane geometry
  - 4.2. Space geometry
5. Sets and maps
  - 5.1. Set theory
  - 5.2. Maps
  - 5.3. Cardinality of sets
6. Differentiation and Integration
  - 6.1. Difference quotients and derivatives
  - 6.2. Partial derivatives
  - 6.3. Integration

*Introduction to Probability and Mathematical Statistics – Bain/Engelhardt*

1. Probability
  - 1.1. Introduction
  - 1.2. Notation and terminology
  - 1.3. Definition of probability
  - 1.4. Some properties of probability
  - 1.5. Conditional probability
  - 1.6. Counting techniques
2. Random Variables and their Distributions
  - 2.1. Introduction
  - 2.2. Discrete random variables
  - 2.3. Continuous random variables
  - 2.4. Some properties of expected values
  - 2.5. Moment generating functions
3. Special Probability Distributions
  - 3.1. Introduction
  - 3.2. Special discrete distributions
  - 3.3. Special continuous distributions
  - 3.4. Location and scale parameters

# Mathematical Analysis 1

## *Mathematical Analysis 1 (reader)*

1. Convergence of Sequences
  - 1.1. Sequences
  - 1.2. The limit of a sequence
  - 1.3. Properties of convergent sequences
  - 1.4. Arithmetic rules for limits of sequences
  - 1.5. Monotone sequences
  - 1.6. The number  $e$
  - 1.7. Mixed exercises
2. Subsequences
  - 2.1. Subsequences
  - 2.2. Convergent subsequences
  - 2.3. Cauchy sequences
  - 2.4. The contraction theorem
  - 2.5. Mixed exercises
3. Limits of Functions
  - 3.1. Limit of a function
  - 3.2. A criterion for the limit of a function
  - 3.3. Arithmetic rules for limits of functions
  - 3.4. Extensions of the concept limit
  - 3.5. Mixed exercises
4. Continuity
  - 4.1. Continuous functions
  - 4.2. Arithmetic rules for continuous functions
  - 4.3. Continuous functions on an interval
  - 4.4. Continuity and the inverse function
  - 4.5. Mixed exercises
5. Derivative
  - 5.1. Differentiable functions
  - 5.2. Arithmetic rules for differentiable functions
  - 5.3. Linear approximation and differential
  - 5.4. Marginality
  - 5.5. Elasticity
  - 5.6. Partial derivatives
  - 5.7. Mixed exercises
6. Differentiable Functions
  - 6.1. Mean Value Theorem
  - 6.2. Monotone and differentiable functions
  - 6.3. Differentiability of the inverse function
  - 6.4. Taylor's Theorem
  - 6.5. Rule of de l'Hopital
  - 6.6. Mixed exercises

7. Summability of a Sequence
  - 7.1. Summable sequences
  - 7.2. Arithmetic rules for summable sequences
  - 7.3. Two summability criteria
  - 7.4. Power sets
  - 7.5. Mixed exercises
8. Integration
  - 8.1. The integral
  - 8.2. The Principal Theorem of Integral Calculus
  - 8.3. A list with primitive functions
  - 8.4. Arithmetic rules for integration
  - 8.5. Indefinite integral
  - 8.6. Mixed exercises

## Mathematical Analysis 2

### *Introduction Analysis 2 (reader)*

1. The n-Dimensional Euclidean Space
  - 1.1. The space  $\mathbb{R}^n$
  - 1.2. Functions on  $\mathbb{R}^n$
  - 1.3. Sets in  $\mathbb{R}^n$
  - 1.4. Sequences in  $\mathbb{R}^n$
  - 1.5. Mixed exercises
2. Limits of Functions and Continuity
  - 2.1. Limit of a function
  - 2.2. Continuity of a function
  - 2.3. Arithmetic rules for continuous functions
  - 2.4. Continuous functions on a compact set
  - 2.5. Mixed exercises
3. Differentiation in  $\mathbb{R}^n$ 
  - 3.1. Partial differentiable functions
  - 3.2. Total differentiable functions
  - 3.3. (Total) differential
  - 3.4. (Arithmetic rules for (total) differentiable functions
  - 3.5. Continuously differentiable functions
  - 3.6. Directional derivative
  - 3.7. Marginality and marginal rate of substitution
  - 3.8. Higher order partial derivatives
  - 3.9. Mixed exercises
4. Equations and implicit functions
  - 4.1. Analysis of an economical model
  - 4.2. Implicit Function Theorem
  - 4.3. Inverse Function Theorem
  - 4.4. Comparative Statics
5. Differentiable Functions
  - 5.1. Convex sets
  - 5.2. Taylor's Theorem
  - 5.3. Quadratic functions
  - 5.4. Concave and convex functions
  - 5.5. Homogeneous functions
  - 5.6. Mixed Exercises
6. Optima of a Differentiable Function
  - 6.1. Free optima
  - 6.2. Concavity and optima
  - 6.3. Least-squares-method
  - 6.4. A constrained optimization problem
  - 6.5. The substitution method
  - 6.6. The method of Lagrange
  - 6.7. The Lagrange multiplier
  - 6.8. Mixed exercises
7. Integration
  - 7.1. Integrals for functions of more variables
  - 7.2. Mixed exercises

*Handout Constrained Optimization*

1. Constrained Optimization: Inequality Constraints
  - 1.1. The Directional Derivative
  - 1.2. Two Variables and One Inequality Constraint
  - 1.3. The General Case
  - 1.4. Some Pathological Examples



## Probability and Statistics

### *Introduction to Probability and Mathematical Statistics – Bain/Engelhardt*

4. Joint Distributions
  - 4.1. Introduction
  - 4.2. Joint discrete distributions
  - 4.3. Joint continuous distributions
  - 4.4. Independent random variables
  - 4.5. Conditional distributions
  - 4.6. Random samples
5. Properties of Random Variables
  - 5.1. Introduction
  - 5.2. Properties of expected values
  - 5.3. Correlation
  - 5.4. Conditional expectation
  - 5.5. Joint moment generating functions
6. Functions of Random Variables
  - 6.1. Introduction
  - 6.2. The CDF technique
  - 6.3. Transformation methods
  - 6.4. Sums of random variables
  - 6.5. Order statistics
7. Limiting Distributions
  - 7.1. Introduction
  - 7.2. Sequences of random variables
  - 7.3. The central limit theorem
  - 7.4. Approximations for the binomial distribution
  - 7.5. Asymptotic normal distributions
  - 7.6. Properties of stochastic convergence
  - 7.7. Additional limit theorems
8. Statistics and Sampling Distributions
  - 8.1. Introduction
  - 8.2. Statistics
  - 8.3. Sampling distributions
  - 8.4. The t, F, and beta distributions
  - 8.5. Large-sample approximations
9. Point Estimation
  - 9.1. Introduction
  - 9.2. Some methods of estimation
  - 9.3. Criteria for evaluating estimators

## Statistics for Econometrics

### *Introduction to Probability and Mathematical Statistics – Bain/Engelhardt*

9. Point estimation
  - 9.4. Large-sample properties
  - 9.5. Bayes and minimax estimators
10. Sufficiency and Completeness
  - 10.1. Introduction
  - 10.2. Sufficient statistics
  - 10.3. Further properties of sufficient statistics
  - 10.4. Completeness and exponential class
11. Interval Estimation
  - 11.1. Introduction
  - 11.2. Confidence intervals
  - 11.3. Pivotal quantity method
  - 11.4. General method (*only basics*)
  - 11.5. Two-sample problems
12. Test of Hypotheses
  - 12.1. Introduction
  - 12.2. Composite hypotheses
  - 12.3. Tests for the normal distribution
  - 12.4. Binomial tests
  - 12.5. Poisson tests
  - 12.6. Most powerful tests
  - 12.7. Uniformly most powerful tests
  - 12.8. Generalized likelihood ratio tests (until k-sample tests)
13. Contingency Tables and Goodness-of-Fit
  - 13.5. R-Sample multinomial
  - 13.6. Test for independence,  $r \times c$  contingency table
14. Nonparametric Methods
  - 14.5. Wilcoxon paired-sample signed-rank test
  - 14.7. Wilcoxon and Mann-Whitney (WMW) tests

### *Slides not covered in the book*

1. Nonparametric methods
  - 1.1. Empirical distribution functions
  - 1.2. Density estimation
  - 1.3. Local behavior
  - 1.4. Global behavior
  - 1.5. Practical bandwidth choices
  - 1.6. Nonparametric regression

## Advanced Linear Algebra

*Linear Algebra and Its Applications – David C. Lay*

- 4. Vector Spaces
  - 4.4. Coordinate Systems
  - 4.8. Applications to Difference Equations
  - 4.9. Applications to Markov Chains
- 5. Eigenvalues and Eigenvectors
  - 5.1. Eigenvectors and Eigenvalues
  - 5.2. The Characteristic Equation
  - 5.3. Diagonalization
  - 5.4. Eigenvectors and Linear Transformations
  - 5.5. Complex Eigenvalues
  - 5.8. Iterative Estimates for Eigenvalues
- 7. Symmetric Matrices and Quadratic Forms
  - 7.1. Diagonalization of Symmetric Matrices
  - 7.2. Quadratic Forms

### *Handouts*

- 1. The Characteristic Polynomial
- 2. Linear Maps
- 3. Euler, Cayley-Hamilton and Jordan
- 4. Difference Equations
- 5. Jordan's Theorem for  $n \times n$  matrices
- 6. Positive and positive definite matrices
- 7. Solution Criteria for Linear Systems

## Differentiation and Integration

*Reader*

### **Integration part**

1. Riemann integration
  - 1.1. Introduction
  - 1.2. Under- and overestimations
  - 1.3. Integration over intervals; the Fundamental Theorem of Calculus
  - 1.4. Integrability criteria
2. Stieltjes integration
  - 2.1. Introduction
  - 2.2. Integration over intervals with respect to an increasing function
  - 2.3. From Stieltjes back to Riemann
  - 2.4. Stieltjes integration and convergence
3. Lebesgue integration
  - 3.1. Introduction
  - 3.2.  $\sigma$ -Fields
  - 3.3. Measures
  - 3.4. Measurable functions
  - 3.5. Integrals of simple functions
  - 3.6. The integrals of non-negative measurable functions
  - 3.7. The integral of measurable functions
  - 3.8. Fubini's Theorem
  - 3.9. Riemann and Stieltjes integration vs. Lebesgue integration
  - 3.10. Two applications in the theory of probability

### **Differentiation part**

1. Introduction
2. Linear first order differential equations
3. The Local Existence Theorem and Euler's method
4. Separable differential equations
5. Homogeneous differential equations
6. Complex numbers
7. Linear second order differential equations
8. Systems of differential equations
  - 8.1. Linear autonomous systems
  - 8.2. The substitution method
9. Stability of stationary solutions

## Econometrics

### *Econometrics – Fumio Hayashi*

1. Finite-Sample Properties of OLS
  - 1.1. The Classical Linear Regression Model
  - 1.2. The Algebra of Least Squares
  - 1.3. Finite-Sample Properties of OLS
  - 1.4. Hypothesis Testing under Normality
2. Single Equation GMM
  - 2.1. Endogeneity Bias: Working's Example
  - 2.2. More examples
  - 2.3. The General Formulation
  - 2.4. Generalized Method of Moments Defined
  - 2.5. Large-Sample Properties of GMM

### *Econometrics Analysis – Greene*

14. Maximum Likelihood Estimation
  - 14.1. Introduction
  - 14.2. The Likelihood Function and Identification of the Parameters
  - 14.3. Efficient Estimation: The Principle of Maximum Likelihood
  - 14.4. Properties of Maximum Likelihood Estimators
  - 14.5. Conditional Likelihoods, Econometric Models and the GMM Estimator
  - 14.6. Hypothesis and Specification Tests and Fit Measures
17. Discrete Choice
  - 17.1. Introduction
  - 17.2. Models for Binary Outcomes
  - 17.3. Estimation and Inference in Binary Choice Models
18. Discrete Choices and Event Counts
  - 18.1. Introduction
  - 18.2. Models for Unordered Multiple Choices
  - 18.3. Random Utility Models for Ordered Choices
19. Limited Dependent Variables – Truncation, Censoring, and Sample Selection
  - 19.1. Introduction
  - 19.2. Truncation
  - 19.3. Censored Data
  - 19.5. Incidental Truncation and Sample Selection