Competition leverage: how the demand side affects optimal risk adjustment[*]

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Abstract

We study optimal risk adjustment in imperfectly competitive health insurance markets when high-risk consumers are less likely to switch insurer than low-risk consumers. First, insurers have an incentive to select even if risk adjustment perfectly corrects for cost differences. To achieve first best, risk adjustment should overcompensate insurers for serving high-risk agents. Second, we identify a trade off between efficiency and consumer welfare. Reducing the difference in risk adjustment subsidies increases consumer welfare by leveraging competition from the elastic low-risk market to the less elastic high-risk market. Third, mandatory pooling can increase consumer surplus further, at the cost of efficiency.

Keywords: health insurance, risk adjustment, imperfect competition, leverage

JEL classification: I11, I18, G22, L13

1. Introduction

Health insurance markets suffer from adverse selection. Buyers of health insurance know more about their expected health care costs than the firms selling insurance. And even if some of their health characteristics are observable, insurers are often prohibited from exploiting this information and practising third-degree price discrimination, by so-called community rating requirements. Insurance companies can however engage in second-degree price discrimination, offering different contracts to high cost and low cost consumers, in order to separate the different types. This separation of consumer types leads to inefficiency in health insurance markets (see Rothschild and Stiglitz (1976)).

An important goal of risk adjustment is to reduce health insurers' incentive to select, thus enhancing efficiency.[1] Countries that use risk adjustment include Belgium, Colombia, Germany, Israel, the Netherlands, Switzerland and the United States. Van de Ven and Ellis (2000), Ellis (2008) and Armstrong et al. (2010) discuss the different ways risk adjustment is used in these countries. These schemes involve significant amounts of money. To illustrate, the Netherlands currently have an elaborate nationwide risk adjustment system with a 2011 budget equal to 18 billion Euro, which amounts to 3% of GDP. This paper analyzes how the money involved should be allocated to maximize the sponsor's objective function.

Traditionally, risk adjustment focuses on the supply side of health insurance. This literature tries to predict the health care costs of an individual in a future period of, for example, one year, based on the observed characteristics of this individual. Variables used include age, gender, previous diagnoses and drug prescriptions. See Van de Ven and Ellis (2000) for an overview of variables used in risk adjustment. Making the transfers to insurance companies a function of these predicted costs tends to increase efficiency in the insurance market.[2] Glazer and McGuire (2000), Glazer and McGuire (2002) and Jack (2006) present results along these lines. These and other papers in the risk adjustment literature have two implications. First, once the prediction of health care costs for an individual are perfect and the risk adjustment system fully corrects for any cost differences (we call this perfect risk adjustment), the insurance companies have no incentive any more to engage in cream skimming, i.e., to select certain risks. Therefore, once the cost predictions are good enough, according to this literature the outcome in the health

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[1]See Cutler and Reber (1998) for an estimate of the inefficiency in case there is no risk adjustment.

[2]Depending on the way that risk adjustment is organized, these transfers to insurers are paid by the government or another sponsor of the scheme, like an employer.

insurance market will be efficient. Second, risk adjustment enhances both efficiency and equity. The reason is the following. People with high expected health care costs tend have a low health status. As risk adjustment pays a higher contribution for such unfortunate high risk people, it is viewed as contributing to fairness.

We claim that both implications are unlikely to hold in reality. Consequently, risk adjustment resources are likely to be wasted if the program is designed on the basis of these premises. The reason is the importance of the demand side in determining health insurance market outcomes. It is well known that people do not only differ in their expected health care costs but also in their tendency to switch insurers. Beaulieu (2002), Buchmueller (2006), Nuscheler and Knaus (2005), Royalty and Solomon (1999), Schut, Gress and Wasem (2004), Strombom, Buchmueller and Feldstein (2002) and Van Vliet (2006) document that old or sick insured are less likely to switch insurer. For example, in a US study, Strombom, Buchmueller and Feldstein (2002) find that young and healthy employees are four times more price-elastic than the oldest employees. Schut, Gress and Wasem (2004) find a similar ratio of elasticities in a comparison between German non-pensioners and pensioners. As both elderly and sick have relatively higher expected costs than the average person in the population, people with high expected costs are less likely to switch insurer.[3] Importantly, the variables used in these studies on consumer type and switching behavior are similar to the ones used in risk adjustment. This connection is overlooked in the traditional literature on risk adjustment.

The main contribution of our paper is to model explicitly this relation between expected costs and tendency to switch insurer. This leads to the following implications for optimal risk adjustment. First, even if risk adjustment perfectly corrects for expected costs, insurers still have an incentive to select. The reason is that risk adjustment also needs to compensate insurers for the foregone mark-ups on the high types. To get an efficient outcome in the insurance market, risk adjustment needs to overcompensate the high risk types. That is, the difference in risk adjustment transfers between the high and low type should be bigger than their difference in expected costs (proposition 1 below), and include the difference in mark-

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[3]This is related to adverse retention as discussed by Cutler, Lincoln and Zeckhauser (2010). Possible explanations for this relation between expected costs and switching behavior include the following. People currently undergoing treatment tend to be reluctant to switch half-way through. People with a history of sickness and elderly people have a relation with their current physicians. Moreover, they chose these physicians themselves in the past. Hence switching to another insurer with another provider network entails high costs for them. These costs can be both psychological but also entail the transaction costs of transferring medical records. Finally, there may be a status quo bias that is stronger for people that have actively interacted with their insurer in the past, for example by filing health care expenditures with their insurer or asking advice about which provider to go to.

ups ($\frac{\text{price}}{\text{elasticity}}$) between the two types. Assuming elasticities differing by a factor of four (see Strombom, Buchmueller and Feldstein (2002) and Schut, Gress and Wasem (2004)), this implies that this correction to traditional risk adjustment is of the same order of magnitude as the mark-up itself. The effect is therefore significant in less than fully competitive insurance markets.

Second, the difference in switching behavior creates a trade off between efficiency and consumer welfare. In particular, starting at first best, introducing inefficiency by biasing the transfers against the high type unambiguously increases average consumer welfare. This is driven by competition leverage. Increasing the transfer of the low risk type relative to the high type induces insurers to compete more vigorously for low types. This has three implications. First, this selection behavior through second-degree price discrimination creates inefficiency. Second, competing more vigorously for low types reduces their insurance premium. Third, via the incentive compatibility (IC) constraint, high-risk types benefit from intensified competition in the low risk market as well. Starting from first best, the latter two effects dominate the former and consumer welfare increases. By biasing the risk adjustment transfers against the high types, competition is leveraged from the low risk market where consumers are likely to switch insurers, to the high risk market where the tendency to switch insurers is lower. To illustrate the power of competition leverage, we derive conditions under which maximizing the utility of the high type implies that the risk adjustment scheme taxes the high risk type and subsidizes the low type (proposition 2). Although this seems an extreme outcome, it does point to the possibility that high risk consumers may be better off without than with "conventional" risk adjustment as the latter may go in the wrong direction.

As a third implication, if people do not differ in their tendency to switch insurer and only differ in costs, optimal risk adjustment implies that each risk type gets efficient insurance. In the Rothschild-Stiglitz model this implies a pooling contract. Hence there is no benefit to enforce mandatory pooling in the health insurance market. Indeed, if people differ on other dimensions, such as taste for risk, enforcing pooling would be inefficient. However, in our model with different switching behavior mandatory pooling also has a benefit. In particular, mandatory pooling tightens the IC-constraint relating high to low type utility. As a result, a given level of competition leverage comes at lower distortion costs. Hence consumer welfare is higher under mandatory pooling than if firms are allowed to engage in second-degree price discrimination (proposition 4). In practice, this welfare gain needs to be weighed against reduced variety in contracts, which may reduce welfare if consumers have different tastes for

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risks.

Our paper is related to the health economics literature on health insurance and risk adjustment and the industrial organization (IO) literature on oligopoly with price discrimination. We discuss each in turn. The risk adjustment literature has two strands. The empirical literature, as mentioned above, tries to find the best prediction of an individual's next period health care costs. It focuses on variables that are easily available, such as age, gender, or zip code, so as to ensure that the estimated model can be used in practice. Theoretical papers, like Glazer and McGuire (2000), Glazer and McGuire (2002) and Jack (2006) analyze how imperfect signals about consumer types should be mapped into risk adjustment transfers. In these papers, absence of noise implies that perfect risk adjustment leads to efficient market outcomes. Our paper differs from these papers by explicitly modeling the relation between risk type and elasticity to switch insurers. As a consequence, even with perfectly informative signals, perfect risk adjustment is inefficient.

In the industrial economics literature there are a number of papers on price discrimination in an oligopoly setting. Examples include Stole (1995), Armstrong and Vickers (2001) and Schmidt-Mohr and Villas-Boas (1999) (see Stole (2007) for an overview). These papers focus on the characterization of the equilibrium. A policy question is whether allowing price discrimination raises total welfare. Below we show that, in health insurance markets, banning price discrimination by mandating a pooling contract can increase consumer welfare. Further, in a risk adjustment context a much richer set of policy instruments exists besides allowing or banning price discrimination. Policy makers can adjust the tax or subsidy per consumer type. We characterize the effects of such taxes and subsidies on efficiency (total welfare) and consumer welfare. Finally, leverage in the I.O. literature refers to firms leveraging market power from one market to the next. Instruments to do this include exclusionary contracts, vertical integration and bundling of products, see Rey and Tirole (2007) for an overview. In this paper, in contrast, we identify a way for the social planner to leverage competition from one market segment to the next.

Our analysis has two main policy implications. First, risk adjustment should not focus on the costs, i.e., the supply side, only. Efficiency cannot be fully restored in this way. Risk adjustment should take on board the literature that shows the relation between consumer type and tendency to switch insurer. We show how predicted costs and switching elasticities should be combined to find the risk adjustment transfers that maximize the planner's objective. Second, policy makers should be clear on their goals. As we demonstrate, if everyone has the

same tendency to switch insurers, efficiency, consumer welfare and the utility of the high risk type are all maximized by implementing the first best. Hence, the exact goal of the risk adjustment scheme is immaterial. However, in the empirically relevant situation the switching elasticity is negatively correlated with expected health care costs. The goal then does matter for the exact risk adjustment transfers. Therefore, it is important that the sponsor of the insurance plan, whether it is a government or an employer, is clear about its objective for risk adjustment.

The set up of our paper is as follows. We first introduce the model in section 2 and discuss conditions for the existence of symmetric equilibria. Section 3 derives optimal risk adjustment in case the planner's objective is total welfare. We then derive in section 4 how risk adjustment changes if, instead, the planner wants to maximize consumer welfare. Section 5 shows that mandatory pooling leads to higher consumer welfare than allowing the firms to price discriminate. As was stressed by Glazer and McGuire (2000), it is important to take into account that the planner only has imperfect information about consumer types. Section 6 shows that our results are robust to such noisy signals. We conclude with a discussion of policy implications.

2. The model

We introduce a model in which consumers have different expected medical costs and heterogeneous preferences over insurers. The former captures the adverse selection problem in insurance markets. The latter implies that insurers have market power. Recent evidence that market power is important in health insurance markets includes Dafny (2010). As discussed in the introduction, we explicitly take into account that people with high expected health care costs are less likely to switch insurer.

We follow the set up in Rothschild and Stiglitz (1976) and Newhouse (1996) to elegantly capture adverse selection and the inefficiency that comes with it. Suppose that there are two types of consumers, h and l, where the fraction of h types is denoted by $\lambda \in \langle 0, 1 \rangle$. A consumer of type $i = l, h$ has expected medical costs θ^i, with $\theta^h > \theta^l$. An insurance contract consists of a price p^i and a coverage q^i, implying that an agent has to pay fraction $1 - q^i$ of medical costs herself.

More generally, q can be interpreted as the generosity of the insurance contract. This can

take the form of copayments and deductibles but also whether the insured is free to choose the provider she wishes. That is, whether the insurance is fee-for-service, a provider from the network has to be chosen (as is common in managed care in the form of Health Maintenance Organizations), or something in between (such as in Preferred Provider Organizations). In each of these cases, higher reimbursement and more freedom is preferable for each consumer, but more so for the high risk type who is more likely to need treatment. Glazer and McGuire (2000), Jack (2001) and Olivella and Vera-Hernández (2007) chose a slightly different set up where the insurer offers contracts with a different mix of treatments for acute and chronic illnesses, respectively. Types differ in their preferences over the mix of treatments. Mathematically, both models are similar, although the interpretation differs slightly. Like in our model, in the Glazer and McGuire (2000) set up the efficient contract is a pooling contract.

To obtain a tractable parametrization, we use mean-variance preferences. The utility for a consumer of type i who buys an insurance contract with price p^i and coverage q^i is as follows

$$u^i = w - (1 - q^i)\theta^i - p^i - \tfrac{1}{2}r\sigma^2(1 - q^i)^2 \tag{1}$$

Here, w denotes the initial wealth of the agent, the variance of a consumer's expected medical costs equals σ^2, and $r > 0$ measures the agent's degree of risk aversion. We only consider insurance contracts with $q \in [0, 1]$. Contracts with $q > 1$ are ruled out as these would invite serious moral hazard problems: a consumer could then earn money by undergoing treatment. Similarly, $q < 0$ is ruled out because consumers would not report any treatment on which they spent money.

We have two symmetric insurers I_a and I_b who face demand on segment $i \in \{h, l\}$ given by $D^i(u_a^i, u_b^i) \geq 0$ and $D^i(u_b^i, u_a^i) \geq 0$, respectively. As demand is written in utility terms we have $D_1^i > 0, D_2^i < 0$, where D_1^i denotes the derivative of I_j's demand $(j = a, b)$ with respect to its own utility offer u_j^i on market segment $i = l, h$. Similarly, D_2^i denotes the derivative with respect to its opponent offer u_{-j}^i.

Because of moral hazard issues (see e.g. Pauly (1974)), an agent can buy at most one insurance contract. The individual rationality constraint can be written as

$$w - p - \theta^i(1 - q) - \tfrac{1}{2}r\sigma^2(1 - q)^2 \geq w - \theta^i - \tfrac{1}{2}r\sigma^2 \tag{2}$$

We will assume that this constraint never binds in equilibrium. That is, we focus on the

case where the relevant outside option for an agent is switching to the competing insurance company. This is called "full scale competition" (Schmidt-Mohr and Villas-Boas (1999)) or "pure competition" (Stole (1995)). As illustrated below, in a Hotelling model this is equivalent to assuming that the whole market is served. Another reason that can justify why we neglect the individual rationality constraint is that in many countries, health insurance is mandatory. The main reason for assuming pure competition is that it simplifies the budget constraint for risk adjustment, see equation (7) below.

These specifications imply that $D^i(u_a^i, u_b^i) + D^i(u_b^i, u_a^i) = \lambda^i$ where $\lambda^h = \lambda$ and $\lambda^l = 1 - \lambda$. Further, $D^i(u, u) = \frac{1}{2}\lambda^i$: if both insurers offer the same utility on segment $i \in \{l, h\}$, consumers split equally between them. With perfect competition on segment i we have $D_1^i(u^i, u^u)|_{u^i = u^u} = +\infty$, $D_2^i(u^i, u^u)|_{u^i = u^u} = -\infty$. In words, starting in a symmetric outcome ($u_a^i = u_b^i = u^i$), a small increase in u_a^i (u_b^i) makes sure that I_a gains (loses) the whole market. With less than perfect competition, say because consumers view insurers as selling differentiated products, the derivatives D_1^i, D_2^i are finite. In this case, insurers have market power.

In addition, we assume that in a symmetric equilibrium the h market is less elastic than the l market:
$$\frac{D_1^h(u^h, u^h)}{\frac{1}{2}\lambda} = \frac{D_1^h(u^h, u^h)}{D^h(u^h, u^h)} < \frac{D_1^l(u^l, u^l)}{D^l(u^l, u^l)} = \frac{D_1^l(u^l, u^l)}{\frac{1}{2}(1 - \lambda)} \tag{3}$$

for each u^h, u^l. This captures the observation that h-type consumers are less likely to switch insurer than l-types. As mentioned in the introduction, reasons why h-consumers are less likely to switch insurers include: they may be in the middle of treatment and do not want to switch insurer (e.g. due to provider network considerations), status quo bias may be stronger if a consumer interacted with the insurer before and low health consumers are more likely to have had such interactions in the past. Finally, $D_1^i(u^i, u^i)$ is independent of u^i ($i \in \{h, l\}$). Let us briefly discuss an example of the demand structure above.

Example 1 *Consider the case where insurer I_a (I_b) is on the far left (right) of the Hotelling beach of length 1. The travel cost t is the same for both consumer types. Consumers are distributed over the beach with different distributions for different types. The symmetric density function on $[0, 1]$ is given by $f(x)$ for a h-type and by $g(x)$ for an l-type, with respective cumulative distribution functions $F(x)$ and $G(x)$. We model the idea that the high types are less elastic in switching insurer by assuming that F has relatively more mass at the extremes and less in the middle. For G it is the other way around. Roughly speaking, θ^l types are willing to buy from any insurer while the high types are "biased" towards the insurer close to them.*

It is routine to verify that for a certain market segment when firm I_a offers utility u_a and I_b offers u_b the indifferent consumer is located at

$$x = \tfrac{1}{2} + \frac{u_a - u_b}{2t} \tag{4}$$

Therefore, firm I_a's market share in the θ^h market is given by $D^h(u_a^h, u_b^h) = \lambda F(\frac{1}{2} + \frac{u_a^h - u_b^h}{2t})$. If both insurers offer the same utility, the market is split fifty-fifty. If I_a offers higher utility, it gains market share from I_b and the lower t the faster this goes. Hence, lower t is interpreted as more intense competition and perfect competition corresponds to $t = 0$. Equation (3) can be written as

$$\frac{f(\tfrac{1}{2})}{F(\tfrac{1}{2})} < \frac{g(\tfrac{1}{2})}{G(\tfrac{1}{2})}$$

which is implied by the assumptions on the density functions f and g described above. Finally, in a symmetric equilibrium we have $D_1^h(u, u) = \lambda f(\frac{1}{2})$ independent from the level of u and similarly for the l-type.

An insurer offering two contracts (one intended for θ^l and one for θ^h)[4] needs to take into account the incentive compatibility constraints for the high and the low type:

$$IC_h : u^h \geq u^l - \Delta\theta(1 - q^l) \tag{5}$$
$$IC_l : u^l \geq u^h + \Delta\theta(1 - q^h) \tag{6}$$

where $\Delta\theta = \theta^h - \theta^l > 0$. It follows from adding the two incentive compatibility constraints that $q^h \geq q^l$. The constraint for the low type then implies that $p^h \geq p^l$.

Equation (5) shows the idea of competition leverage. If by stimulating competition on the l-market, the planner can raise u^l (by reducing p^l) the h-type benefits from this as well. Insurers will reduce q^l in response to limit the increase in u^h. Competition leverage benefits h-types if the former effect outweighs the latter.

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[4]Note that we consider second degree price discrimination, and assume third degree price discrimination to be prohibited. In the terminology of the health economics literature, we have community rating. If, say, the h-type would buy the contract intended for the l-type, he would buy the contract at the same price as the l-type. In section 5 we consider the case where second degree price discrimination is banned.

2.1. Risk adjustment

We assume that the government wants to use risk adjustment to improve the outcome in the insurance market. In particular, an insurer receives ρ^h (ρ^l) for each h (l) customer that it has. Here we assume that the government can perfectly observe each customer's type.[5] In section 6 we show that our results generalize to the case with imperfect observation of types.

To simplify notation, we assume that the overall budget for risk adjustment equals zero.[6] That is, the budget equation can be written as

$$\lambda\rho^h + (1-\lambda)\rho^l = 0 \tag{7}$$

Using equation (1), we can write the profit margin π^i (price minus expected cost plus the risk adjustment contribution) for type $i = l, h$ as

$$\pi^i(u^i, q^i) = p^i - q^i\theta^i + \rho^i = w - u^i - \theta^i + \rho^i - \tfrac{1}{2}r\sigma^2(1-q^i)^2 \tag{8}$$

Thus, risk adjustment affects market outcomes by changing the margin that health care insurers make on different types of consumers.

2.2. An insurer's optimization problem

As oligopoly models with price discrimination are not straightforward (see Stole (2007) for an overview), we need to be careful in characterizing equilibrium outcomes. This section derives sufficient conditions for the insurers' optimization problem to be well behaved.

When insurer I_b chooses u_b^h, u_b^l, we write insurer I_a's optimization problem as

$$\max_{u_a^h, u_a^l, q_a^h, q_a^l} \Pi(u_a^h, u_a^l, q_a^h, q_a^l; u_b^h, u_b^l) \tag{$P_{u_b^h, u_b^l}$}$$

subject to $q_a^h, q_a^l \in [0,1]$ and the IC constraints for the high-type and the low type (5) and (6),

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[5]This assumption is usually justified by assuming that the government has (ex post) more information than the insurer ex ante. Hence the insurer is not able to (explicitly) select risks ex ante but the government can perfectly risk adjust ex post. Alternatively, the insurer has the relevant information but is prevented by law to act upon this information. To illustrate, in the Netherlands an insurer cannot refuse a customer who wants to buy a certain insurance contract.

[6]In particular, below we use $d\rho^h = -(1-\lambda)/\lambda d\rho^l$. In words, we consider changes in risk adjustment for a given risk adjustment budget. As we do not analyze changes in the budget, we can normalize the budget to zero without loss of generality.

and where profits are given by

$$\Pi(u_a^h, u_a^l, q_a^h, q_a^l; u_b^h, u_b^l) = D^h(u_a^h, u_b^h)\pi^h(u_a^h, q_a^h) + D^l(u_a^l, u_b^l)\pi^l(u_a^l, q_a^l) \qquad (9)$$

Note that I_b's choice of q_b^h, q_b^l does not affect I_a's profits (for given u_b^h, u_b^l). Below, we focus on symmetric Nash equilibria of this optimization problem, defined as follows.

Definition 1 *The vector $(u^{h*}, u^{l*}, q^{h*}, q^{l*})$ forms a symmetric Nash equilibrium if $(u^{h*}, u^{l*}, q^{h*}, q^{l*})$ solves $(P_{u^{h*},u^{l*}})$; that is, I_b chooses $u_b^h = u^{h*}$ and $u_b^l = u^{l*}$.*

Directly analyzing this problem is not straightforward for two reasons. First, given I_b's strategy (u_b^h, u_b^l), I_a optimizes over four variables $(u_a^h, u_a^l, q_a^h, q_a^l)$. Second, it is routine to verify that insurer's optimization problem $(P_{u_b^h, u_b^l})$ is not concave in its four variables.[7] The problem is, however, considerably simplified if we assume that the incentive compatibility constraint for the high type is binding and that the incentive compatibility constraint for the low type is non binding. In that case, we can focus on the following problem.

$$\max_{u_a^h, u_a^l} \Pi(u_a^h, u_a^l, 1, 1 - \frac{u_a^l - u_a^h}{\Delta\theta}; u_b^h, u_b^l) \qquad (\hat{P}_{u_b^h, u_b^l})$$

This is an optimization problem with only two variables (for given u_b^h, u_b^l). Further, as we illustrate below, it is straightforward to derive sufficient conditions such that a stationary point of $(\hat{P}_{u_b^h, u_b^l})$ gives the solution to this problem. We first derive such a stationary point and then give conditions under which this point forms the solution to the original problem $(P_{u_b^h, u_b^l})$.

In our analysis below, we focus on the solution (u^{h*}, u^{l*}) characterized by the following

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[7]This can be seen as follows. $\Pi_{11} = D_{11}^h \pi^h - 2D_{11}$, $\Pi_{33} = -D^h r\sigma^2$ and $\Pi_{13} = D_1^h r\sigma^2(1 - q^h)$. Hence the requirement on the Hessian that $\Pi_{11}\Pi_{33} - (\Pi_{13})^2 > 0$ cannot be satisfied for values u^h so low that $D^h \approx 0$. Note that –strictly speaking– the Hessian does not need to be negative semi-definite. Because profits are maximized under constraints, the relevant requirements are on the bordered Hessian.

equations:

$$D_1^h(u^{h*}, u^{h*})\pi^h(u^{h*}, 1) - D^h(u^{h*}, u^{h*}) + D^l(u^{l*}, u^{l*})r\sigma^2\frac{u^{l*} - u^{h*}}{(\Delta\theta)^2} = 0 \tag{10}$$

$$D_1^l(u^{l*}, u^{l*})\pi(u^{l*}, q^{l*}) - D^l(u^{l*}, u^{l*}) - D^l(u^{l*}, u^{l*})r\sigma^2\frac{u^{l*} - u^{h*}}{(\Delta\theta)^2} = 0 \tag{11}$$

$$q^{h*} = 1 \tag{12}$$

$$q^{l*} = 1 - \frac{u^{l*} - u^{h*}}{\Delta\theta} \in [0, 1] \tag{13}$$

These equations give us the stationary point of $(\hat{P}_{u_b^h, u_b^l})$ for the case where $u_b^h = u^{h*}, u_b^l = u^{l*}$. As discussed above, we are not interested in parameter values which lead to $q^{l*} \notin [0, 1]$.

Sufficient conditions for problem $(\hat{P}_{u_b^h, u_b^l})$ to be concave are straightforward to derive.[8] We get the following for the elements of the Hessian:

$$\frac{\partial^2 \Pi_a}{\partial (u_a^h)^2} = D_{11}^h(u_a^h, u^{h*})\pi^h(u_a^h, 1) - 2D_1^h(u_a^h, u^{h*}) - D^l(u_a^l, u^{l*})\frac{r\sigma^2}{(\Delta\theta)^2} \tag{14}$$

$$\frac{\partial^2 \Pi_a}{\partial (u_a^l)^2} = D_{11}^l(u_a^l, u^{l*})\pi^l(u_a^l, 1 - \frac{u_a^l - u_a^h}{\Delta\theta}) - 2D_1^l(u_a^l, u^{l*})(1 + r\sigma^2\frac{u_a^l - u_a^h}{(\Delta\theta)^2}) - D^l(u_a^l, u^{l*})\frac{r\sigma^2}{(\Delta\theta)^2} \tag{15}$$

$$\frac{\partial^2 \Pi_a}{\partial u_a^l \partial u_a^h} = D_1^l(u_a^l, u^{l*})r\sigma^2\frac{u_a^l - u_a^h}{\Delta\theta} + D^l(u_a^l, u^{l*})\frac{r\sigma^2}{(\Delta\theta)^2} \tag{16}$$

To illustrate, we show that if demand is linear $(D_{11}^i = 0)$ sufficient conditions for concavity imply a restriction on the relative slopes D_1^h/D_1^l. With linear demand, it is immediate that $\frac{\partial^2 \Pi_a}{\partial (u_a^h)^2}, \frac{\partial^2 \Pi_a}{\partial (u_a^l)^2} < 0$. The determinant of the Hessian can be written as

$$\begin{aligned}
\frac{\partial^2 \Pi_a}{\partial (u_a^h)^2}\frac{\partial^2 \Pi_a}{\partial (u_a^l)^2} - \left(\frac{\partial^2 \Pi_a}{\partial u_a^l \partial u_a^h}\right)^2 = {} & 4D_1^h D_1^l + 2D_1^h D^l\frac{r\sigma^2}{(\Delta\theta)^2} + 2D^l D_1^l\frac{r\sigma^2}{(\Delta\theta)^2} \\
& + D_1^l r\sigma^2\frac{u_a^l - u_a^h}{(\Delta\theta)^2}\left(4D_1^h - D_1^l r\sigma^2\frac{u_a^l - u_a^h}{(\Delta\theta)^2}\right)
\end{aligned} \tag{17}$$

For profits to be concave in u_a^h, u_a^l, the determinant of the Hessian must be positive for all values u_a^h, u_a^l that satisfy the incentive compatibility constraints. A sufficient condition for this

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[8]In fact, quasi-concavity of Π is already sufficient for a stationary point being the global maximum of Π. Olivella and Vera-Hernández (2007) give an example of a Hotelling model where the symmetric equilibrium is characterized by equations (10)–(13).

is $D_1^h/D_1^l > r\sigma^2/4$.

Below we focus on equations (10)–(13) as solution to problem $(\hat{P}_{u_b^h,u_b^l})$. The following lemma gives sufficient conditions under which such a solution is a symmetric Nash equilibrium. In order to state and discuss the intuition of the lemma, we define $\tilde{u}^i$ as follows:

$$\tilde{u}^i = \arg\max_u D^i(u, u^{i*})\pi^i(u, 1)$$

In words, $\tilde{u}^i$ maximizes profits when the insurer focuses on the i market (where its opponent offers u^{i*} as determined by equations (10) and (11)) and ignores the other market.

Lemma 1 *Assume that equations (10)–(13) solve problem $(\hat{P}_{u^{h*},u^{l*}})$ and that*

$$D^h(\tilde{u}^l, u^{h*})(w - \theta^h + \rho^h - \tilde{u}^l) \geq 0.$$

Then $(u^{h*}, u^{l*}, 1, q^{l*})$ solves $(P_{u^{h*},u^{l*}})$ and hence is a symmetric Nash equilibrium.

The assumption makes sure that focusing on the l-market leads to non-negative profits on the h market. As shown in the proof, this excludes the possibility that a corner solution dominates an interior stationary point in terms of profits.[9]

2.3. Total welfare

Throughout this paper, we will consider both total and consumer welfare. Papers that consider an insurance market with zero profits, for example due to perfect competition, cannot make this distinction. In that case, by necessity, consumer welfare equals total welfare. However, in our case the insurers have market power and the difference between consumer and total welfare is important. As shown below, optimal risk adjustment depends on whether the planner wants to maximize total welfare (efficiency) or consumer surplus.

In symmetric equilibrium, total welfare is defined as

$$W = \lambda(w - \theta^h - \tfrac{1}{2}r\sigma^2(1 - q^h)^2) + (1 - \lambda)(w - \theta^l - \tfrac{1}{2}r\sigma^2(1 - q^l)^2) \tag{18}$$

The first term on the right hand side corresponds to total utility of the high type plus the profit of the insurers on those types. The second term corresponds to total profit on, and utility of,

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[9]See Olivella and Vera-Hernández (2007) for a numerical analysis of the case with corner solutions.

the low type. Note that we focus on symmetric equilibria. Hence the (dis)utility a consumer experiences when buying one brand rather than the other, the travel costs in our Hotelling example, does not change in the comparative static exercises that we do below. Therefore we ignore this term in the definition of welfare.

3. Total welfare

We first consider a market where the insurers can use second degree price discrimination by offering two contracts: one contract for the h-type and one for the l-type. Section 5 considers the case where each insurer is forced by the government to offer only one contract (mandatory pooling).

We write problem $(\hat{P}_{u_b^h, u_b^l})$ here as follows:

$$
\begin{aligned}
\max_{u_a^h, u_a^l, q_a^h, q_a^l} D^h(u_a^h, u_b^h)(w - \theta^h + \rho^h - \tfrac{1}{2}r\sigma^2(1 - q_a^h)^2 - u_a^h) \\
+ D^l(u_a^l, u_b^l)(w - \theta^l + \rho^l - \tfrac{1}{2}r\sigma^2(1 - q_a^l)^2 - u_a^l) \\
+ \mu(u_a^h - u_a^l + \Delta\theta(1 - q_a^l))
\end{aligned}
\tag{19}
$$

with a similar expression for insurer I_b. μ denotes the shadow price of the incentive compatibility constraint. Following lemma 1, we characterize the symmetric Nash equilibrium with the first order conditions, which can be written as

$$
q^h = 1 \tag{20}
$$
$$
-D^h(u^h, u^h) + D_1^h(u^h, u^h)(w - \theta^h + \rho^h - u^h) + \mu = 0 \tag{21}
$$
$$
-D^l(u^l, u^l) + D_1^l(u^l, u^l)(w - \theta^l + \rho^l - \tfrac{1}{2}r\sigma^2(1 - q^l)^2 - u^l) - \mu = 0 \tag{22}
$$
$$
D^l(u^l, u^l)r\sigma^2(1 - q^l) = \mu\Delta\theta \tag{23}
$$

We immediately get that the h-type receives full insurance. Coverage for the low type is given by

$$
1 - q^l = \frac{\mu\Delta\theta}{\tfrac{1}{2}(1 - \lambda)r\sigma^2}
\tag{24}
$$

where we use that $D^l(u, u) = \tfrac{1}{2}(1 - \lambda)$. Hence, the lower μ (i.e., the less tightly the incentive compatibility constraint binds), the higher the coverage for the low type.

Using the first order conditions in equations (20)–(23), total welfare can be written as

$$W = w - \lambda\theta^h - (1-\lambda)\theta^l - \frac{(\mu\Delta\theta)^2}{\frac{1}{2}(1-\lambda)r\sigma^2}$$

This expression shows that total welfare increases as μ decreases. This result obtains because total welfare is only affected by efficiency. Prices are a pure transfer between consumers and insurers. As μ decreases, q^l increases and hence efficiency as well as total welfare rise.

As mentioned in the introduction, the existing risk adjustment literature focuses on equalizing costs. At first sight, this seems equivalent to equalizing price cost margins. As the h-type is less elastic in switching insurer than the l-type, the mark up is higher for the h-type. Naively, one might expect that the higher mark up reduces the compensation to be paid for higher cost consumers to make insurers indifferent. Efficient risk adjustment would then undershoot the cost difference: $\rho^h - \rho^l < \theta^h - \theta^l$. However, the next result shows that this intuition is incorrect.

Proposition 1 *There exist ρ^h, ρ^l that implement first best as an equilibrium outcome where both types buy the same contract with efficient insurance ($q^{l,h} = 1$). It has $\rho^h > \rho^l$, that is, risk adjustment in the standard direction, and $\rho^h - \rho^l > \theta^h - \theta^l$, that is, risk adjustment has to overshoot the difference in expected costs between types.*

The intuition behind this result is the following. It follows from the IC-constraints (5) and (6) that we can only get efficiency ($q^l = 1$) when the utilities u^h, u^l are equalized. Which risk adjustment $\rho^h - \rho^l$ causes insurers to offer both types the same utility? To achieve $u^h = u^l$ we need to compensate insurers both for the higher expected cost $\theta^h - \theta^l$ and for the missed higher mark up $\frac{\lambda}{D_1^h(u^h,u^h)} - \frac{1-\lambda}{D_1^l(u^l,u^l)} > 0$ for the h-type.[10] If insurers are forced to offer both types the same utility, they face the opportunity cost of the higher mark up on the h-type. If this opportunity cost is overlooked (such that $\rho^h - \rho^l = \theta^h - \theta^l$) insurers have an incentive to separate the types by extracting rents from the inelastic h-type and competing more vigorously for the elastic l-type, leading to $q^l < 1$.

In other words, the current risk adjustment models with their exclusive focus on the cost/supply side will not restore efficiency in health insurance markets. Even if one would perfectly compensate for cost differences between types, ignoring their differing demand elasticities leads to selection behavior by insurers and hence inefficient health insurance.

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[10]Note that this expression equals the difference in price over elasticity for each type.

If risk adjustment corrects for both the difference in costs and the difference in mark up, and consequently overshoots the standard risk adjustment ($\rho^h - \rho^l > \theta^h - \theta^l$), we find $\mu = 0$. We call this *first best* or *efficient risk adjustment*. Efficient risk adjustment implies that the insurer can optimize the contracts of each type separately without worrying about IC and the resulting contracts will satisfy IC. Optimizing each contract separately induces the insurer to give each type efficient insurance.

Note that in the equilibrium in proposition 1, insurance companies offer pooling contracts. Hence making pooling mandatory will not affect the outcome in this case. However, as shown in section 5, this does not imply that mandatory pooling is an irrelevant instrument for the planner.

4. Consumer welfare

The analysis above on efficiency and first best is the standard analysis of risk adjustment in papers like Glazer and McGuire (2002) and Jack (2006). However, as mentioned in the introduction, it is not obvious that this is the appropriate analysis in a context with imperfect competition. Competition authorities and regulators around the world explicitly claim that their objective is consumer welfare, not total welfare. Further, in the health care sector, policy motivations for interventions such as risk adjustment usually focus on consumers and solidarity between different types of consumers, rather than on efficiency.[11]

In order to analyze the consequences for risk adjustment of such a consumer focus, we introduce the following objective function for the social planner:

$$CS_\omega = \omega u^h + (1 - \omega)u^l \tag{25}$$

with $\omega \in [\lambda, 1]$. With $\omega = \lambda$, the planner maximizes consumer surplus. With $\omega > \lambda$ the planner gives relatively more weight to the (unfortunate) h-type than to the (lucky) l-type. This is a simple way to formalize solidarity with the h-type.[12]

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[11]Relatedly, in his overview paper Ellis (2008) mentions "the concept of 'optimal risk adjustment' in which the sponsor's goal is to maximize consumer welfare rather than to just break even."

[12]The underlying assumption is that the h-type was born with, say a chronic disease like diabetes. Hence θ^h's high expected health care costs are exogenously given. Then fairness or solidarity considerations can lead the planner to give a higher weight ($\omega > \lambda$) to the h-type in the objective function. Alternatively, high health care costs can be endogenous due to, for example, smoking behavior, food habits, drug use etc. See Van de Ven

Let us first consider consumer utilities as a function of the shadow price μ of the incentive compatibility constraint. Using (21)–(23), we write

$$u^h = \frac{1}{D_1^h}(\mu - \tfrac{1}{2}\lambda) + w - \theta^h + \rho^h \tag{26}$$

$$u^l = -\frac{1}{D_1^l}(\mu + \tfrac{1}{2}(1 - \lambda)) + w - \theta^l + \rho^l - \frac{(\mu\Delta\theta)^2}{\tfrac{1}{2}r\sigma^2(1-\lambda)^2}. \tag{27}$$

where we use the shorthand notation $D_1^i = D_1^i(u, u)$ ($i \in \{l, h\}$) when this does not cause confusion. The social planner can adjust $\rho^{h,l}$, subject to the budget constraint (7), to optimize consumer welfare CS_ω. But note that a change in risk adjustment also affects the shadow price of incentive compatibility, μ. In particular, an increase in ρ^l (at the cost of reducing ρ^h) makes it more attractive for insurers to compete for the l-type. This increases u^l and hence tightens incentive compatibility for the high type. As a consequence, μ increases. We identify competition leverage with this increase in μ as it shows the extent to which the planner uses IC constraint (5) to increase u^h. The increase in ρ^l and the increase in μ have opposite effects on the utilities, as is clear from equations (26) and (27). Depending on the relative strength of the direct effect (reduced ρ^h) and the indirect effect (increased μ), the high types might actually win if subsidies towards them are reduced from their first-best values.

The idea of competition leverage is to use the binding IC constraint (5) to let the h-type benefit from the more intense competition in the l-market. To find the effect of μ on u^h we add λ times equation (26) to $(1 - \lambda)$ times equation (27). This eliminates $\rho^{h,l}$ and the result can be written as:

$$u^h = \mu\left(\frac{\lambda}{D_1^h} - \frac{1-\lambda}{D_1^l}\right) - \frac{\mu^2(\Delta\theta)^2}{\tfrac{1}{2}r\sigma^2(1-\lambda)} - (1-\lambda)(u^l - u^h) + \text{constant} \tag{28}$$

where the constant does not depend on μ and

$$u^l - u^h = \Delta\theta(1 - q^l) = \mu\frac{(\Delta\theta)^2}{\tfrac{1}{2}r\sigma^2(1-\lambda)}$$

using equations (5) and (24). Note that the constant in equation (28) equals u^h in case $\mu = 0$. Equation (28) shows that an increase in μ affects u^h through three channels that we label margin effect (ME), distortion effect (DE) and inequality effect (IE). We discuss each of these

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and Ellis (2000) for a discussion of the limits to solidarity due to such moral hazard effects.

in detail.

We define the *margin effect* as

$$ME = \left(\frac{\lambda}{D_1^h} - \frac{1-\lambda}{D_1^l} \right) > 0 \tag{29}$$

This effect is the driving force of competition leverage. As the l-market is more competitive than the h-market, heating up competition between insurers in the l-market by increasing ρ^l leads to higher u^l. By the IC constraint this increases u^h and μ as well. Clearly, if both markets would be equally competitive, then $ME = 0$.

We define the *distortion effect* as

$$DE = 2\frac{1}{(1-\lambda)}\frac{\mu(\Delta\theta)^2}{\frac{1}{2}r\sigma^2} > 0 \tag{30}$$

The total distortion in the market equals the fraction $(1-\lambda)$ of the population affected times their dis-utility from $q^l < 1$:

$$(1-\lambda)\tfrac{1}{2}r\sigma^2 \left(\frac{\mu\Delta\theta}{\frac{1}{2}r\sigma^2(1-\lambda)} \right)^2 \tag{31}$$

The distortion effect equals the derivative of this expression with respect to μ. Note that although $q^h = 1$, equation (28) shows that the distortion does reduce u^h. This effect is driven by IC constraint (5) linking the utilities of the two types. Through this constraint the distortion reduces u^h as well. Clearly, with $\mu = 0$ there is no distortion and by the envelope theorem a small increase in μ has no first-order effect. However, this does not imply that the utilities of the two types remain equal after an increase in μ. This brings us to the last effect.

Starting from $\mu = 0$, an increase in μ creates or increases a wedge between u^h and u^l. Once $q^l < 1$, l-types can always mimic h-types and they are still better off as their expected expenditure is lower. This is the *adverse selection* or *inequality effect*:

$$IE = \frac{(\Delta\theta)^2}{\frac{1}{2}r\sigma^2(1-\lambda)} > 0 \tag{32}$$

This observation that l-types are always weakly better off than h-types no matter which contracts are offered is the motivation for solidarity. The unfortunate h-type is worse off than the l-type and it is not possible to make the h-type strictly better off than the l-type. Increasing μ increases the wedge between the utilities: $d(u^h - u^l)/d\mu = IE > 0$.

We can do a similar analysis for the effect of μ on u^l. The following proposition summarizes these decompositions of the effect of competition leverage on utilities. Moreover, to illustrate the strength of competition leverage, we derive a condition under which u^h is maximized by having $\rho^h < 0$.

Proposition 2 *We can decompose the effect of μ on the agents' utilities and CS_ω as follows*

$$\frac{du^h}{d\mu} = ME - (DE + (1 - \lambda)IE) \tag{33}$$

$$\frac{du^l}{d\mu} = ME - (DE - \lambda IE) \tag{34}$$

$$\frac{dCS_\omega}{d\mu} = ME - (DE + (\omega - \lambda)IE) \tag{35}$$

If $\omega = \lambda$, optimization of CS_λ leads to $\mu > 0$ and $q^l < 1$.
If $ME > (1-\lambda)IE$, then an increase in μ away from its first-best value (leveraging competition) increases CS_ω for each $\omega \in [\lambda, 1]$. In fact, both types win from such an increase in μ.
For IE close enough to zero, we find that u^h is maximized by setting $\rho^h < 0$.

First of all, in the absence of a margin effect ($ME = 0$), as the existing risk adjustment literature assumes, the proposition implies that efficient risk adjustment ($\mu = 0$) maximizes both u^h and CS_ω for each $\omega \in [\lambda, 1]$. In this sense, the existing literature is correct in focusing on first best because efficiency and solidarity go hand in hand. However, as discussed in the introduction it is well documented that h-types are less likely to switch insurer compared to l-types. This difference in switching behavior implies that $ME > 0$ and therefore that efficiency and consumer welfare start to diverge.

Starting from first best (where $\mu = 0$ and hence $DE = 0$) an increase in μ increases CS_λ. Competition leverage raises consumer surplus while reducing efficiency. Hence, from an (average) consumer perspective, efficiency ($q^l = 1$) is, in fact, not optimal. If, moreover, $ME > (1 - \lambda)IE$, we see that $du^h/d\mu|_{\mu=0} > 0$. Since the low type's utility u^l always increases with μ at $\mu = 0$, in that case $\frac{dCS_\omega}{d\mu}|_{\mu=0} > 0$ for each $\omega \in [\lambda, 1]$. Roughly speaking, if the margin effect exceeds the inequality effect, even the h-types benefit from competition leverage. Increasing ρ^l (and hence reducing ρ^h) leads to higher u^h and hence to higher CS_ω.

The intuition is that higher ρ^l intensifies competition on the l-market. In order to pocket the higher ρ^l for more low-type consumers, insurers raise u^l. Via intra-brand competition (IC)

this forces them to raise u^h as well. This effect is partly undone by increasing the distortion (reducing q^l) but under the assumption that IE is sufficiently small compared to the difference in mark ups, the intra-brand effect outweighs the inequality effect, such that u^h increases with ρ^l.

For IE close enough to zero, the ρ^h maximizing u^h is negative. The intuition for the condition that IE should be small is the following. At first best, $\mu = 0$, we have $\rho^h > 0$ (see proposition 1) hence μ needs to fairly high for ρ^h to turn negative. When maximizing u^h, μ will be high if the costs of increasing μ are relatively small. As both DE and IE are driven by the term $\frac{(\Delta\theta)^2}{\frac{1}{2}r\sigma^2(1-\lambda)}$, the cost of increasing μ is small if IE is small. This leads to such a high μ that ρ^h actually turns negative in maximizing u^h.

Note that the last result implies that for IE close to zero, all consumers are better off without risk adjustment than with efficient risk adjustment. Indeed, the efficient risk adjustment goes in the wrong direction. Although this result may be extreme, it does point to the following more general policy implication. Even though from an efficiency point of view, risk adjustment is desirable, a planner that invokes consumer solidarity may want to abstain from risk adjustment if consumers are sufficiently risk averse (high $r\sigma^2$) or the difference between types ($\Delta\theta$) is small. We are not aware of empirical studies trying to quantify IE so that it can be compared to the difference in elasticities found in papers like Royalty and Solomon (1999) and Schut, Gress and Wasem (2004). Nevertheless, $\Delta\theta$ seems more likely to be small for an employer offering health insurance to its employees than for a whole country combining risk adjustment with mandatory insurance.

5. Mandatory pooling

As shown in proposition 1, efficient risk adjustment implies that insurers offer both types the same contract. This might suggest that there is no role for mandatory pooling, as long as risk adjustment is chosen optimally. In this section, we show that this intuition is incorrect. Mandating pooling leads to higher CS_ω than allowing for second-degree price discrimination (separating contracts). Moreover, the consumer welfare maximizing insurance contract features inefficient insurance ($q < 1$).

In this section, we assume that each insurer is allowed to offer only a single contract with some price p and a co-payment $1 - q$. Hence, the insurer cannot price discriminate between

types. The margin π^i on type $i = l, h$ therefore equals

$$\pi^i = p - q\theta^i + \rho^i.$$

Given a price and a co-payment, the utility of the h-type satisfies

$$p = w - u^h - (1-q)\theta^h - \tfrac{1}{2}r\sigma^2(1-q)^2 \tag{36}$$

and equation (5) holding with equality determines the utility that the l-type receives given the utility of the h-type. However, the latter is not an IC constraint here, as there is only one contract, but an "accounting identity", which relates the utilities of the h-type and the l-type consumer. Put differently, this equation ensures that $p^h = p^l = p$ when $q^h = q^l = q$.

Hence, insurer I_a's optimization problem can be written as

$$\max_{u_a^h, u_a^l, q} D^h(u_a^h, u_b^h)(w - \theta^h + \rho^h - \tfrac{1}{2}r\sigma^2(1-q)^2 - u_a^h)$$
$$+ D^l(u_a^l, u_b^l)(w - \theta^l + \rho^l - \tfrac{1}{2}r\sigma^2(1-q)^2 - u_a^l) + \mu(u_a^h - u_a^l + (1-q)\Delta\theta)$$

The optimization problem is similar to equation (19), with the additional requirement that $q^l = q^h$. The first order conditions for q, u^h and u^l in symmetric equilibrium can be written as[13]

$$\tfrac{1}{2}r\sigma^2(1-q) = \mu\Delta\theta \tag{37}$$

$$-D^h(u^h, u^h) + D_1^h(u^h, u^h)(w - \theta^h + \rho^h - \tfrac{1}{2}r\sigma^2(1-q)^2 - u^h) + \mu = 0 \tag{38}$$

$$-D^l(u^l, u^l) + D_1^l(u^l, u^l)(w - \theta^l + \rho^l - \tfrac{1}{2}r\sigma^2(1-q)^2 - u^l) - \mu = 0 \tag{39}$$

Writing again $D_1^i = D_1^i(u^i, u^i)$, the types' utilities in the pooling equilibrium are given by

$$u^h = \frac{1}{D_1^h}(\mu - \tfrac{1}{2}\lambda) + w - \theta^h + \rho^h - \frac{(\mu\Delta\theta)^2}{\tfrac{1}{2}r\sigma^2} \tag{40}$$

$$u^l = -\frac{1}{D_1^l}(\mu + \tfrac{1}{2}(1-\lambda)) + w - \theta^l + \rho^l - \frac{(\mu\Delta\theta)^2}{\tfrac{1}{2}r\sigma^2}. \tag{41}$$

---

[13]Note that if equations (20)–(23) solve (P_{u^h, u^l}), then (37)–(39) solve (P_{u^h, u^l}) with the additional constraint that $q_a^h = q_a^l = q_a$. Intuitively, by limiting possible deviations for insurers (by requiring $q^h = q^l$) it becomes easier to move from local conditions to a global optimum.

Note the difference with equation (26), where the last term in the utility of the h-type consumers is missing because they receive full insurance ($q^h = 1$).

It is obvious that with mandatory pooling, the regulator could again choose risk adjustment as given by (51) in the appendix, i.e., to optimize efficiency. Clearly, then $\mu = 0$ and consumers are fully insured, $q = 1$ (equation (37)). However, as shown below, if the objective is to maximize weighted consumer surplus CS_ω, mandatory pooling leads to lower ρ^h, lower q and higher μ than their respective first best values.

Adding λ times equation (40) to $(1 - \lambda)$ times (41) yields

$$u^h = \mu \left(\frac{\lambda}{D_1^h} - \frac{1 - \lambda}{D_1^l} \right) - \frac{\mu^2 (\Delta\theta)^2}{\frac{1}{2} r\sigma^2} - (1 - \lambda)(u^l - u^h) + \text{constant} \tag{42}$$

where the constant[14] does not depend on μ and

$$u^l - u^h = \Delta\theta(1 - q) = \mu \frac{(\Delta\theta)^2}{\frac{1}{2} r\sigma^2}$$

Although DE and IE could be defined differently in the pooling case, we choose to use the definitions in equations (30) and (32) to facilitate the comparison of the two cases in proposition 4 below.

Proposition 3 *We can decompose the effect of μ on the agents' utilities and CS_ω as follows*

$$\frac{du^h}{d\mu} = ME - (1 - \lambda)(DE + (1 - \lambda)IE) \tag{43}$$

$$\frac{du^l}{d\mu} = ME - (1 - \lambda)(DE - \lambda IE) \tag{44}$$

$$\frac{dCS_\omega}{d\mu} = ME - (1 - \lambda)(DE + (\omega - \lambda)IE) \tag{45}$$

If $\omega = \lambda$, optimization of CS_λ leads to less than perfect risk adjustment: $q < 1$.
If $ME > (1-\lambda)^2 IE$, optimization of consumer surplus CS_ω (for any $\omega \in [\lambda, 1]$) with mandatory pooling leads to less than perfect risk adjustment: $q < 1$.

Hence, we find again that efficient risk adjustment is not in the interest of consumers. At $\mu = 0$, CS_λ is increasing in μ: competition leverage raises consumer surplus. In fact, comparing

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[14]The constant equals u^h in case $\mu = 0$. Moreover, in case $\mu = 0$ proposition 1 leads to the same outcome as with mandatory pooling. Hence, the constant in equation (42) has the same value as the one in (28).

the expressions for $du^h/d\mu$ and $dCS_\omega/d\mu$ in the proposition above with the expressions in proposition 2 shows that the benefits of increasing μ (ME) are the same in both cases, but the costs (DE and IE) are a factor $1 - \lambda < 1$ smaller in the pooling case. This shows that competition leverage is less costly in the pooling case and can be used to a bigger extent (higher μ). Indeed, the following result shows that under mandatory pooling CS_ω cannot be lower than when allowing for second degree price discrimination and is strictly higher if $q < 1$.

Proposition 4 *If $\omega = \lambda$ then $CS_\lambda^{pool} > CS_\lambda^{sep}$.*
If $ME > (1 - \lambda)^2 IE$, then mandatory pooling leads to higher CS_ω than second degree price discrimination:

$$CS_\omega^{pool} > CS_\omega^{sep}$$

for each $\omega \in [\lambda, 1]$.

If $\omega = 1$ then $ME < (1 - \lambda)^2 IE < (1 - \lambda)IE$ implies that efficient insurance ($\mu = 0$) is optimal in both proposition 2 and proposition 3. Even if separation is allowed, the outcome is pooling and hence utilities and welfare are the same in both cases.

However, if either $\omega = \lambda$ or $ME > (1 - \lambda)^2 IE$ then competition leverage is used to a bigger extent under pooling because the costs of doing so are smaller than in the case where price discrimination is allowed. Hence consumers are better off under mandatory pooling than under separation.

The intuition for this is as follows. As the planner increases ρ^l, insurers compete more fiercely for l-types thereby reducing p^l and increasing u^l. The IC constraint (5) then implies that either u^h increases as the planner intended, or coverage q falls because this is the insurers' instrument to keep u^h low. When second degree price discrimination is allowed, reducing coverage q^l is relatively cheap for insurers. Indeed, q^l only affects profits on the l-market. However, with mandatory pooling, reducing q affects profits on the both the l- and h-market segment. This makes it expensive for insurers to use q as a "defense mechanism" against competition leverage. Hence firms reduce q less (compared to q^l) in response to competition leverage μ. Since the reduction in q is the planner's cost of competition leverage, the cost terms (DE and IE) are smaller in proposition 3 compared to proposition 2. Due to the lower cost, if competition leverage is used under mandatory pooling, it leads to higher consumer welfare than second degree price discrimination.

Finally, by solving $dCS_\omega/d\mu = 0$ for the optimal q in both the case where separation (q^{sep})

is allowed and with mandatory pooling (q^{pool}) leads to the following result.

Corollary 1 *If $\omega = \lambda$ then $(1 - q^l)^{sep} = (1 - q)^{pool}$. If $\omega > \lambda$ and $q^{pool} < 1$ then $(1 - q^l)^{sep} < (1 - q)^{pool}$.*

In words, in the case where $\omega = \lambda$, we know from proposition 4 that mandatory pooling leads to higher consumer surplus than allowing for second degree price discrimination because more competition leverage (higher μ) is used under mandatory pooling. Since both μ and the efficiency loss at given q grow with the same factor when switching from separation to pooling, for utility function (1) CS_λ is optimized at the same $1 - q$ under both separation and pooling. Under separation, this distortion only applies to the l-type while with mandatory pooling this distortion applies to everyone. If $\omega > \lambda$ and some distortion is introduced under mandatory pooling (to rule out the case where $q^{pool} = q^{sep} = 1$), then the distortion with mandatory pooling is actually higher. In both cases, the $(1 - q)^{pool}$ distortion leads to lower costs of competition leverage and hence $CS_\omega^{pool} > CS_\omega^{sep}$.

6. Imperfect observation of risk

Above, we assume that the planner can ex post perfectly observe each customer's type. In reality, the planner may only have a noisy signal of a customer's θ. The question is: are the results above robust to the imperfect observation of risk?

Following Glazer and McGuire (2000), let us assume that the planner cannot perfectly observe the consumers' types. Rather, the planner observes an imperfect signal (H, L) of consumers' types (h, l). With probability P^h, the h-type will give a high signal H, and with probability $1 - P^h$ the high type's signal will be L. Likewise, with probability P^l the l-type's signal will be H (and with $1 - P^l$ it will be L). Of course, for the signals to be informative at all we require

$$0 \le P^l \le \tfrac{1}{2} \le P^h \le 1.$$

Thus the h-type will more likely produce the H-signal, and the l-type will more likely produce signal L.

The planner can now base the risk adjustment only on the observed signal (H, L). Consumers, however, know their types and base their choices on their actual types (h, l). Call the

risk adjustment based on the signal $\rho^{H,L}$. Then an insurer will get an effective subsidy on the high types it attracts equal to

$$\tilde{\rho}^h = P^h \rho^H + (1 - P^h)\rho^L,$$

and likewise the effective subsidy for the low types will equal

$$\tilde{\rho}^l = P^l \rho^H + (1 - P^l)\rho^L.$$

Note that the $\tilde{\rho}$'s interpolate between the ρ's.

To analyze the optimal $\rho^{H,L}$ in this case, note that our analysis when signals are perfect fully carries through with ρ replaced by $\tilde{\rho}$. Consequently, the optimal subsidies that we found in that case are the optimal effective subsidies $\tilde{\rho}$. The subsidies the planner should give based on the signals (H, L) should therefore be an exaggeration of the optimal subsidies, with positive subsidies being increased and negative ones being decreased. This observation is independent of whether the planner enforces mandatory pooling or not. Hence we get in both cases:

$$\rho^H = \frac{1}{P^h - P^l}\left((1 - P^l)\tilde{\rho}^h - (1 - P^h)\tilde{\rho}^l\right), \tag{46}$$

$$\rho^L = \frac{1}{P^h - P^l}\left(-P^l\tilde{\rho}^h + P^h\tilde{\rho}^l\right). \tag{47}$$

To illustrate, when it is optimal to subsidize the low types ($\tilde{\rho}^l > 0$) at the expense of the high types (proposition 2), imperfect observation of types only makes this stronger in the sense that $\rho^H < \tilde{\rho}^h < 0 < \tilde{\rho}^l < \rho^L$. We conclude that our results are, indeed, robust to the introduction of noisy signals about consumers' types.

7. Conclusion

In several countries, imperfect competition in markets for health care insurance goes hand in hand with risk adjustment. The resources allocated to risk adjustment are significant; e.g. approximately 3% of GDP in the Netherlands. Conventional risk adjustment focuses on the supply side and tries to correct as well as possible for cost differences between high-cost and low-cost consumers. Existing theoretical papers argue that this improves efficiency by reducing

insurers' selection incentives, while at the same time guaranteeing access to health insurance for high-risk consumers by lowering insurance premiums.

We study the consequences of the demand side for optimal risk adjustment. In particular, we take into account that consumers' price elasticities are negatively correlated with their expected health care costs. We find that the difference in switching behavior creates a trade off between efficiency and consumer welfare. From an efficiency point of view, to nullify selection incentives, risk adjustment needs to overcompensate the high risk types (relative to their costs) to get an efficient outcome in the insurance market. From the point of view of consumer welfare, reducing the level of risk adjustment as compared to the first best increases consumer welfare by leveraging competition from the elastic low-risk market to the less elastic high-risk market. Mandatory pooling can increase consumer surplus even further, at the cost of efficiency.

What are the policy implications of our findings? First, an exclusive focus on increasing the accuracy of cost forecasts in refining risk adjustment systems is misguided. Risk adjustment resources will be partially wasted by this focus on costs. Risk adjustment systems should take into account the impact they have on insurer competition, and the possibility of leveraging competition that arises because of the relation between consumer type and tendency to switch insurers. Our model shows how the literature studying this relation can be taken on board. Accounting for the demand side of health insurance may be easier than it seems, because the empirical literature studying consumers' brand sensitivity uses many of the same explanatory variables as the literature on risk adjustment. Our analysis also shows that nullifying selection incentives using risk adjustment does not benefit consumers. Thus, risk selection by insurers should not be viewed as bad per se, as seems the case in policy debates.

Second, policy makers should be clear on their goals. When setting risk adjustment levels, policy makers should decide whether their goal is to optimize total welfare or consumer welfare. Although conventional risk adjustment most likely promotes efficiency, it will reduce consumer surplus under the – in our view – realistic assumptions that competition in health-insurance markets is imperfect and healthy consumers are more price elastic than high-risk consumers. If the aim of policy makers is to reduce high-risk consumers' costs in a market with adverse selection, risk adjustment should 'undershoot' relative to the first best. In extreme cases, risk adjustment may even run in the opposite direction. That is, high risk types are better off if they are taxed to finance a subsidy for the low risk types.

On a more general level, our analysis points out the possibility of leveraging competition

from one market to the other. In imperfectly competitive markets with second degree price discrimination, we show that policy intervention in the form of taxes and subsidies may increase competition by capitalising on incentive compatibility constraints. In markets for health insurance, a framework for such a policy, i.e. the risk adjustment system, already exists and can be used to this effect. One may wonder whether the idea could be used in other markets, too. As an example, Dessein (2003) considers competition between telecom networks in markets with different types of consumers, those with high and low demand for calls. When price elasticities differ among those types, Dessein finds that interconnection fees (the fees which networks charge each other to put through calls to their consumers) may be used to influence the level of competition in a way that is related to the mechanism we describe in this paper. Regulators wanting to influence network competition can take this mechanism into account when regulating interconnection fees.

References

Armstrong, J., F. Paolucci, H. McLeod and W. Van de Ven. 2010. "Risk equalisation in voluntary health insurance markets: a three country comparison." *Health Policy* 98:39–49.

Armstrong, M. and J. Vickers. 2001. "Competitive price discrimination." *RAND Journal of Economics* 32(4):579–605.

Beaulieu, Nancy Dean. 2002. "Quality information and consumer health plan choices." *Journal of Health Economics* 21(1):43–63.

Buchmueller, Thomas. 2006. "Price and the health plan choices of retirees." *Journal of Health Economics* 25(1):81–101.

Cutler, D., B. Lincoln and R. Zeckhauser. 2010. "Selection stories: understanding movement across health plans." *Journal of Health Economics* 29(5):821–838. Harvard Kennedy School.

Cutler, D. and S. Reber. 1998. "Paying for health insurance: the tradeoff between competition and adverse selection." *Quarterly Journal of Economics* 113(2):433–466.

Dafny, L. 2010. "Are health insurance markets competitive?" *American Economic Review* 100(4):1399–1431.

Dessein, Wouter. 2003. "Network Competition in Nonlinear Pricing." *RAND Journal of Economics* 34:593–611.

Ellis, Randall P. 2008. Risk adjustment in health care markets: concepts and applications, in Lu, Mingshan, and Jonnson, Egon, Eds, *Paying for Health Care: New Ideas for a Changing Society*. Wiley-VCH publishers Weinheim , Germany.

Glazer, Jacob and Thomas G. McGuire. 2000. "Optimal Risk Adjustment in Markets with Adverse Selection: An Application to Managed Care." *The American Economic Review* 90(4):1055–1071.

Glazer, Jacob and Thomas G. McGuire. 2002. "Setting health plan premiums to ensure efficient quality in health care: minimum variance optimal risk adjustment." *Journal of Public Economics* 84(2):153–173.

Jack, William. 2001. "Controlling selection incentives when health insurance contracts are endogenous." *Journal of Public Economics* 80(1):25–48.

Jack, William. 2006. "Optimal risk adjustment with adverse selection and spatial competition." *Journal of Health Economics* 25(5):908–926.

Newhouse, J. 1996. "Reimbursing health plans and health providers: selection versus efficiency in production." *Journal of Economic Literature* 34(3):1236–1263.

Nuscheler, Robert and Thomas Knaus. 2005. "Health Care Financing Risk selection in the German public health insurance system." *Health Economics* 14 (12):1253–1271.

Olivella, Pau and Marcos Vera-Hernández. 2007. "Competition among differentiated health plans under adverse selection." *Journal of Health Economics* 26(2):233–250.

Pauly, M. 1974. "Overprovision and public provision of insurance." *The Quarterly Journal of Economics* 88:541–562.

Rey, P. and J. Tirole. 2007. A primer on foreclosure, in Armstrong, M. and Porter, R., Ed., *Handbook of industrial organization*. North Holland pp. 2145–2220.

Rothschild, Michael and Joseph Stiglitz. 1976. "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information." *The Quarterly Journal of Economics* 90(4):629–649.

Royalty, Anne Beeson and Neil Solomon. 1999. "Health Plan Choice: Price Elasticities in a Managed Competition Setting." *The Journal of Human Resources* 34(1):1–41.

Schmidt-Mohr, U. and M. Villas-Boas. 1999. "Oligopoly with Asymmetric Information: Differentiation in Credit Markets." *RAND Journal of Economics* 30:375–396.

Schut, Frederik T., Stefan Gress and Juergen Wasem. 2004. "Consumer Price Sensitivity and Social Health Insurer Choice in Germany and the Netherlands." *International Journal of Health Care Finance and Economics* 3:117–138.

Stole, L. 1995. "Nonlinear Pricing and Oligopoly." *Journal of Economics and Management Strategy* 4:529–562.

Stole, L. 2007. Price discrimination and competition in Armstrong, M. and Porter, R., Ed., *Handbook of industrial organization*. North Holland pp. 2221–2299.

Strombom, Bruce A., Thomas C. Buchmueller and Paul J. Feldstein. 2002. "Switching costs, price sensitivity and health plan choice." *Journal of Health Economics* 21(1):89–116.

Van de Ven, W.P.M.M. and R.P. Ellis. 2000. Risk adjustment in competitive health care markets, in Culyer, A.P. and Newhouse, J. P., Ed., *Handbook of health economics*. North Holland pp. 754–845.

Van Vliet, R. 2006. "Free choice of health plans combined with risk-adjusted capitation payments: are switchers and new enrolees good risks?" *Health Economics* 15:763–774.

A. Appendix

Proof of lemma 1 In order to simplify notation, we write

$$\Pi^*(u^h, u^l, q^h, q^l) = \Pi(u^h, u^l, q^h, q^l; u^{h*}, u^{l*}) \tag{48}$$

$$\hat{\Pi}(u^h, u^l) = \Pi^*(u^h, u^l, 1, 1 - \frac{u^l - u^h}{\Delta\theta}) \tag{49}$$

Suppose, by contradiction, that the claim is not correct. That is, there exist u^h, u^l, q^h, q^l such that

$$\Pi^*(u^h, u^l, q^h, q^l) > \hat{\Pi}(u^{h*}, u^{l*}) \tag{50}$$

We consider three cases:

(I) $D^h(u^h, u^{h*}) > 0$ and $D^l(u^l, u^{l*}) > 0$:

- without loss of generality $q^h = 1$
  - Suppose not, that is $q^h < 1$, then increasing q^h increases profits ($\partial\Pi^*/\partial q^h > 0$) and relaxes ($IC_l$)
- without loss of generality $q^l = 1 - \frac{u^l - u^h}{\Delta\theta}$; suppose not:
  - if $q^l > 1 - \frac{u^l - u^h}{\Delta\theta}$, ($IC_h$) is violated
  - if $q^l < 1 - \frac{u^l - u^h}{\Delta\theta}$, ($IC_l$) is slack and increasing q^l raises profits

  but then inequality (50) contradicts that (u^{h*}, u^{l*}) solves $\hat{P}_{u^{h*}, u^{l*}}$.

(II) $D^h(u^h, u^{h*}) > 0$ and $D^l(u^l, u^{l*}) = 0$: we have the following inequalities:

$$\hat{\Pi}(u^{h*}, u^{l*}) \geq \max_u \hat{\Pi}(u, \bar{u}^l) \geq D^h(u^h, u^{h*})\pi(u^h, q^h) = \Pi^*(u^h, u^l, q^h, q^l)$$

where $\bar{u}^i = \max\{u | D^i(u, u^{i*}) \leq 0\}$. This contradicts equation (50).

(III) $D^h(u^h, u^{h*}) = 0$ and $D^l(u^l, u^{l*}) > 0$: we have the following inequalities:

$$\hat{\Pi}(u^{h*}, u^{l*}) \geq \hat{\Pi}(u^{h*}, \tilde{u}^l) \geq \Pi^*(u^h, u^l, q^h, q^l)$$

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where the second inequality follows from the assumption that $D^h(\tilde{u}^l, u^{h*})(w - \theta^h + \rho^h - \tilde{u}^l) \geq 0$. Again we find a contradiction of equation (50).   Q.E.D.

Proof of proposition 1 Assume that there is a choice of $\rho^{h,l}$ that gives an efficient solution: $q^{h,l} = 1$. Note that from the first order condition (23) it follows that $q^l = 1$ implies that $\mu = 0$. The incentive compatibility constraints for the high and the low type read

$$IC_h : u^h \geq u^l - \Delta\theta(1 - q^l)$$
$$IC_l : u^l \geq u^h + \Delta\theta(1 - q^h).$$

Therefore, efficiency implies $u^h = u^l$. Since also $\mu = 0$ in an efficient equilibrium, we have from the first order conditions

$$u^h = w - \theta^h + \rho^h - \tfrac{1}{2}\frac{\lambda}{D_1^h}$$
$$u^l = w - \theta^l + \rho^l - \tfrac{1}{2}\frac{(1 - \lambda)}{D_1^l}$$

where we write $D_1^i = D_1^i(u^i, u^i)$ to ease notation. So, $u^h = u^l$ leads to

$$\rho^h - \theta^h - \tfrac{1}{2}\frac{\lambda}{D_1^h} = \rho^l - \theta^l - \tfrac{1}{2}\frac{1 - \lambda}{D_1^l}. \tag{51}$$

Conversely, setting the values for ρ such that this holds evidently solves the first order conditions plus the IC-constraints. Note that there are multiple combinations of ρ^h and ρ^l that yield the first-best outcome. The budget constraint $\lambda\rho^h + (1 - \lambda)\rho^l = 0$ pins down a unique pair ρ^l, ρ^h. As $\theta^h > \theta^l$ and $\frac{\lambda}{D_1^h} > \frac{1-\lambda}{D_1^l}$ (by assumption (3)), the first-best implementing ρ^h and ρ^l satisfies $\rho^h > \rho^l$ and $\rho^h - \theta^h > \rho^l - \theta^l$.   Q.E.D.

Proof of proposition 2 Both in this proof and the proof of proposition 3, we use a slightly different way to compute the effect of μ on u^h, u^l and CS_ω than the one given in the main text. The reason is that, for reference below, we need the relation between $\rho^{h,l}$ and μ.

To obtain an expression relating μ and $\rho^{h,l}$, subtract the first-order conditions for the utilities, (22) and (21) and then use the one for the low type's coverage q^l (24) and the binding incentive compatibility constraint. This results in

$$-\frac{\lambda}{D_1^h} + \frac{1 - \lambda}{D_1^l} - \Delta\theta - \frac{\rho^l}{\lambda} + \frac{(\Delta\theta)^2\mu}{(1 - \lambda)\frac{1}{2}r\sigma^2}\left(1 + \frac{\mu}{1 - \lambda}\right) + \left(\frac{1}{D_1^h} + \frac{1}{D_1^l}\right)\mu = 0$$

where $D_1^i = D_1^i(u^i, u^i)$. It is now convenient to look at the total derivative of the shadow price μ, taking ρ^l as function of μ, and substituting $\rho^h = -\frac{1-\lambda}{\lambda}\rho^l$. From the implicit function theorem, it now follows that

$$\frac{d\rho^l}{d\mu} = \lambda\left[\left(\frac{1}{D_1^h} + \frac{1}{D_1^l}\right) + \frac{(\Delta\theta)^2}{\frac{1}{2}r\sigma^2(1-\lambda)}\left(1 + \frac{2\mu}{1-\lambda}\right)\right]. \tag{52}$$

This allows us to compute the total effect of changing μ on the individual types' utilities:

$$\begin{aligned}\frac{du^h}{d\mu} &= \frac{\partial u^h}{\partial\mu} + \frac{\partial u^h}{\partial\rho^l}\frac{\partial\rho^l}{\partial\mu} \\ &= \frac{\lambda}{D_1^h} - \frac{1-\lambda}{D_1^l} - \frac{2\mu(\Delta\theta)^2}{\frac{1}{2}r\sigma^2(1-\lambda)} - \frac{(\Delta\theta)^2}{\frac{1}{2}r\sigma^2},\end{aligned} \tag{53}$$

and

$$\frac{du^l}{d\mu} = \frac{\lambda}{D_1^h} - \frac{1-\lambda}{D_1^l} - \frac{2\mu(\Delta\theta)^2}{\frac{1}{2}r\sigma^2(1-\lambda)} + \frac{\lambda}{1-\lambda}\frac{(\Delta\theta)^2}{\frac{1}{2}r\sigma^2}.$$

Consequently, at first best ($\mu = 0$), low types always gain from increasing μ (i.e. the direct transfers ρ^l outweigh the drag from the incentive compatibility constraint). More remarkably, under the assumption that $ME > (1-\lambda)IE$, also high types gain from transferring premiums to low types, starting from the first best.

More generally, adding the types' utilities with weights ω and $1 - \omega$, we find

$$\frac{dCS_\omega}{d\mu} = -\frac{\omega - \lambda}{1-\lambda}\frac{\Delta\theta^2}{\frac{1}{2}r\sigma^2} + \frac{\lambda}{D_1^h} - \frac{1-\lambda}{D_1^l} - \frac{2\mu(\Delta\theta)^2}{\frac{1}{2}r\sigma^2(1-\lambda)}. \tag{54}$$

which is then also positive at $\mu = 0$ under the assumption that $ME > (1-\lambda)IE$. We proceed to find the optimum value $\mu^*(\omega)$. Solving $dCS_\omega/d\mu = 0$ for μ gives us

$$\mu^*(\omega) = -\tfrac{1}{2}(\omega - \lambda) + \frac{(1-\lambda)\frac{1}{2}r\sigma^2}{2(\Delta\theta)^2}\left(\frac{\lambda}{D_1^h} - \frac{1-\lambda}{D_1^l}\right) \tag{55}$$

It follows that $d\mu^*/d\omega < 0$. Hence, as long as $ME > (1-\lambda)IE$, we have that $\mu^*(\omega) > 0$ for all ω.

By definition, u^h is maximized for $\mu = \mu^*(1)$. We proceed to show that ρ^h can be negative

for this value of μ. Since $\rho^h = -\frac{1-\lambda}{\lambda}\rho^l$ and in the first best

$$\rho^h - \theta^h - \frac{1}{2}\frac{\lambda}{D_1^h} = \rho^l - \theta^l - \frac{1}{2}\frac{1-\lambda}{D_1^l},$$

we get that in the first best ($\mu = 0$)

$$\rho^l(0) = -\lambda\left(\Delta\theta + \frac{1}{2}\left(\frac{\lambda}{D_1^h} - \frac{1-\lambda}{D_1^l}\right)\right). \tag{56}$$

This is negative, of course: risk adjustment in the "standard" direction. We now show that at the point where u^h is maximized ($\mu = \mu^*(1)$) ρ^l is a positive number (and then ρ^h is negative, as a consequence). We write

$$\rho^l(\mu^*(1)) = \rho^l(0) + \int_0^{\mu^*(1)} \frac{d\rho^l}{d\mu}d\mu \tag{57}$$

with $\rho^l(0)$ given in equation (56) and from the derivative of ρ^l (equation (52)) we compute

$$\int_0^{\mu^*(1)} \frac{d\rho^l}{d\mu}d\mu = \lambda\left[\mu^*(1)\left(\frac{1}{D_1^h} + \frac{1}{D_1^l}\right) + \mu^*(1)\frac{(\Delta\theta)^2}{\frac{1}{2}r\sigma^2(1-\lambda)} + \frac{\Delta\theta^2\mu^*(1)^2}{\frac{1}{2}r\sigma^2(1-\lambda)^2}\right].$$

Finally, it is convenient to write equation (55) as

$$2\frac{\mu^*(1)(\Delta\theta)^2}{\frac{1}{2}(1-\lambda)r\sigma^2} = \left(\frac{\lambda}{D_1^h} - \frac{1-\lambda}{D_1^l}\right) - \frac{(\Delta\theta)^2}{\frac{1}{2}r\sigma^2}, \tag{58}$$

Combining this and using equation (58) in the second and third step we find:

$$\begin{aligned}
\frac{\rho^l(\mu^*(1))}{\lambda} &= -\Delta\theta - \frac{1}{2}\left(\frac{\lambda}{D_1^h} - \frac{1-\lambda}{D_1^l}\right) + \mu^*(1)\left(\frac{1}{D_1^h} + \frac{1}{D_1^l}\right) + \mu^*(1)\frac{(\Delta\theta)^2}{\frac{1}{2}r\sigma^2(1-\lambda)} + \frac{(\Delta\theta)^2\mu^*(1)^2}{\frac{1}{2}r\sigma^2(1-\lambda)^2} \\
&= -\Delta\theta - \frac{(\Delta\theta)^2}{r\sigma^2} + \frac{1}{2}\mu^*(1)\left(\frac{2-\lambda}{1-\lambda}\frac{1}{D_1^h} + \frac{1}{D_1^l}\right) - \frac{\tilde{\mu}(\Delta\theta)^2}{(1-\lambda)r\sigma^2} \\
&= -\Delta\theta - \frac{(\Delta\theta)^2}{2r\sigma^2} + \frac{1}{2}\mu^*(1)\left(\frac{2-\lambda}{1-\lambda}\frac{1}{D_1^h} + \frac{1}{D_1^l}\right) - \frac{1}{4}\left(\frac{\lambda}{D_1^h} - \frac{1-\lambda}{D_1^l}\right) \tag{59}
\end{aligned}$$

It follows from equation (55) that $\mu^*(1)$ is increasing in $r\sigma^2$ and that $\mu^*(1) \to +\infty$ as $r\sigma^2 \to +\infty$ and/or $\Delta\theta \to 0$. Hence we find that $\rho^l(\mu^*(1)) > 0$ for $IE > 0$ small enough. $\qquad$ Q.E.D.

Proof of proposition 3 We start by finding the relation between risk adjustment $\rho^{h,l}$ and

the shadow price μ. Solving for μ from (38),(39) leads to

$$\mu \left[\left(\frac{1}{D_1^h} + \frac{1}{D_1^l} \right) + \frac{(\Delta\theta)^2}{\frac{1}{2}r\sigma^2} \right] = \frac{1}{2} \left(\frac{\lambda}{D_1^h} - \frac{1-\lambda}{D_1^l} \right) + \Delta\theta - \Delta\rho. \tag{60}$$

Hence we find

$$\frac{\partial \rho^l}{\partial \mu} = \lambda \left(\frac{1}{D_1^h} + \frac{1}{D_1^l} + \frac{(\Delta\theta)^2}{\frac{1}{2}r\sigma^2} \right) \tag{61}$$

We can now compute the total derivatives of the utilities $u^{h,l}$ with respect to μ,

$$\frac{du^h}{d\mu} = -2\mu \frac{(\Delta\theta)^2}{\frac{1}{2}r\sigma^2} + \left(\frac{\lambda}{D_1^h} - \frac{(1-\lambda)}{D_1^l} \right) - (1-\lambda) \frac{(\Delta\theta)^2}{\frac{1}{2}r\sigma^2}, \tag{62}$$

$$\frac{du^l}{d\mu} = -2\mu \frac{(\Delta\theta)^2}{\frac{1}{2}r\sigma^2} + \left(\frac{\lambda}{D_1^h} - \frac{(1-\lambda)}{D_1^l} \right) + \lambda \frac{(\Delta\theta)^2}{\frac{1}{2}r\sigma^2} \tag{63}$$

and consumer surplus varies with μ according to

$$\frac{dCS_\omega}{d\mu} = -2\mu \frac{(\Delta\theta)^2}{\frac{1}{2}r\sigma^2} + \left(\frac{\lambda}{D_1^h} - \frac{(1-\lambda)}{D_1^l} \right) - (\omega - \lambda) \frac{(\Delta\theta)^2}{\frac{1}{2}r\sigma^2}. \tag{64}$$

Consumer surplus optimization then yields

$$\mu^*(\omega) = \frac{\frac{1}{4}r\sigma^2}{(\Delta\theta)^2} \left(\frac{\lambda}{D_1^h} - \frac{1-\lambda}{D_1^l} \right) - \frac{1}{2}(\omega - \lambda) \tag{65}$$

Hence, for equally-weighted consumer surplus, $\omega = \lambda$, equation (3) translates into positive $\mu^*(\lambda)$. It is obvious that $\mu^*(\omega)$ falls as ω increases. $ME > (1-\lambda)^2 IE$ implies that $\mu^* > 0$ even if $\omega = 1$. That is, even when trying to maximize the utility of the θ^h-type, the planner still distorts $q < 1$. $\hfill Q.E.D.$

Proof of proposition 4 At $\mu = 0$, comparing equation (60) and (51) shows that under both separation and pooling we have

$$\Delta\rho = \Delta\theta + \frac{1}{2} \left(\frac{\lambda}{D_1^h} - \frac{1-\lambda}{D_1^l} \right)$$

Hence ρ^h (and ρ^l) are the same for $\mu = 0$ under separation and pooling. This implies that $CS_\omega^{pool}(0) = CS_\omega^{sep}(0)$. Subtracting equation (64) under pooling from the similar equation (54)

under separation yields

$$\frac{dCS_\omega^{pool}}{d\mu} - \frac{dCS_\omega^{sep}}{d\mu} = \frac{(\Delta\theta)^2}{\frac{1}{2}r\sigma^2}\frac{\lambda}{1-\lambda}(2\mu + \omega - \lambda) > 0$$

for each $\omega \geq \lambda, \mu > 0$. Since under pooling $\mu^*(\omega) > 0$ if either $\omega = \lambda$ or for each $\omega \in [\lambda, 1]$ if $ME > (1-\lambda)^2 IE$, the result follows. *Q.E.D.*