Servicing Securitisation through Excessive Foreclosure*

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Abstract

How does securitisation distort the foreclosure decision of non-performing mortgages? In a model of mortgage-backed securitisation with endogenous foreclosure, we find that securitisers optimally adopt an excessive foreclosure policy towards future delinquent mortgages while retaining the junior tranche, in order to signal the quality of their assets to investors in the senior tranche. Ex post excessive foreclosure is therefore ex ante optimal because it mitigates informational frictions in securitisation. When the securitisers cannot commit to such a foreclosure policy, they can effectively do so by outsourcing the foreclosure decisions to mortgage servicers who are intrinsically “tough” or are provided with “biased” servicing contracts. Our model predictions are consistent with empirical findings on foreclosures in the subprime mortgage crises. Finally we demonstrate that policies which aim to restore ex post efficient foreclosures have the unintended consequence of lowering mortgage originators’ screening effort and thus social welfare. (JEL D8, G21, G23, G24)

Keywords. Security design, mortgage-backed securities, mortgage foreclosure, mortgage servicers, asymmetric information, commitment

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1 Introduction

The wave of mortgage foreclosures in the aftermath of the subprime mortgage crisis in the United States has raised concerns from the general public and policy makers.\footnote{According to a recent report by RealtyTrac (2015), there are more than 14 millions US properties with foreclosure filings from 2008-2014.} Reports and empirical studies have argued that foreclosures often result in significant losses for both the lenders and the borrowers, and impose substantial negative externalities to the broader society.\footnote{See for example Pennington-Cross (2006) for a survey on the deadweight loss on foreclosure. Using data from 1987 to 2009 in Massachusetts, Campbell et al. (2011) estimate foreclosure discounts as large as 27 percent on average.} In response to the unfolding foreclosure crisis started in 2008, the U.S. government has developed the Home Affordable Modification Program (HAMP), a large-scale intervention to incentivise mortgage renegotiation instead of foreclosure.\footnote{HAMP provides direct monetary incentives to mortgage servicers for each successfully renegotiated delinquent mortgage. For a detailed description and an empirical evaluation of HAMP, see Agarwal et al. (2012).}

Recent empirical studies about the subprime mortgage crisis have suggested that securitisation could have contributed to the severity of the foreclosure crisis. For instance, using different data sets and identification strategies, Piskorski et al. (2010), Agarwal et al. (2011a), and Krueger (2014) have shown that conditional on being non-performing (delinquent), mortgages in a securitised pool are more likely to be foreclosed than otherwise similar mortgages on bank portfolios. Yet, the economic forces that link mortgage foreclosures to securitisation are under-studied in the literature. This paper provides a framework to understand the equilibrium implications of mortgage foreclosures for the incentives in mortgage origination, servicing and securitisation.

We develop a model of mortgage-backed securitisation with two crucial features: 1) Security design under asymmetric information: Ex ante an informed mortgage pool owner, due to liquidity reasons, wants to raise cash by designing and selling mortgage-backed securities (MBS) to uninformed investors; 2) Endogenous foreclosure decision: Ex post, some mortgages may become non-performing and must be either foreclosed or modified.\footnote{For simplicity, we do not distinguish modification from forbearance, i.e. simply continuing the mortgage contract with the defaulted borrowers. As it will soon be clear, there is no qualitative difference in the interpretation of our model.} The foreclosure policy deciding how much of these
delinquent mortgages to foreclose thus endogenously affects the distribution of the mortgage pool’s final cash flow. While these two features are particularly relevant in a mortgage securitisation context, the economic setting of the model is more general: An informed owner of an asset or a firm tries to raise funding by selling claims on existing asset while the initial owner can take a non-contractible action to affect the asset/firm’s cash flow distribution.

The main result of the paper is that securitisers optimally adopt excessive foreclosure policies in order to signal the quality of their mortgage pools to uninformed investors. Specifically, the informed mortgage pool owner (henceforth “the securitiser”) of high-quality assets ex ante optimally commits to an ex post excessive, value-reducing foreclosure policy. This policy, along with the retention of the junior tranche, serves as a signal of her favourable private information, increasing the proceeds from the sale of the senior tranche to uninformed investors in a separating equilibrium. The optimal excessive foreclosure policy thus trades off this benefit of reduced signalling costs against the loss in the loan value due to inefficient foreclosures. While the costly retention of junior securities as a signal is established in the security design literature (e.g. Leland and Pyle, 1977; DeMarzo and Duffie, 1999), the signalling role of excessive foreclosure policies is novel. Our model provides an information-based explanation for the observed higher foreclosure rate in securitised mortgages compared to bank-held mortgages. Bank-held mortgages are not sold to outsiders and hence do not suffer from the information asymmetry problem.

Foreclosure policy as a signal can mitigate frictions arising from information asymmetry because foreclosure affects the distribution of the mortgage pool’s cash flow in two specific ways. First, the foreclosure policy only matters in the bad state in which some mortgages default. Second, foreclosure, as opposed to modification, lowers the exposure of the mortgage pool to the aggregate risk of borrower re-defaults. As a result, given that some mortgages have defaulted, foreclosing a delinquent mortgage and selling the underlying property increases (decreases) the mortgage

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5We abstract from strategic defaults by the borrowers in our model to highlight the effect of informational frictions in securitisation on mortgage foreclosure policies. Our main results are robust to considerations of strategic defaults, since the incentives for borrowers to strategically default are not directly affected by whether their mortgages are securitised. Our result that informational friction in securitisation leads to excessive foreclosures, however, implies an indirect attenuating effect of securitisation on the borrowers incentives to strategically default.

6Such aggregate risk of borrower re-default may be driven by uncertainties in future unemployment and future property prices.
pool cash flows in case the borrower would re-default (recover).

These properties of mortgage foreclosure lead to the result that excessive foreclosure reduces the signalling costs incurred by the securitiser. Consider the problem faced by the securitiser with a high-quality pool (or the high-type securitiser): She wants to maximise the proceeds from the sale of the MBS to the investors, which are limited by the low-type securitiser’s mimicking incentives. By designing the same foreclosure policy and MBS as the high type’s, the low-type securitiser can sell the MBS at a premium, as if it is backed by a high-quality mortgage pool. This premium, and hence the mimicking incentives, are smaller if the high-type securitiser increases the payoff offered by the MBS in the bad states, because the low-quality mortgage pool has a higher default probability (and thus also a higher unconditional probability of re-default) than the high-quality mortgage pool. The high-type securitiser achieves this by i) offering a debt security, and ii) enforcing an excessive foreclosure policy. While the debt security pays off all the cash flows in the worst re-default state to the investors, an excessive foreclosure policy further increases the cash flow available to the MBS in the re-default state. Consequently, excessive foreclosure by the high-type securitiser reduces the signalling cost by lowering the premium the low type receives when mimicking. In equilibrium, the high type optimally incurs the cost of excessive foreclosure to reduce her signalling cost, realising a larger gain from securitisation.

The mechanism of excessive foreclosures as a costly signal implicitly relies on the securitiser’s commitment power, i.e. the ability to implement this ex ante optimal foreclosure policy ex post when defaults occur. Without commitment power, the securitiser ex post would deviate to an overly lenient foreclosure policy as she holds the residual junior tranche, which gives rise to risk-shifting incentives.

Our paper suggests an economic rationale for mortgage servicers as a commitment device. In practice, mortgage servicers feature prominently in the mortgage industry and they have discretion over the foreclosure decision of delinquent mortgages. By contracting with a mortgage servicer, the securitiser can effectively commit to an excessive foreclosure policy by either choosing a servicer known to be tough, or providing the servicer with an incentive contract that is biased towards excessive}

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7The servicer performs duties including collecting the payments, forwarding the interest and principal to the lenders, and negotiating new terms if the debt is not being paid back, or supervising the foreclosure process.
foreclosure. The first interpretation corroborates with the economically important servicer fixed effect in predicting foreclosure probability as in Agarwal et al. (2011a). The second interpretation can explain why compensation contracts are endogenously biased towards foreclosures. Such biases have been documented by Thompson (2009) and Krueger (2014). Finally, it is plausible that investors of MBS infer information from the identity of the servicer and the mortgage servicing contract, as they are provided to investors in the Pooling and Servicing Agreement (PSA) alongside the prospectus of the MBS issue.

We extend our model to investigate how foreclosure policies can affect the quality of mortgage pool and welfare in general, by endogenising the securitiser’ ex ante screening effort choice. We find that while information asymmetry leads to under-provision of screening effort, allowing the securitiser to distort foreclosure policy can ameliorate this under-provision and hence increase overall welfare, in spite of ex post excessive mortgage foreclosures. In terms of policy implication, our model suggests that policies that aim to restore ex post foreclosure efficiency such as HAMP can create the unintended consequence of lowering the securitiser’s screening effort and worsen the average quality of mortgage pools in the economy. Finally, we provide several novel, testable empirical implications of our model regarding the relationship between mortgage pool quality and foreclosure policies as well as servicer-specific characteristics.

Our paper closely relates to the empirical works on the causal relationship between securitisation and foreclosures. Agarwal et al. (2011a), Piskorski et al. (2010), and Krueger (2014) find that private securitisation increases the foreclosure probability and decreases modification probability for delinquent mortgages. This paper provides a theoretical explanation for this causal relationship, based on the information friction in the securitisation process.

Our results contribute to the understanding of the role of servicers and their incentive contracts. Agarwal et al. (2011b) find that when servicers hold the junior tranche, they act to maximise the value of their claim potentially at the expense of investors who are the senior claimants. Levitin and Goodman (2009) and Krueger (2014) argue that servicers’ financial incentives are biased towards foreclosures. Our paper can explain why mortgage servicers are hired in the first place and are provided a biased incentive contract—to overcome the time-inconsistency problem.
documented in Agarwal et al. (2011b).

A recent paper by Mooradian and Pichler (2014) also studies the role of servicers in the mortgage-backed securitisation industry. The authors argue that the servicers need to be provided with incentives to exert effort to gather information following a loan default, in order to offer loan renegotiation efficiently. The authors then study the asset composition (pooling) and show that non-diversified mortgage pool can alleviate the servicer’s moral hazard problem. Our paper instead focuses on the securitisation (tranching) problem under asymmetric information for a mortgage pool of given quality and shows that it is \textit{ex ante} optimal to have an \textit{ex post} inefficient foreclosure policy. We therefore propose a different mechanism for how securitisation distorts foreclosures and can rationalise the hiring decision of mortgage servicers and their compensation contracts.

Our paper also relates to the study of optimal loan modification and foreclosure policy. Our paper highlights that securitisation can be another important factor affecting foreclosure decisions. Wang et al. (2002) show that when a lender (bank) has a high screening cost to ascertain whether a borrower is in distress, it could be optimal for the bank to randomly reject loan workout requests to deter the non-distressed borrower from opportunistically applying for a loan modification. Riddiough and Wyatt (1994) study the case in which the lender’s foreclosure cost is private information and the borrowers will infer this cost from past loan foreclosure decisions and consequently decide their default decision and concession request. The lender thus may costly foreclose many loans today to reduce future expected default and loan modification costs. Gertner and Scharfstein (1991) focus on the free-riding problem among multiple creditors and show that when the cost of debt concessions is private but the benefit is shared, a creditor’s incentive to grant concessions to a distressed firm is reduced.

More generally this paper belongs to the growing body of literature on the incentive problems associated with mortgage securitisation. Various studies argue that securitisation relaxes the \textit{ex ante} lending standards. Keys et al. (2010, 2012), using evidence from securitised subprime loans, show that the ease of securitisation reduces lenders’ incentives to carefully screen the mortgage borrowers and that mortgages with higher likelihood to be securitised have higher default rates. Mian and Sufi (2009) find that securitisation of subprime loans is associated with credit
expansion and, as a result, counties with a high proportion of subprime mortgages face a larger number of defaults. Elul (2011) also finds securitised prime loans have a higher default rates than otherwise comparable portfolio loans. Hartman–Glaser et al. (2012) and Malamud et al. (2013) study ex ante loan originators’ screening effort and the design of compensation contract. Chemla and Hennessy (2014) and Vanasco (2014) analyse how securitisation and the liquidity in the MBS market affect ex ante loan originators’ screening effort. Our work complements this literature by studying the decision of ex post mortgage foreclosures in relation to securitisation.

The rest of the paper is organised as follows. Section 2 describes the model setup. Section 3 characterises the first-best and the full-information benchmarks. Section 4 carries out the main analysis to solve for the equilibrium with endogenous foreclosure policy. Section 5 highlights the importance of commitment over foreclosure policy and discusses the role of mortgage servicers in enabling such commitment power. Section 6 extends the model to consider ex ante screening incentives of the securitiser in relation to the subsequent foreclosure policy. Section 7 lists the model’s empirical implications. Section 8 concludes.

## 2 Model setup

This section sets up the model and comments on the assumptions which are central to the model.

There are four dates: 0, 1, 2 and 3. The model’s participants consist of a bank and a continuum of outside investors. The main analysis of this paper (Section 3–5) concerns only $t = 1, 2, 3$. We extend the model to an ex ante stage $t = 0$ only in Section 6.

All agents are risk neutral. The outside investors are deep pocketed and competitive. The banks are impatient and have a discount factor $\delta < 1$ between $t = 1$ and $t = 3$. This follows the assumption of DeMarzo and Duffie (1999) and can be interpreted as the bank’s liquidity needs to raise capital by securitising parts of their long term assets as they have access to some positive return investment opportunities. The outside investors have no such discount. Hence, there are gains from trade between the banks and the investors.
Mortgage pool and foreclosures

The underlying asset in our model is a pool containing a continuum of ex ante identical mortgages that pay off at $t = 3$. For the main analysis, we focus on the foreclosure of the mortgages when they become delinquent, as detailed below. In Section 6 we extend the model to consider the ex ante screening effort choice, which also has an effect on the cash flow of the mortgage pool.

Because of the diversification benefit of pooling the mortgages, that the mortgage pool is only exposed to aggregate risks, which affect the ability for the borrowers to repay. With probability $\pi$, the mortgage pool is in a good state ($G$) and no borrowers default. In state $G$, the value of the mortgage pool is $Z_G$. With probability $1 - \pi$, the mortgage pool is in a bad state ($B$) and a fixed portion of the mortgages becomes delinquent. This can be interpreted as a well diversified portfolio with only a systemic component of default risk. We normalise the measure of the delinquent mortgages in the pool to 1. The remaining performing mortgages continue to repay and amount to a value of $Z_B < Z_G$ at $t = 2$. Since mortgage delinquency only occurs in the bad state, we will focus primarily on the sequence of events after a realisation of the bad state to study mortgage foreclosures.

When a mortgage becomes delinquent at $t = 2$, it can be foreclosed or renegotiated.\footnote{Throughout the paper, we use “mortgage renegotiation” and “mortgage forbearance” interchangeably. Because we abstract from the renegotiation process between the mortgage lender and the borrower, one can interpret the cash flows to the mortgage pool following a decision of no foreclosure as the un-modelled optimal renegotiation outcome.} In the case of foreclosure, the collateral property is repossessed and sold to outside investors. Alternatively, if the delinquent mortgage is renegotiated, it pays off $X$ with probability $\theta$ at $t = 3$ (recovery) or zero otherwise (re-default). For simplicity, we assume that the recovery of delinquent mortgages are perfectly correlated. It can be interpreted as capturing the aggregate nature of the risk of mortgage recoveries (e.g. aggregate property prices and employment opportunities for borrowers).\footnote{An alternative interpretation is that due to diversification benefits, only the correlated risk in the recover of the delinquent mortgages affects the over cash flow of the mortgage pool.} Finally, we further assume that $Z_G \geq Z_B + X$, so that the value of a mortgage in the good state is at least as high as in a bad state, even if all delinquent mortgages resume payments in the bad state. Intuitively, the difference accounts for the value of the temporary missing payments.

The exposure to the aggregate risk of a mortgage pool is characterised by the
probability of entering state $G$. This probability $\pi \in \{\pi_H, \pi_L\}$, where $\pi_H > \pi_L$, is mortgage-pool specific and is the source of information asymmetry between the bank and outside investors, as detailed in the next section. At $t = 1$ all model participants have the prior belief that $\pi = \pi_H$ with probability $\gamma$.\footnote{In Section 6 we endogenise this probability $\gamma$ in an ex ante stage $t = 0$ through the bank’s screening effort choice.} We interpret $\pi_i$ as the “quality” of the mortgage pool (subscript “H” stands for “High” and “L” for “Low”), arising from the pool’s exposure to the systematic default risk of the borrowers. We make the following assumption to ensure that the bad state ($B$) arises with sufficient probability to guarantee its relevance, which ensures the concavity of the objective function in the analysis.

**Assumption 1.** $\pi_L < \pi_H < 1 - (1 - \delta) \frac{\theta}{1 - \theta}$, where $1 - \delta < \frac{1 - \theta}{\theta}$.

The focus of the paper is to study what proportion of the delinquent mortgages is chosen to be foreclosed in equilibrium and how securitisation affects this decision variable. The foreclosure policy can be summarised by $\lambda_i \in [0, 1]$, the fraction of delinquent mortgages foreclosed in a mortgage pool of quality $i$ (i.e. $1 - \lambda$ fraction of delinquency mortgages renegotiated). Denote by $L(\lambda_i)$ the total liquidation proceeds from repossessed properties. For a given foreclosure policy, the overall cash flow from a type $i$ mortgage pool at $t = 3$ is then $Z_G$ with probability $\pi_i$ (the “Good” state), $Z_B + L(\lambda_i) + (1 - \lambda_i)X$ with probability $(1 - \pi_i)\theta$ (the “Recovery” state), and $Z_B + L(\lambda_i)$ with probability $(1 - \pi_i)(1 - \theta)$ (the “Re-default” state), as illustrated in Fig 1.

The exact functional form of the liquidation proceeds $L(\lambda)$ depends on the characterisation of the market for distressed properties as well as the direct and indirect costs associated with foreclosures. We abstract from these considerations to keep the analysis general and make the following intuitive assumption on the foreclosure technology.

**Assumption 2.** For $\lambda \in [0, 1]$, (i) $L(\lambda)$ is strictly increasing and concave; (ii) $\frac{\partial L(\lambda)}{\partial \lambda} \in [0, X]$; and (iii) $\frac{\partial L(0)}{\partial \lambda} > \theta X > \frac{\partial L(1)}{\partial \lambda}$.

Assumption 2 states that, first, $L(\lambda)$ is strictly increasing and concave in $\lambda$. The decreasing marginal liquidation value of the foreclosed loans could be due to either scarce capital or scarce expertise in making the renovation needed to
realise the value of the properties. Secondly, the marginal liquidation value of the mortgage is below the full repayment value of the mortgage \( \frac{\partial L}{\partial \lambda} \in [0, X] \) for any \( \lambda \in [0, 1] \). Intuitively, there are costs associated with liquidating a mortgage, due to, for example, renovation and repair costs associated with investing in distressed property, as well as other outstanding liabilities such as unpaid fees and taxes. The last part of this assumption is a technical assumption to ensure an interior optimal foreclosure policy in the first-best case.

### Securitisation

Because of the liquidity discount \( \delta \), at \( t = 1 \), the bank who owns the mortgage pool would like to design and sell securities backed by the cash flow of the mortgage pool at \( t = 3 \) to outside investors. We will henceforth refer to the bank as the “securitiser” and the security as the mortgage-backed securities (MBS). The securitiser thus receives the cash from selling the MBS at \( t = 1 \), and retains any residual cash flow from the mortgage pool after paying off the investors at \( t = 3 \).

At the beginning of \( t = 1 \), the securitiser receives private information regarding the recovery probability of potential future delinquent loans \( \pi_i \in \{\pi_H, \pi_L\} \). The source of private information could come from new information produced during the process of structuring the individual mortgages into a pool for securitisation, as in DeMarzo and Duffie (1999).\(^{11}\)

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\(^{11}\)DeMarzo and Duffie (1999) solves the ex ante security design problem, whereas we solve for the ex post security design problem after the banks learn about their private information. As shown...
The focus of this paper is the foreclosure decision, which interacts with the securitisation process because the foreclosure policy affects the cash flows of the mortgage pool. We therefore model the securitisation process as follows. The securitiser with information $\pi_i$ offers the outside investors a security $F_i$ and a promised foreclosure policy $\lambda_i$. In the baseline model analysed in Section 4, we assume that the securitiser is able to commit to the foreclosure policy promised at $t = 1$ and then implement it as $t = 2$, when defaults occur. The resulting equilibrium is constrained efficient, allowing us to isolate the effect of asymmetric information on the foreclosure policy. In Section 5 we discuss how the securitiser can establish this commitment power over her foreclosure policy by contracting with a mortgage servicer.

Observing the offer $(F_i, \lambda_i)$, the competitive investors form a posterior belief $\hat{\pi}$ regarding the private information of the securitiser, and bids the price of the security $p$ to its fair value, taking the foreclosure policy in equilibrium as given. After repaying investors according to $F_i$, the securitiser retains the residual cash flow from the mortgage pool.

The security $F_i$ is contracted upon the cash flows at $t = 3$, specifying payments to the MBS investors for each realisation of the cash flow. We restrict our attention to monotone securities. That is, a higher realisation of the mortgage pool cash flow should leave both the outside investors and the securitiser a (weakly) higher payoff.\footnote{Although this implies some loss of generality, it is not uncommon in the security design literature, e.g. Innes (1990) and Nachman and Noe (1994). One potential justification provided by DeMarzo and Duffie (1999) is that, the issuer has the incentive to contribute additional funds to the assets if the security payoff is not increasing in the cash flow. Similarly, the issuers has the incentive to abscond from the mortgage pool if the security leaves the issuer a payoff that is not increasing in the cash flow. If such actions cannot be observed, the monotonicity assumption is without loss of generality.}

The securitisation process could be hindered by the securitiser’s private information regarding the mortgage pool, as the classic lemon’s problem in Akerlof (1970). When the securitiser is uninformed, there is symmetric information between the securitiser and the investors. Under symmetric information a simple “pass-through” security or selling the whole mortgage pool to the investors is optimal as it exhausts all the gains from trade, and results in a first-best foreclosure policy that maximises the value by DeMarzo et al. (2015), similar intuition carries through in the ex post problem, although the problem becomes more complicated as the design itself becomes a signal.
of the mortgage pool (Section 3). If the securitiser is better informed than outside investors, however, the presence of information asymmetry prompts the securitiser to not only design the MBS strategically, but also distorts her foreclosure decision, in order to mitigate the information friction. We study the optimal securitisation problem of an informed securitiser in Section 4.

Time line and the equilibrium concept

The timeline of the model is summarised in Table 1. The main analysis carried out in Section 3–5 concerns only $t = 1, 2, 3$. We extend the model to an ex ante stage $t = 0$ in Section 6.

| $t = 0$ | Securitiser exerts screening effort (Section 6 only) |
| $t = 1$ | Securitiser observes $\pi_i$ and offers $(F_i, \lambda_i)$ |
| $t = 2$ | Mortgage defaults in state $B$ |

Implement foreclosure policy $\lambda_i$ |

| $t = 3$ | Final payoffs realise |

The equilibrium concept in this model is the perfect Bayesian equilibrium (PBE). Formally, a PBE consists of a security $F_i$ issued by the securitiser of each type $i \in \{H, L\}$, the foreclosure policy $\lambda_i$ of the securitiser of each type, and a system of beliefs such that i) the securitiser chooses the security and the foreclosure policy to maximise her expected payoff, given the equilibrium choices of the other agents and the equilibrium beliefs, and ii) the beliefs are rational given the equilibrium choices of the agents and are formed using Bayes’ rule (whenever applicable). As there can be multiple equilibria in games of asymmetric information, we invoke the Intuitive Criterion of Cho and Kreps (1987) to eliminate equilibria with unreasonable out-of-equilibrium belief. This allows us to restrict attention to only the least cost separating equilibrium.

3 First-best and the full-information benchmarks

In this section we study two benchmark cases – first best and full information. The case of full information only differs from the first best due to the non-contractibility
of the foreclosure rate $\lambda_i$. We first characterise the first-best foreclosure policy. We then analyse the equilibrium under full information, and show that the first best is achieved in the full-information equilibrium.

The first-best foreclosure policy maximises the value of the mortgage pool $V_i(\lambda)$.

$$
\lambda_i^{FB} = \arg \max_{\lambda \in [0,1]} V_i(\lambda)
$$

(1)

where

$$
V_i(\lambda) \equiv \pi_i Z_G + (1 - \pi_i)[Z_B + L(\lambda) + (1 - \lambda)\theta X]
$$

(2)

The solution is characterised by the first order condition, $\frac{\partial L(\lambda^{FB})}{\partial \lambda} = \theta X$. That is, since the marginal value obtained from foreclosure is decreasing with the fraction of foreclosed loans, the first-best level of foreclosure is determined such that the the margin value from foreclosure is equal to the expected recovery value given forbearance. Furthermore, as the $H$ and $L$ type mortgage pools only differ in the default probability, the first-best level of foreclosure is identical across types, condition on delinquency. Denote the solution to this first order condition $\lambda^{FB}$.

We now characterise the equilibrium under full information. Firstly consider the optimal security issued in the securitisation process at $t = 1$. Since any retention of the cash flows by the securitiser incurs a liquidity discount, it is optimal for the security issued to be a full equity pass through security to the investors, since all securitise are fairly priced given full information. Secondly, given that the entire cash flow is securitised, the securitiser at $t = 2$ is indifferent between all foreclosure policies. As a tie break convention, we focus on the Pareto dominating equilibrium, in which the securitiser chooses the foreclosure policy that maximises the value of the mortgage pool.

The following proposition thus summarises the full-information benchmark results.

All proofs are in Appendix unless stated otherwise.

**Lemma 1.** In the full-information benchmark, the securitiser of both types securitises the mortgage pool by issuing a pass through equity security backed by the cash flows, and chooses the first-best foreclosure policy $\lambda^{FB}$.

We would like to conclude this section by stressing the fact that the first-best foreclosure policy is achieved in the full-information benchmark equilibrium. Therefore any inefficiency in the foreclosure policy discussed in the subsequent sections are
due to the asymmetric information problem between the securitiser and the outside investors. Denote henceforth the expected payoff to a type $i$ securitiser in the full-information benchmark as $U_i^{FB} \equiv V_i(\lambda^{FB})$.

4 Securitisation under asymmetric information

In this section we study the case of securitisation under asymmetric information. In order to highlight the main result of excessive foreclosure policy, we first solve the model under the simplifying assumption that the equilibrium securities are debt securities with endogenous face values. We then show that indeed debt is the optimal security in this model.

4.1 Excessive foreclosure policy as a costly signal

In this subsection, we characterise the properties of the optimal foreclosure policy, while restricting the securities issued to debt securities. This simplifying assumption allows us to highlight the key intuition underlying our main result, namely that securitisation under asymmetric information leads to excessive foreclosure in equilibrium.

At $t = 1$, the securitiser with private information $\pi_1$ issues an MBS $F_i$ backed by the cash flows of the mortgage pool, and promises a foreclosure policy $\lambda_i$. Observing the offer from the securitiser $(F_i, \lambda_i)$, the investors form a belief about the quality $\hat{\pi}$ of the mortgage pool.

Let’s start the analysis with the securitiser who owns a low quality mortgage pool (high default probability). In a separating equilibrium, the low-type securitiser always receives the fair price on any security issued. Therefore the securitiser maximises her expected payoff by selling the entire cash flow from the mortgage to outside investors, and promising the first-best level of foreclosure policy. There is no distortion in the form of inefficient retention or inefficient foreclosure for the low type. Denote with $U_i^*$ the expected payoff to a type $i$ securitiser in equilibrium. The payoff to the low-type securitiser in a separating equilibrium is thus equal to the first-best level, $U_L^* = U_L^{FB}$, while her foreclosure policy in equilibrium is $\lambda_L^* = \lambda^{FB}$. We denote henceforth with superscript * all equilibrium quantities.

The high-type securitiser, on the other hand, has to issue a security and promise a foreclosure policy such that in equilibrium it is not profitable for the low type to
deviate and mimic. Since we restrict the security to debt, we denote with $F_H$ the face value of the security issued by the high-type securitiser. We focus on the case in which $F_H \in [Z_B + \mathcal{L}(\lambda_H), Z_G]$, i.e. a risky debt, for any $\lambda_H$ that the securitiser promises. We show in Section 4.2 that indeed the optimal security is a risky debt (Proposition 3).

In the least cost separating equilibrium, the high-type securitiser maximises the proceeds from securitisation plus the residual cash flow by choosing the face value of her debt and her promised foreclosure policy $\lambda_H$.

$$U_H^* = \max_{(F_H, \lambda_H)} \quad p(F_H, \lambda_H) + \delta [\pi_H(Z_G - F_H)] + (1 - \pi_H)\theta \max \{Z_B + \mathcal{L}(\lambda_H) + (1 - \lambda_H)X - F_H, 0\}$$

s.t. (MC) $p(F_H, \lambda_H) = \pi_H F_H + (1 - \pi_H)\theta \min \{Z_B + \mathcal{L}(\lambda_H) + (1 - \lambda_H)X, F_H\} + (1 - \pi_H)(1 - \theta)[Z_B + \mathcal{L}(\lambda_H)]$

$U^*_L \geq p(F_H, \lambda_H) + \delta [\pi_L(Z_G - F_H)] + (1 - \pi_L)\theta \max \{Z_B + \mathcal{L}(\lambda_H) + (1 - \lambda_H)X - F_H, 0\}$ (3)

where (MC) is the market clearing condition given that the market observes $(F_H, \lambda_H)$ believes that the issuer of the security with face value $F_H$ who promises a foreclosure policy of $\lambda_H$ is of the high type, and (IC) is the incentive compatibility constraint for the low type not to mimic the offer $(F_H, \lambda_H)$ of the high type. Denote with $(F_H^*, \lambda_H^*)$ the unique solution to the above optimisation programme.

The following proposition highlights a key property of the equilibrium foreclosure policy, which is the main result of the paper.

**Proposition 1.** In the least cost separating equilibrium, the high-type securitiser adopts a (weakly) excessive foreclosure policy in equilibrium, whereas the low-type securitiser adopts the first-best foreclosure policy. That is,

$$\lambda_H^* \geq \lambda^{FB} = \lambda_L^*$$ (4)

The weak inequality is strict if and only if

$$G(\lambda^{FB}) = \frac{(\pi_H - \pi_L)}{\pi_L(1 - \delta)}(1 - \theta)(1 - \lambda^{FB})X - Z_G + Z_B + \mathcal{L}(\lambda^{FB}) + (1 - \lambda^{FB})X > 0$$ (5)
As shown in the Appendix, the condition $G(\lambda^{FB}) > 0$ given by Eq. 5 implies that the equilibrium face value of the debt satisfies $F^*_H < Z + L(\lambda^*_H) + (1 - \lambda^*_H)X$, that is, the optimal debt security that does not default in the recovery state. Proposition 1 states that, in an equilibrium in which the face value of the debt issued by the high type is relatively low, the equilibrium foreclosure policy of the high type deviates from the first-best policy and is qualitatively excessive. $G(\lambda^{FB}) > 0$ is more likely to hold when i) the discount factor $\delta$ is high, and/or ii) the extent of asymmetric information as measured by $\frac{\pi_H - \pi_L}{\pi_L}$ is high, so that a large fraction of the cash flows must be retained by the high type in order to signal her quality.

To clarify the trade-off faced by the high-type securitiser when choosing her foreclosure policy in equilibrium, we can rewrite the expected payoff to the high type issuer as follows, which consists of two components.

$$\delta V_H(\lambda_H) + (1 - \delta)p(F_H, \lambda_H)$$

The first term represents the discounted value of the mortgage pool enjoyed by the securitiser without securitisation, and the second term represents the gains from securitising given the security choice and the foreclosure policy.

The equilibrium foreclosure policy $\lambda^*_H > \lambda^{FB}$ trades off the efficiency loss associated with excessive foreclosure $\frac{\partial V_H(\lambda^*_H)}{\partial \lambda} < 0$ against increased gains from securitisation. The latter force is stated by the following corollary. Denote with $\hat{p}(\lambda_H) \equiv \max_{F_H} p(\lambda_H, F_H)$ s.t. $(MC)\&(IC)$ the highest securitisation proceeds the high-type securitiser can obtain by optimising the face value of the debt issued in a separating equilibrium in which her foreclosure policy is $\lambda_H$.

**Corollary 1.** In a separating equilibrium, excessive foreclosure allows the high-type securitiser to receive higher securitisation proceeds. That is, $\frac{\partial \hat{p}(\lambda_H)}{\partial \lambda} \bigg|_{\lambda_H = \lambda^{FB}} \geq 0$, where the inequality is strict if and only if Eq. 5 holds.

The high-type securitiser commits to excessive foreclosure in equilibrium, because an excessive foreclosure policy reduces the signalling cost she must incur at the securitisation stage. We shall illustrate the intuition behind such signalling cost reduction effect as follows: a marginally more excessive foreclosure policy relaxes the low type’s no-mimicking, incentive compatibility constraint, which can be re-written

\[16\]
as

\[
\delta V_L(\lambda_H) + \left( p(F_H, \lambda_H) - \delta p_L(F_H, \lambda_H) \right) \leq U_L^{FB}
\]

(7)

where \( p_L(F_H, \lambda_H) \equiv \pi_L F_H + (1 - \pi_L) \theta \min \{ Z_B + \mathcal{L}(\lambda_H) + (1 - \lambda_H) X, F_H \} + (1 - \pi_L)(1 - \theta)[Z_B + \mathcal{L}(\lambda_H)] \)

(8)

\( \delta p_L(F_H, \lambda_H) \) is the low-type securitiser’s valuation of the MBS that she issued when she mimics the high type’s action \((F_H, \lambda_H)\).

Relaxing the incentive constraint is equivalent to lowering the low type’s mimicking payoff, i.e. the left-hand-side of (Eq. 7), as \( U_L^{FB} \) is not affected by the high type’s action \((F_H, \lambda_H)\). The mimicking payoff comprises of two parts – i) the discounted value of the low type’s portfolio \( \delta V_L(\lambda_H) \) following deviation in her foreclosure policy from the first-best to that of the high type, \( \lambda_H \), and ii) the premium (in utility units) from securitising under the terms offered by the high type, where the low-type securitiser receives \( p(F_H, \lambda_H) \) from the investors whilst only giving up a security that is worth \( \delta p_L(F_H, \lambda_H) \) to her. We exclusively focus on the premium \( p(\cdot) - \delta p_L(\cdot) \) in the following discussion of the intuition because the expected value of the pool \( V_L \) is not affected by a marginal deviation from \( \lambda^{FB} \) as \( \frac{\partial V_L(\lambda^{FB})}{\partial \lambda} = 0 \).

The key driving forces behind the result that excessive foreclosure reduces the premium \( p(\cdot) - \delta p_L(\cdot) \) come from the sensitivity differential of \( p(\cdot) \) and \( p_L(\cdot) \) with respect to changes in foreclosure rate \( \lambda_H \) and face value \( F_H \). More precisely, the low type’s valuation of the MBS is more sensitive to an increase in \( \lambda \) but less sensitive to an increase in \( F_H \) than the high type’s, as shown in

\[
(1 - \pi_L)(1 - \theta) = \frac{\partial p_L(F, \lambda)}{\partial \lambda} > \frac{\partial p(F, \lambda)}{\partial \lambda} = (1 - \pi_H)(1 - \theta)
\]

(9)

\[
1 - (1 - \pi_L)(1 - \theta) = \frac{\partial p_L(F, \lambda)}{\partial F} < \frac{\partial p(F, \lambda)}{\partial F} = 1 - (1 - \pi_H)(1 - \theta)
\]

(10)

Equipped with these properties of the MBS valuation, we can show that the high type does strictly better under an excessive foreclosure policy than under the first-best one. Starting from the separating offer \((\lambda^{FB}, F_H^{FB})\) that satisfies \((MC)\) and binds \((IC)\), we construct an alternative offer by with a marginal increase in \( \lambda_H \) and a simultaneous marginal decrease in \( F_H \) such that \( p(\cdot) \) remains unchanged.
This alternative offer overall increases \( p_L(\cdot) \) as it increases more with the increase in \( \lambda_H \) and decreases less with the decrease in \( F_H \) than \( p(\cdot) \), thereby reducing the premium \( p(\cdot) - \delta p_L(\cdot) \) and relaxing the \((IC)\). Finally, as proven in the Appendix (as part of the proof of Proposition 3), a slack \((IC)\) implies that the high type can issue an MBS with lower retention that gives her strictly higher ex ante utility, given her (marginally) excessive foreclosure policy. Since the low-type securitiser always receive the same payoff in any separating equilibrium, excessive foreclosure by the high-type not only improves the high type’s equilibrium payoff, but also achieves an equilibrium allocation that \textit{Parato-dominates} any allocation with first-best foreclosure.

The final missing piece in understanding the intuition behind excessive foreclosures reducing signalling cost is what drives the sensitivity differential properties in Eq (9) and (10). It comes from several important features of the model. First, thanks to information asymmetry, the optimal MBS issued by the high type is a debt security of which the value put more weights on the mortgage pool cash flow realisation in bad states. Particularly, when Eq. 5 holds, the MBS has a low face value and its payout depends on the mortgage pool cash flow only in the re-default state. Second, an increase in the foreclosure rate of delinquent loans limits the mortgage pool exposure to the re-default risk by effectively transferring cash flow from the recovery state to the re-default state. Finally, as the low-quality pool has a higher probability of default (and a higher unconditional probability of re-default), the valuation of the MBS backed by a low-quality pool is more sensitive to a change in foreclosure policy and less to a change in face value, relative to the same MBS backed by a high-quality pool.

13 A more formal argument of this alternative offer: increase \( \lambda_H \) by a small positive amount \( \epsilon \) and decrease \( F_H \) by \( \frac{\partial p}{\partial F_H} \). By construction, the price of the MBS \( p(F_H, \lambda_H) \) is the same because by total differentiation and first-order approximation

\[
dp(F_H, \lambda_H) \approx \frac{\partial p}{\partial F_H} dF_H + \frac{\partial p}{\partial \lambda_H} d\lambda_H = \frac{\partial p}{\partial F_H} \left( -\frac{\partial \lambda_H}{\partial F_H} \right) + \frac{\partial p}{\partial \lambda_H} \epsilon = 0
\]

Meanwhile, this deviation increases the low type’s valuation of the MBS \( p_L(F_H, \lambda_H) \) as

\[
dp_L(F_H, \lambda_H) \approx \frac{\partial p_L}{\partial F_H} dF_H + \frac{\partial p_L}{\partial \lambda_H} d\lambda_H = \left[ \frac{\partial p_L}{\partial F_H} \left( -\frac{\partial \lambda_H}{\partial F_H} \right) + \frac{\partial p_L}{\partial \lambda_H} \right] \epsilon > 0
\]

This offer thus relaxes \((IC)\) while leaving the high type indifferent.
To summarise, the high-type securitiser commits to excessive foreclosure in equilibrium $\lambda_H > \lambda^{FB}$, in order to generate greater gains from securitisation. This is because excessive foreclosure reduces the exposure of the mortgage pools to the risk of re-default of the delinquent mortgages, which discourages mimicking by the low type, reducing the signalling cost the high type must incur at the time of securitisation.

This effect is further illustrated by the following comparative statics: An increase in the quality of the high type, $\pi_H$, exacerbates the information asymmetry because it creates greater mimicking incentives. As a result, there is more distortion towards excessive foreclosure in equilibrium.

**Proposition 2.** The equilibrium foreclosure policy of the high-type securitiser, and hence the distortion in the foreclosure policy, is increasing in her quality. That is, $\frac{\partial \lambda_H^*}{\partial \pi_H} \geq 0$, where the inequality is strict if and only if $G(\lambda^{FB})$ given by Eq. 5 is non-negative.

### 4.2 The optimality of debt

In this subsection we endogenise the security design problem of the securitisers, and show that indeed risky debt is the optimal security in this model.

As analysed in Section 4.1, the low-type securitiser optimally issues a full pass-through security to outside investors, because she always receives the fair price on her security in any separating equilibrium.

We therefore focus our analysis on the high-type securitiser. In conjunction with promising a foreclosure policy $\lambda_H$, the high type designs a security $F_H = (f_1, f_2, f_3)$, which maps the realisation of the mortgage pool cash flows to a set of payoffs to the outside investors, as summarised in Table 2, given the foreclosure policy $\lambda_H$.

**Table 2:** Payoffs of the security issued by the high-type securitiser with commitment over foreclosure policy

<table>
<thead>
<tr>
<th>Realisation of cash flow $c_i$</th>
<th>Security payoff $F_H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1 \equiv Z_G$</td>
<td>$f_1$</td>
</tr>
<tr>
<td>$c_2 \equiv Z_B + \mathcal{L}(\lambda_H) + (1 - \lambda_H)X$</td>
<td>$f_2$</td>
</tr>
<tr>
<td>$c_3 \equiv Z_B + \mathcal{L}(\lambda_H)$</td>
<td>$f_3$</td>
</tr>
</tbody>
</table>

In the least cost separating equilibrium, the high-type securitiser maximises the
proceeds from securitisation plus the residual cash flow by choosing the security $F_H$ to offer and her promised foreclosure policy $\lambda_H$, while ensuring that it is not profitable for the low type to deviate and mimic.

$$\max_{(F_H, \lambda_H)} \quad p(F_H, \lambda_H) + \delta (\pi_H (c_1 - f_1) + (1 - \pi_H) \theta (c_2 - f_2) + (1 - \theta) (c_3 - f_3))$$

subject to

- **MC**
  $$p(F_H, \lambda_H) = \pi_H f_1 + (1 - \pi_H) [\theta f_2 + (1 - \theta) f_3]$$

- **IC**
  $$U^B_L \geq p(F, \lambda_H) + \delta (\pi_L (c_1 - f_1) + (1 - \pi_L) \theta (c_2 - f_2) + (1 - \theta) (c_3 - f_3))$$

- **LL**
  $$f_j \leq c_j \quad \forall \ j \in \{1, 2, 3\}$$

- **MNO**
  $$f_1 \geq f_2 \geq f_3 \geq 0$$

- **MNI**
  $$c_1 - f_1 \geq c_2 - f_2 \geq c_3 - f_3 \geq 0$$ (13)

The optimisation programme given by Eq. 13 takes into account the security design problem. Therefore it has three additional constraints regulating the security design when compared to Eq. 3: (LL) is the limited liability condition for each cash flow realisation, (MNO) is the outside investors’ monotonicity constraint, and (MNI) is the insider residual claim’s monotonicity constraint.

The following proposition establishes the optimality of debt, which was taken as given in the discussion in Section 4.1.

**Proposition 3.** The high-type securitiser optimally issues a debt security backed by the mortgage pool with face value $F_H \in [c_3, c_1]$. That is, given any promised foreclosure policy $\lambda_H$, the security $F_H$ is given by $f_j = \min\{c_j, F_H\} \forall\ j \in \{1, 2, 3\}$.

This result is consistent with the literature on the pecking order of outside financing, e.g. Myers (1984) under asymmetric information. In order to discourage the low type from mimicking, the optimal security issued by the high-type is a debt security ($F < c_1$). Moreover, the high-type exhausts her capacity of issuing risk-free debt ($F \geq c_3$), since risk-free securities are free from the adverse problem.

The high-type securitiser’s retention of residual claims of future cash flow could be seen as a necessary signalling cost in order to separate from the low type, a well-established result in the security design literature such as Leland and Pyle (1977) and DeMarzo and Duffie (1999). As discussed in the previous section, the
main result of this paper is to show that excessive foreclosure helps to mitigate such signalling cost for the high-type securitiser. The extent of excessive foreclosure in equilibrium thus trades off the direct cost of inefficient foreclosure against the greater gains from securitisation.

4.3 Discussion of some key assumptions

As demonstrated in Section 4.1, the key properties of the model which give rise to excessive foreclosure in equilibrium are given by Eq. 9–10. In this section we discuss the intuition behind these properties and their robustness.

Eq. 9–10 state that, the value of a debt security issued by the high type, when compared to the low type, is more sensitive to a change in the face value than the low type (Eq. 10), whereas it is less sensitive to an increase in the foreclosure policy (Eq. 9). These two properties are directly implied by the following two ingredients, which we discuss in details below.

(i) For any given foreclosure policy, the cash flows of the mortgage pool satisfies hazard rate ordering (HRO).

Definition 1 (Hazard Rate Ordering (HRO)). For all type $i \in \{H,L\}$, the cash flows $C_i$ has a common support; and $\frac{Pr(C_L \geq c)}{Pr(C_H \geq c)}$ is decreasing in $c$, for $c$ in the support of $C$ up to the upper boundary of the support.

(ii) Increasing foreclosure rate $\lambda$ decreases the exposure of the mortgage pool to the risk of re-default of the delinquent mortgages.

(i) Hazard rate ordering

The (HRO) property immediately implies Eq. 10, as a debt security’s sensitivity to the face value is equal to the probability of full repayment, which is higher for the high type than for the low type. The (HRO) property is weaker than the Monotone Likelihood Ratio Property (MLRP), which is commonly assumed in signalling environments. In particular, DeMarzo et al. (2015) show that this is a sufficient condition to ensure the optimality of debt security in a signalling framework with liquidity needs. In Section 4.2 we show that this result also holds true in our model with endogenous foreclosure decisions, i.e. the optimal security is debt.
(ii) The effects of foreclosure decision on cash flows

Our specification of the foreclosure technology relative to forbearance affects the cash flow of the mortgage pool in two important ways: 1) the expected cash flow initially increases and then decreases in foreclosures; 2) the risk exposure of the cash flow to re-defaults decreases in foreclosures. The first feature about the total expected cash flow is a standard decreasing-returns-to-scale assumption imposed to guarantee interior optimal level of foreclosures. It is also economically realistic; For example, some delinquent mortgages are easier to foreclose than others because for instance some underlying properties are cheaper to refurbish and/or to re-sell in the market. Alternatively, the marginal cost of foreclosure is likely to be increasing in the amount of foreclosures due to the servicers’ limited expertise and/or human capital for large amount of foreclosures in a timely manner.14

The second feature is of crucial importance to our results. Compared to forbearance, foreclosing a delinquent mortgage reduces risk exposure of the mortgage pool to aggregate risk of re-default in the economy. Modifying a defaulted mortgage borrower exposes the mortgage pool to significant aggregate uncertainty of re-default in the future: when property prices drop further, and/or when income decreases due to unemployment and general business cycle. In contrast, foreclosure implies a relatively quick liquidation of the property, hence it only exposes to short-term uncertainty and delivers a more stable payoff than that of forbearance. In our model, this intuition is captured by the way the cash flows are affected by foreclosure policy. Following the realisation of the bad state in which some mortgages default, the delinquent mortgages can either re-default, or recover. This captures the aggregate risk the mortgage pool is exposed to. Increasing the fraction of delinquent mortgages reduces such exposure. As a result, increasing foreclosure rate increases the cash flow received by the mortgage pool in the re-default state, while reducing the cash flow in the recover state.

This intuitive assumption together with (HRO) implies Eq. 9. When the face value of the debt issued in equilibrium is low, the value of the debt security is

14The “robo-signing” controversy could be illustrative of the limited foreclosure capacity problem. In the fall of 2010, employees of several major servicing companies and banks allegedly signed large amount of documents and swore to their accuracy as legally required in the bankruptcy process, without verifying the information i.e. signing like a robots. The goal of the practice was to speed up the foreclosure procedure of delinquent mortgages. See for example, http://www.wsj.com/articles/j-p-morgan-justice-department-reach-50-million-robo-signing-settlement-1425399145
sensitive to the foreclosure policy only when the delinquent mortgages re-default. Since the low-quality mortgage pools have higher unconditional probability of entering the re-default state (HRO), they are more sensitive to an increase in the foreclosure rate.

5 Mortgage servicers and commitment

Thus far we have assumed that the securitiser is able to commit to a foreclosure policy at $t = 1$ and then implement it at $t = 2$ when defaults occur. We devote this section to discussing the role of mortgage servicers in enabling the securitisers to commit to the foreclosure policy, which improves efficiency as we show below.

5.1 The efficiency of commitment

While we have assumed the ability for the securitiser to commit to a foreclosure policy, we formally establish in the subsection that the equilibrium is more efficient with commitment than without.

Table 3: Model timeline without commitment

| $t = 1$ | Securitiser observes $\pi_i$ and offers $F_i$ |
| $t = 2$ | Mortgage defaults in state $B$ |
|         | Securitiser chooses foreclosure policy $\lambda_i$ |
| $t = 3$ | Final payoffs realise |

In order to examine the role of commitment, we modify the sequence of events in the model to account for the lack of commitment. Specifically, the securitiser chooses the foreclosure policy of the mortgage pools at $t = 2$ when defaults occur, instead of at $t = 1$, as indicated in Table 3. This implies that at $t = 2$, the securitiser has the incentive to choose a foreclosure policy that maximises her residual claim to the mortgage pool. We defer the full characterisation of the equilibrium without commitment to the Appendix, and discuss the main intuition below. Denote with $U_i'$ the expected payoff to a securitiser with a type $i$ mortgage pool in the least cost separating equilibrium without commitment.

23
Proposition 4. Compared to the scenario without commitment, in the least cost separating equilibrium with commitment, the low-type securitiser is equally well off whereas the high-type securitiser is better off. That is,

\[ U_{LB}^F = U_{L}^* = U_{L}^* = U_{HB}^F > U_{H}^* \geq U_{H}^* \]  

(14)

where the inequality is strict if and if Eq. 5 holds.

While it is generally not surprising that the securitiser can be not worse off with commitment power, Proposition 4 show that commitment power strictly improves the efficiency of securitisation, when Eq. 5 holds. This is the case when the face value of the debt issued by the high type in the equilibrium is relatively low, as discussed in Section 4.1.

In this case, the equilibrium foreclosure policy \( \lambda_{H}^* \) described in Section 4 requires commitment. Given the security issued at \( t = 1 \), the high-type securitiser at \( t = 2 \) following a realisation of state \( B \) has the incentive to choose a foreclosure policy to maximise the expected value of her residual claim in the mortgage pool. When the face value of the debt issued is low, the securitiser receives a residual payoff only when the delinquent mortgage recover, and zero otherwise.

\[ \max_{\lambda_{H}} \theta [Z_B + \mathcal{L}(\lambda_{H}) + (1 - \lambda_{H})X - F_{H}^*] \]  

(15)

This levered-equity claim gives the securitiser risk-shifting incentives at \( t = 2 \). Since foreclosing delinquent mortgages reduces risk, the securitiser prefers a lenient foreclosure policy with zero foreclosure at \( t = 2 \), if she were not to implemented the \( t = 1 \) committed foreclosure policy of \( \lambda_{H}^* \).

5.2 Mortgage servicers as commitment devices

When the securitiser cannot commit to a foreclosure policy, they can effective do so by outsourcing the foreclosure decision to specialist mortgage servicers at the time of securitisation (\( t = 1 \)). This interpretation of mortgage servicers as a commitment device provides a raison d’être of hiring mortgage servicers, which is a common practice in the mortgage securitisation industry. We also believe that this interpretation corroborates with some empirical findings regarding the servicers’
impact on foreclosures and their compensation contracts.

We propose two potential ways in which the securitiser can commit to a foreclosure policy through contracting with a mortgage servicer. First, if servicers have known heterogeneity in foreclosure capacity or modification capacity, the high-type securitiser can effectively commit to a “tough” foreclosure policy by hiring a servicer with high foreclosure capacity (or low modification capacity). It is commonly accepted that both foreclosing and modifying a delinquent mortgage require substantial human capital, see for example Thompson (2009). Therefore ex post in the default state, it is difficult for a servicer to alter their capacity to foreclose. This interpretation of heterogeneity in mortgage servicers corroborates well with the economically and statistically significant servicer fixed effect in predicting foreclosure probability in Agarwal et al. (2011a).

Alternatively, the securitiser can hire a servicer and provide an incentive contract that is biased towards foreclosure. This interpretation can explain why biased servicers’ incentives are offered in the first place, which for example Thompson (2009) and Krueger (2014) argue are an important friction leading to excessive foreclosures. The key assumption behind this interpretation is that there are substantial frictions preventing the mortgage servicing contract from being renegotiated in the foreclosure stage \( t = 2 \). Krueger (2014) has shown indeed in practice mortgage servicing contracts are rarely renegotiated, mainly because by law, it requires the consent of the securitiser, the servicer, and the dispersed MBS investors.

### 6 Ex ante screening effort and welfare

So far we have treated the ex ante probability \( \gamma \) of the mortgage pool being high quality as exogenous. In this section, we extend the model to incorporate an ex ante stage \( t = 0 \), at which time the securitiser can endogenously exert non-verifiable costly effort to increase the probability of receiving a high-quality at \( t = 1 \). The main finding is that while information asymmetry leads to underinvestment in screen effort, committing to an ex post excessive foreclosure policy mitigates this underinvestment problem and the associated inefficiency.

At \( t = 0 \), the securitiser is endowed with $1 and can invest in a mortgage pool. If the securitiser exerts non-contractible effort to affect \( \gamma \in [\tilde{\gamma}, \bar{\gamma}] \), the probability
that the mortgage pool is of high quality, before investing at $t = 0$. For example, the mortgage originator spends more time and attention to screen out borrowers with suspicious income or the investment bank performs more due diligence to form better quality mortgage pool. The effort cost function has a quadratic form $\frac{1}{2}k(\gamma - \gamma)^2$. We assume $k$ is high enough to guarantee interior optimal effort level, and $U_L^{FB} \geq 1$ so that investing in the mortgage pool is always efficient.

### 6.1 Optimal screening effort

In this section we solve for the optimal screening effort of the originator in equilibrium. The securitiser is willing to exert costly effort because the expected payoff of being a high type $U_H$ is higher than that of being a low type $U_L$. Since $U_H$ and $U_L$ will be potentially affected by the information environment, the security design, and the foreclosure policy in the subsequent stages of the model, the optimal screening effort chosen by the securitiser will indirectly be affected.

Notice that since the securitisation stage is in the least cost separating equilibrium, the equilibrium outcome does not depends on $\gamma$, the prior probability that the mortgage pool is of high quality. We can therefore consider any generic pair of $\{U_H, U_L\}$ that represents the payoffs to the securitiser in the separating equilibrium at securitisation stage $t = 1$. At $t = 0$, the securitiser chooses the optimal level of effort to maximise her ex ante expected payoff

$$
\max_\gamma \gamma U_H + (1 - \gamma) U_L - \frac{1}{2} k(\gamma - \gamma)^2
$$

The optimal effort thus is

$$
\gamma^*(U_H, U_L) = \gamma + \frac{U_H - U_L}{k}
$$

The optimal effort chosen by the securitiser is increasing in the difference in the expected payoff $(U_H - U_L)$ between a high-quality and a low-quality pool. We will look at how this difference changes under symmetric and asymmetric information, and under different foreclosure policy.

First note that the low-type securitiser can always attain the highest possible payoff given her type, i.e. $U_L = U_L^{FB}$, because she suffers no information friction.
and hence optimally chooses the efficient foreclosure policy $\lambda^{FB}$ and sells a full pass-through security. On the other hand, the high type may be strictly worse off under asymmetric information because of the information friction (Proposition 4). When this is the case, the securitiser exerts strictly less effort as a result of asymmetric information.

**Proposition 5.** Comparing to the symmetric information case, the securitiser expends less screening effort under asymmetric information. The ability to commit to a foreclosure policy at $t = 1$ enhances screening effort at $t = 0$. That is,

$$\gamma^*(U^F_H, U^F_L) > \gamma^*(U^*_H, U^*_L) \geq \gamma^*(U'_H, U'_L)$$  \hspace{1cm} (18)

where the inequality is strict if and only if Eq. 5 holds.

As commitment power over foreclosure policy not only improves efficiency (Proposition 4) but also enhances the ex ante screening effort as stated in the above proposition, this result further points to an additional benefit of mortgage servicers as commitment devices.

### 6.2 Foreclosure policy and screening effort

Next we turn to the question of how regulatory interventions of foreclosure policies can affect the screening effort. As our main result has shown that committing to an excessive foreclosure policy allows the high-type securitiser to separate from the low type at lower costs, the securitiser ex ante also has a stronger incentive to acquire a high-quality mortgage pool and hence optimally expends greater screening effort. This also implies a higher social welfare. The following proposition summarises the effect of a regulatory intervention of the foreclosure policy on ex ante screening efforts and its welfare implication.

**Proposition 6.** If the government imposes a foreclosure policy $\lambda^H$ different from the equilibrium policy $\lambda^*_H$, including the ex post efficient policy $\lambda^{FB}$, the securitiser exerts less screening effort at $t = 0$, hence reducing the total welfare.

Proposition 6 highlights a novel unintended consequence of government regulation of the foreclosure decision in the mortgage securitisation market. Due to information
asymmetry, imposing any foreclosure policy different from $\lambda_H^*$ on the securitiser reduces her payoff in the case of receiving a high-quality mortgage pool. This in turn lowers her incentive to exert screening effort to achieve a high-quality pool. The under-provision of value-enhancing screening effort decreases social welfare as a whole.

7 Empirical implications

This section summarises the novel empirical implications of our model related to the role of foreclosure policies in the mortgage-backed securitisation industry.

1. Securitised mortgage pools on average have a higher foreclosure rate conditional on delinquency than comparable bank-held mortgages. Conditioning on the quality, the high-quality mortgages drive the difference. This is the main result of the model (Proposition 1) that the high-type securitiser distorts foreclosure policy to mitigate the information friction in the process of securitisation, while there is no distortion for the low-quality pool and the bank-held mortgages. This provides an economic explanation of the existing empirical finding of Piskorski et al. (2010), Agarwal et al. (2011a) and Krueger (2014). In particular, Piskorski et al. (2010) show that while securitised loans on average have a 3% to 7% higher foreclosure rate in absolute terms than bank-held loans, the effects are larger among borrowers with better credit quality.

2. Foreclosing the marginal delinquent mortgage in a securitised pool returns on average less than the mortgage’s expected recovery value, or the marginal delinquent mortgage in a bank-held portfolio. As foreclosures in a securitised pool on average are excessive, the foreclosure proceeds of the marginal delinquent mortgage are lower than the loan’s expected recovery value. The excessive foreclosure in securitised pools is driven by informational frictions in the securitisation process. Since bank-held mortgages are free of this friction, they are foreclosed at the ex post efficient rate, where the value of the marginal foreclosure is equated to the loan’s expected recovery value.

3. Servicers of securitised mortgages on average have compensation contracts that are biased towards foreclosure. Conditioning on the quality of the mortgage
pool, the servicers of high-quality mortgage pools drive the difference. Our model suggests that securitisers would offer optimal incentive contracts to servicers in order to implement the ex ante optimal foreclosure policy. Since the optimal policy appears excessive comparing to the ex post efficient benchmark, the servicers’ incentives have to be biased accordingly towards foreclosure. Thompson (2009) and Krueger (2014) document biases in the servicers’ incentives consistent with our prediction.

4. Intrinsic servicer-specific biases towards foreclosure are positively related to the quality of the mortgage pools. Our model allows also the interpretation that securitisers of high-quality mortgage pools seek intrinsically “tough” servicers as commitment to excessive foreclosure. While works by Agarwal et al. (2011b) and Agarwal et al. (2012) uncover the importance of servicer-specific factors relating to their pre-existing organisational capabilities towards foreclosure, our model predicts a testable relationship between such servicer-specific characteristics and the quality of the mortgage pools.

8 Conclusion

This paper studies the relationship between the foreclosure decision of delinquent mortgages and the securitisation of mortgages. We propose a novel mechanism in which excessive foreclosure policy, in addition to the retention of junior securities, serves as costly signals to reduce informational frictions inherent in the securitisation process. We list the empirical predictions coming from our model, some of which explain several important observed patterns and empirical findings in the mortgage securitisation industry.

Our paper also suggests that mortgage servicers could have the important role as commitment devices, allowing the securitiser to commit to ex post excessive foreclosure policies. As a result, the mortgage servicing contracts appear to have incentives biased towards foreclosure, and the servicer-specific capacity related to foreclosures can be informative of mortgage pool quality. These results are broadly consistent with empirical findings and yield new predictions for future empirical work.

For a normative perspective, our results caution that policies attempting to
restore foreclosure efficiency can have the unintended consequence of reducing the securitisers’ ex ante screening effort, thereby worsening the average quality of mortgage pool and reducing social welfare.

We conclude with some conjecture of directions for future work and extensions. First, this framework can be extended to a setting with multiple securitisers to study the spillover effects of foreclosure. For instance, it will be interesting to study the interaction between the excessive foreclosure due to securitisation and the fire-sale externality in the distressed property market. It could also be fruitful to analyse, in a general equilibrium, the potential impact of securitisation on the quantity, quality, and the prices of mortgages originated. Finally, a dynamic framework could shed lights on how the excessive foreclosure due to securitisation interacts with property prices across business cycles.
References


Appendices

A  Proofs

A.1 Proof of Lemma 1

This result follows immediately from the discussion.

A.2 Proof of Proposition 1

The least cost separating equilibrium is characterised by Eq. 3. We prove this proposition by solving the optimisation programme and then highlighting the properties of the equilibrium foreclosure policy.

Firstly, we establish that any optimiser of the programme must bind the (IC).

We prove this by contradiction. Suppose there exists \((F_H, \lambda_H)\) that is an optimiser of the programme such that the (IC) is slack. Then there exists \(F'_H > F_H\) such that the (IC) is still satisfied at \((F'_H, \lambda_H)\). However, the objective function is strictly greater at \((F'_H, \lambda_H)\) than at \((F_H, \lambda_H)\). This contradicts with the supposition that \((F_H, \lambda_H)\) is an optimiser of the programme. Therefore any optimiser of the programme must bind the (IC).

We then solve this optimisation given a binding (IC) by considering two separate cases.

(i) \(F_H \in [Z_B + \mathcal{L}(\lambda_H) + (1 - \lambda_H)X, Z_G]\)

(ii) \(F_H \in [Z_B + \mathcal{L}(\lambda_H), Z_B + \mathcal{L}(\lambda_H) + (1 - \lambda_H)X]\)

Case (i): \(F_H \in [Z_B + \mathcal{L}(\lambda_H) + (1 - \lambda_H)X, Z_G]\)

The solution in this case can be expressed as follows

\[
\max_{(F_H, \lambda_H)} \quad p(F_H, \lambda_H) + \delta \pi_H (Z_G - F_H) \\
\text{s.t.} \quad \mu(F_H, \lambda_H) = \pi_H F_H + (1 - \pi_H) \delta [Z_B + \mathcal{L}(\lambda_H) + (1 - \lambda_H)X] \\
\] 

\[
+ (1 - \pi_H)(1 - \theta)[Z_B + \mathcal{L}(\lambda_H)] \\
\] 

\[
(\text{IC}) \quad U_L^* = p(F_H, \lambda_H) + \delta \pi_L (Z_G - F_H) \quad (19)
\]
The binding \((IC)\) implies that the implicit derivative of \(F_H\) w.r.t. \(\lambda_H\) is

\[
\frac{\partial F_H}{\partial \lambda_H} = -\frac{\partial p(F_H, \lambda_H)}{\partial F_H} - \delta \pi_L
\]  

(20)

We then substituting the \((MC)\) and \((IC)\) into the objective function to eliminate \(F_H\). We confirm that the second order condition is satisfied for the resulting one-dimensional optimisation programme over \(\lambda_H\):

\[
SOC_{(i)} = \frac{\delta (1 - \pi_H)(\pi_H - \pi_L)}{\pi_H - \delta \pi_L} \mathcal{L}''(\lambda_H) < 0
\]  

(21)

The solution \(\lambda_H^*\) is then characterised by the first order condition:

\[
FOC_{(i)} = \frac{\partial p(F_H, \lambda_H)}{\partial \lambda_H} + \left( \frac{\partial p(F_H, \lambda_H)}{\partial F_H} - \delta \pi_H \right) \frac{\partial F_H}{\partial \lambda_H}
\]  

(22)

The above first order condition is equal to zero at \(\lambda_{FB}\) because \(\left. \frac{\partial p(F_H, \lambda_H)}{\partial \lambda_H} \right|_{\lambda_H = \lambda_{FB}} = 0\).

Therefore for case (i), \(\lambda^* = \lambda_{FB}\). \(F_H^*\) is given by the binding \((IC)\) at \(\lambda_{FB}^*\):

\[
F_H^* = Z_G - \frac{V_H(\lambda_{FB}) - V_L(\lambda_{FB})}{\pi_H - \delta \pi_L}
\]  

(23)

We then provide the condition for this case to exist. That is, \(F_H^* \geq Z_B + \mathcal{L}(\lambda_{FB}) + (1 - \lambda_{FB})X\). Expanding the expression and collecting terms yields the following, which is the complimentary case of Eq. 5.

\[
\frac{(\pi_H - \pi_L)(1 - \theta)(1 - \lambda_{FB})}{\pi_L(1 - \delta)} X \leq Z_G - [Z_B + \mathcal{L}(\lambda_{FB}) + (1 - \lambda_{FB})X]
\]  

(24)

Case (ii): \(F_H \in [Z_B + \mathcal{L}(\lambda_H), Z_B + \mathcal{L}(\lambda_H) + (1 - \lambda_H)X]\)

The solution in this case can be expressed as follows

\[
U_H^* = \max_{(F_H, \lambda_H)} p(F_H, \lambda_H) + \delta [\pi_H(Z_G - F_H) + (1 - \pi_H)\theta (Z_B + \mathcal{L}(\lambda_H) + (1 - \lambda_H)X - F_H)]
\]

s.t. \((MC)\) \(p(F_H, \lambda_H) = [\pi_H + (1 - \pi_H)\theta]F_H + (1 - \pi_H)(1 - \theta)[Z_B + \mathcal{L}(\lambda_H)]\)

\((IC)\) \(U_L^* = p(F_H, \lambda_H) + \delta \pi_L [(Z_G - F_H) + (1 - \pi_L)\theta (Z_B + \mathcal{L}(\lambda_H) + (1 - \lambda_H)X - F_H)]\)  

(25)
The binding \((IC)\) implies that the implicit derivative of \(F_H\) w.r.t \(\lambda_H\) is
\[
\frac{\partial F_H}{\partial \lambda_H} = -\frac{\partial p(F_H, \lambda_H)}{\partial \lambda_H} + (1 - \pi_L)\theta[\mathcal{L}'(\lambda_H) - X] + \frac{\partial p(F_H, \lambda_H)}{\partial F_H} - \delta[\pi_L + (1 - \pi_L)\theta]
\] (26)

We then substitute the \((MC)\) and \((IC)\) into the objective function to eliminate \(F_H\). We confirm that the second order condition is satisfied for the resulting one-dimensional optimisation programme over \(\lambda_H\), given Assumption 1:
\[
SOC_{(ii)} = \left[(1 - \pi_H) - \theta(1 - \delta + 1 - \pi_H)\right] \times \frac{\delta(\pi_H - \pi_L)}{(1 - \theta)(\pi_H - \delta \pi_L) + \theta(1 - \delta)} \mathcal{L}''(\lambda_H) < 0
\] (27)

The solution \(\lambda_H^*\) is then characterised by the first order condition:
\[
FOC_{(ii)} = \frac{\partial p(F_H, \lambda_H)}{\lambda_H} + (1 - \pi_H)\theta[\mathcal{L}'(\lambda_H) - X]
+ \left(\frac{\partial p(F_H, \lambda_H)}{\partial F_H} - \delta[\pi_H + (1 - \pi_H)\theta]\right)\frac{\partial F_H}{\partial \lambda_H}
\] (28)

At \(\lambda^{FB}\), the above first order condition is strictly greater than zero:
\[
FOC_{(ii)}\bigg|_{\lambda_H=\lambda^{FB}} = \theta \left[\mathcal{L}'(\lambda_H) - X\right]
\times \left[(1 - \pi_H) - \frac{(1 - \delta)[\pi_H + (1 - \pi_H)\theta]}{[\pi_H + (1 - \pi_H)\theta] - \delta[\pi_L + (1 - \pi_L)\theta]}(1 - \pi_L)\right] > 0
\] (29)

This implies that the solution for case (ii) is \(\lambda_H^* > \lambda^{FB}\). The face value of the debt issued in the case \(F_H^*\) is given by the binding \((IC)\) at \(\lambda_H^*:\)
\[
F_H^* = \frac{U_L^* - \delta \pi_L Z_G - \delta (1 - \pi)\theta[Z_B + \mathcal{L}(\lambda_H^*)] - (1 - \pi_H)(1 - \theta)[Z_B + \mathcal{L}(\lambda_H^*) + (1 - \lambda_H^*)X]}{[\pi_H + (1 - \pi_H)\theta] - \delta[\pi_L + (1 - \pi_L)\theta]}
\] (30)

We now provide the condition for this case to exist. That is, \(F_H^* < Z_B + \mathcal{L}(\lambda_H^*) + (1 - \lambda_H^*)X\), which is equivalent to \(G(\lambda_H^*) > 0\) where \(G(\cdot)\) is defined in Eq. 5. As \(G(\lambda_H^*)\) is strictly increasing in \(\lambda_H^*\) for all \(\lambda_H^* \geq \lambda^{FB}\), the condition Eq. 5 implies
that Case (ii) exists as $G(\lambda^*_H) \geq G(\lambda^{FB}) > 0$.

Finally, while a solution in Case (ii) may still exist when Eq. 5 does not hold, the existence of a solution in Case (i) implies that the solution to the overall optimisation programme given by Eq. 3 is the solution in Case (i), which provides the high-type securitiser with a strictly higher payoff than any solution in Case (ii), given that the Case (i) solution has both an efficient foreclosure policy $\lambda^*_H = \lambda^{FB}$ and a higher face value of the debt issued.

To summarise, the equilibrium foreclosure policy by the high-type securitiser is such that $\lambda^*_H \geq \lambda^{FB}$, where the inequality is strict if and only Eq. 5 holds.

A.3 Proof of Corollary 1

We establish this corollary considering the two separate cases discussed in Appendix A.2. Notice that $\frac{\partial \hat{p}(\lambda_H)}{\partial \lambda_H}$ can be expressed as $\frac{\partial p(F_H, \lambda_H)}{\partial F_H} \frac{\partial F_H}{\partial \lambda_H} + \frac{\partial p(F_H, \lambda_H)}{\partial \lambda_H}$, where $\frac{\partial F_H}{\partial \lambda_H}$ is the implicit derivative of $F_H$ w.r.t. $\lambda_H$ given by a binding (IC).

In Case (i), Eq. 19–20 imply that $\left.\frac{\partial \hat{p}(\lambda_H)}{\partial \lambda_H}\right|_{\lambda_H = \lambda^{FB}} = 0$.

In Case (ii), Eq. 25–26 imply that

$$\left.\frac{\partial \hat{p}(\lambda_H)}{\partial \lambda_H}\right|_{\lambda_H = \lambda^{FB}} \geq 0, \text{ with the inequality strict if and only if Eq. 5 holds.}$$

The face value of the debt issued in the least cost separating equilibrium binds the (IC). The equilibrium is given by Case (ii) if and only if Eq. 5 holds. We therefore have $\left.\frac{\partial \hat{p}(\lambda_H)}{\partial \lambda_H}\right|_{\lambda_H = \lambda^{FB}} \geq 0$, with the inequality strict if and only if Eq. 5 holds.

A.4 Proof of Proposition 2

We prove this proposition considering the two separate cases discussed in Appendix A.2.

In Case (i), Eq. 5 does not hold, and $G(\lambda^{FB}) \leq 0$. The equilibrium foreclosure policy is equal to the first-best level $\lambda^*_H = \lambda^{FB}$ and remains unaffected by a change in $\pi_H$ when $G(\lambda^{FB}) < 0$. For the case where $G(\lambda^{FB}) = 0$, notice that increasing $\pi_H$ increases $G(\lambda^{FB})$. This suggests that a marginal increase in $\pi_H$ causes $\lambda^*_H$ to change from Case (i) to Case (ii). This corresponds to a strict increase in the foreclosure
rate in equilibrium.

In Case (ii) \( G(\lambda^{FB}) > 0 \), the equilibrium foreclosure policy is implicitly defined by the first order condition (Eq. 28). After some algebraic manipulation, the equilibrium foreclosure policy can be implicitly defined by

\[
L'(\lambda_H^*) = \frac{\delta - [\theta(1 - \pi_H) + \pi_H]}{(1 - \delta)\theta + (1 - \pi_H)(1 - \theta)}X
\]

(32)

Implicitly differentiating the above equation yields that \( \frac{\partial \lambda_H^*}{\partial \pi_H} > 0 \), because the RHS of the above equation is strictly decreasing in \( \pi_H \).

To summarise, the equilibrium foreclosure policy is such that \( \frac{\partial \lambda_H^*}{\partial \pi_H} > 0 \) if and only if \( G(\lambda^{FB}) \geq 0 \).

A.5 Proof of Proposition 3

The proof is constructed by establishing several claims in succession. Let us call a security \( \mathcal{F}_H \) permissible if it satisfies (IC), (MNO), and (MNI). For any given \( \lambda_H \), an optimal security is a permissible security that maximises the high-type securitiser’s expected payoff

\[
\delta U_H(\lambda) + (1 - \delta)p(\mathcal{F}_H, \lambda_H)
\]

or since \( U_H(\lambda) \) is not affected by the security design, a security \( \mathcal{F}_H \) that maximises the selling proceeds \( p(\mathcal{F}_H, \lambda_H) = \pi_H f_1 + (1 - \pi_H)[\theta f_2 + (1 - \theta)f_3] \). Since \( \lambda_H \) plays no role in this proof, we suppress the selling proceeds to \( p(\mathcal{F}_H) \) for the ease of notation.

**Claim 1.** For any optimal security \( \mathcal{F}_H^* = \{f_1^*, f_2^*, f_3^*\}, f_1^* < c_1 \).

*Proof.* If \( f_1^* = c_1 \), by (MCI), \( f_2^* = c_2 \) and \( f_3^* = c_3 \). This security (full equity) violates (IC). \( \square \)

**Claim 2.** For any optimal security \( \mathcal{F}_H^* \), the (IC) must bind.

*Proof.* Suppose instead the (IC) is slack for some optimal security \( \mathcal{F}_H^* = \{f_1^*, f_2^*, f_3^*\} \). By Claim 1, \( f_1^* < c_1 \). Unless \( c_1 - f_1^* = c_2 - f_2^* \), there exists another permissible security \( \tilde{\mathcal{F}} = \{\tilde{f}_1, f_2^*, f_3^*\} \) with \( \tilde{f}_1 > f_1^* \) that satisfies (IC). As \( p(\cdot) \) strictly increases with \( f_1, p(\tilde{\mathcal{F}}) > p(\mathcal{F}_H^*) \), contradicting the assumption that \( \mathcal{F}_H^* \) is optimal.

If \( f_1^* < c_1 \) and \( c_1 - f_1^* = c_2 - f_2^* \), one can increase the objective function \( p(\cdot) \) by increasing both \( f_1^* \) and \( f_2^* \) by some \( \epsilon > 0 \) without violating (IC), unless \( f_2^* = c_2 \) or \( c_2 - f_2^* = c_3 - f_3^* \). Note that \( f_2^* = c_2 \) implies \( f_1^* = c_1 \) hence violates Claim 1.
Suppose now $f_1^* < c_1$ and $c_1 - f_1^* = c_2 - f_2^* = c_3 - f_3^*$, similarly one can increase all $f_1^*, f_2^*, f_3^*$ without violating (IC) to strictly increase $p(\cdot)$, unless $f_3^* = c_3$. And $f_3^* = c_3$ implies $f_1^* = c_1$ hence violates Claim 1.

Since we have shown that any security with a slack (IC) can be improved upon, the (IC) must be binding at any optimal security.

**Claim 3.** The optimal security $\mathcal{F}_H^*$ has either

1. $f_1^* = f_2^* > f_3^* = c_3^*$ or
2. $f_1^* > f_2^* = c_2^* > f_3^* = c_3^*$

i.e. the optimal security is a risky debt with face value $F_H$ equals to $f_2^*$ in case 1 or $f_1^*$ in case 2.

**Proof.** Consider a security $\mathcal{F} = \{f_1, f_2, f_3\}$ with a binding (IC) constraint. Using the (IC), write $f_1$ as a function of $f_2$ and $f_3$

$$f_1(f_2, f_3) = \frac{(1 - \delta)U_L - [(1 - \pi_H) - \delta(1 - \pi_L)](\theta f_2 + (1 - \theta)f_3)}{\pi_H - \delta \pi_L} \quad (33)$$

Substitute this $f_1$ into the objective function. After some algebraic manipulation, the objective function becomes

$$\delta U_H + (1 - \delta) \left[ \frac{\pi_H}{\pi_H - \delta \pi_L}(1 - \delta)U_L + \frac{\pi_H - \pi_L}{\pi_H - \delta \pi_L} (\theta f_2 + (1 - \theta)f_3) \right] \quad (34)$$

which is strictly increasing in $f_2$ and $f_3$. Since $f_2$ is bounded above by either $c_2$ or $f_1$ and $f_3$ only by $c_3$, any optimal security $\mathcal{F}_H^*$ must have $f_3^* = c_3$ and $f_2^* = \min\{f_1^*, c_2\}$.

**A.6 Proof of Proposition 4**

In order to establish this result, we first characterise fully the least cost separating equilibrium without commitment. We denote henceforth with $'$ all equilibrium quantities for the case without commitment.

We again start the analysis with the low-type securitiser. Since the low type issues a full pass-through equity security in a separating equilibrium, she retains no cash flow. Maintaining the assumption that in this case she makes the first-best
foreclosure decision to maximise the value of the mortgage pool, \( \lambda'_{L} = \lambda'^{FB}_{L} \), the payoff to a low-type securitiser is therefore equal to \( U'_L = U'^{FB}_{L} \).

The high-type securitiser chooses an optimal security \( \mathcal{F}_H \) at \( t = 1 \) to maximise her expected payoff, taking into account the subsequent foreclosure policy \( \lambda_H \) chosen at \( t = 2 \) given the security issued. Although in this case there are still only 3 possible cash flow realisations conditional on \( \lambda \), each cash flow changes with \( \lambda \). The security \( \mathcal{F}_H \) therefore must specify a payoff to the investors for each possible cash flows, as summarised in Table 4. In the least cost separating equilibrium, the optimal security is given by

\[
\max_{(\mathcal{F}_H, \lambda_H)} \quad p(\mathcal{F}_H, \lambda_H) + \delta \pi_H (c_1 - f_1) \\
+ \delta (1 - \pi_H) (\theta [c_2(\lambda_H) - f_2(\lambda_H)] + (1 - \theta) [c_3(\lambda_H) - f_3(\lambda_H)])
\]

s.t.  
*(MC)* \( p(\mathcal{F}_H, \lambda_H) = \pi_H f_1 + (1 - \pi_H) [\theta f_2(\lambda_H) + (1 - \theta) f_3(\lambda_H)] \)

*(IC)* \( U'_L \geq p(\mathcal{F}_H, \lambda_H) + \delta \pi_L (c_1 - f_1) \\
+ \delta (1 - \pi_L) (\theta [c_2(\lambda_H) - f_2(\lambda_H)] + (1 - \theta) [c_3(\lambda_H) - f_3(\lambda_H)]) \)

*(IC*) \( \lambda_H = \arg \max_{\lambda} \theta [c_2(\lambda) - f_2(\lambda)] + (1 - \theta) [c_3(\lambda) - f_3(\lambda)] \)

*(LL)* \( f_1 \leq c_1, \ f_2(\lambda) \leq c_j(\lambda) \ \forall \ j \in \{2, 3\} \)

*(MNO)* \( f_1 \geq f_2(\lambda) \geq f_2(\hat{\lambda}) \geq f_3(\lambda) \ \forall \ \hat{\lambda} \geq \lambda \)

*(MNI)* \( c_1 - f_1 \geq c_2(\lambda) - f_2(\lambda) \geq c_2(\hat{\lambda}) - f_2(\hat{\lambda}) \geq c_3(\hat{\lambda}) - f_3(\hat{\lambda}) \ \forall \ \hat{\lambda} \geq \lambda \)

(35)

where *(MC)* is the market clearing condition given that the market believes the issuer of the securitisation offer \( (\mathcal{F}_H, \lambda_H) \) is of the high type, *(IC)* is the incentive compatibility constraint for the low type not to mimic the security issued by the high type given market beliefs, *(IC)* is the servicing incentive compatibility constraint that the promised foreclosure policy associated the high type’s security is credible, i.e. following issuing the security \( \mathcal{F}_H, \lambda_H \) maximises the expected value of the securitiser’s residual claim, *(LL)* is the limited liability condition for all cash flow realisation, *(MNO)* is the outside investors’ monotonicity constraint, and *(MNI)* is the insider residual claim’s monotonicity constraint.

This security design problem for the high-type securitiser without commitment is similar to the case with (Eq. 13), apart from one additional constraint, namely
Table 4: Payoffs of the security issued by the high type without commitment

<table>
<thead>
<tr>
<th>Realisation of cash flow</th>
<th>Security payoff $\mathcal{F}_H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1 \equiv Z_G$</td>
<td>$f_1$</td>
</tr>
<tr>
<td>$c_2(\lambda) \equiv Z_B + \mathcal{L}(\lambda) + (1 - \lambda)X$</td>
<td>$f_2(\lambda)$</td>
</tr>
<tr>
<td>$c_3(\lambda) \equiv Z_B + \mathcal{L}(\lambda)$</td>
<td>$f_3(\lambda)$</td>
</tr>
</tbody>
</table>

$(IC^*)$. This constraint reflects the fact that, the securitiser does not have commitment power over the choice of her foreclosure policy. Instead, the equilibrium foreclosure policy for the high type $\lambda_H$ must be incentive compatible at $t = 2$ given the security issued. Similarly, should the low-type securitiser mimic the security issued by the high type, she also subsequently choose the same incentive compatible foreclosure policy $\lambda_H$. This is reflected in the low type’s no-mimicking constraint $(IC)$.

We can now establish the results of this proposition. Because Eq. 35 is a more constrained optimisation problem than Eq. 13, the solution to the latter also satisfies all constraints of the former programme. The solution to Eq. 13 thus implies at least as high an expected payoff to the high-type securitiser as the solution to Eq. 35, i.e. $U^*_H \geq U'_L$.

We next show that this inequality is strict if and only if Eq. 5 holds. Consider firstly the case where Eq. 5 does not hold. We show that in this case $(\mathcal{F}^*_H, \lambda^*_H)$ is also a solution to Eq. 35, therefore $U'_H = U'_H$. Specifically, in this case, $\lambda^*_H = \lambda^{FB}$ and the high-type securitiser with commitment power issues in equilibrium a debt security with face value $F^*_H > Z_B + \mathcal{L}(\lambda^{FB}) + (1 - \lambda^{FB})X$. Notice that since the securitiser retains no cash flow in state $B$ when defaults occur, it is incentive compatible for her to choose the first-best foreclosure policy $\lambda'_H = \lambda^{FB}$ at $t = 2$, satisfying $(IC^*)$.

Finally, consider the case when Eq. 5 holds. We show that in this case $(\mathcal{F}^*_H, \lambda^*_H)$ is not a solution to Eq. 35, therefore $U'_H < U'_H$. To see this, notice that the high-type securitiser with commitment power issues in equilibrium a security that pays off $F^*_H$ both in state $G$ with no default, and in state $B$ after the delinquent mortgages recover, i.e. $f_1 = f_2(\lambda^*_H)$. In the case of non-recovery, the securitiser receives no residual cash flows, i.e. $c_3(\lambda^*_H) = f_3(\lambda^*_H)$. This implies that in state $B$ when defaults occur, the high-type securitiser can deviate to a marginally lower
foreclosure policy $\hat{\lambda}_H < \lambda^*_H$ and receive a higher payoff.

$$
\theta [c_2(\hat{\lambda}_H) - f_2(\hat{\lambda}_H)] + (1 - \theta) [c_3(\hat{\lambda}_H) - f_3(\hat{\lambda}_H)] \\
> \theta [c_2(\lambda^*_H) - f_2(\lambda^*_H)] + (1 - \theta) [c_3(\lambda^*_H) - f_3(\lambda^*_H)]
$$

(36)

Notice that $c_2(\hat{\lambda}_H) > c_2(\lambda^*_H)$ and $c_3(\hat{\lambda}_H) < c_3(\lambda^*_H)$, because foreclosure reduces the risk exposure of the mortgage pool to borrow re-defaults. The above inequality then follows. Firstly, $f_1 = f_2(\lambda^*_H)$ implies that $f_2(\lambda) = f_1 \forall \lambda \leq \lambda^*_H$ by (MNO). This leads to $c_2(\hat{\lambda}_H) - f_2(\hat{\lambda}_H) = c_2(\hat{\lambda}_H) - f_2(\lambda^*_H) > c_2(\lambda^*_H) - f_2(\lambda^*_H)$. Secondly, $c_3(\lambda^*_H) - f_3(\lambda^*_H) = 0$ implies $c_3(\hat{\lambda}_H) - f_3(\hat{\lambda}_H) = 0$ by (MNI). This reasoning suggests that the unique maximiser of Eq. 13, $(F^*_H, \lambda^*_H)$, cannot be an optimiser of Eq. 35 because the (IC") is violated. Therefore the less constrained problem delivers a strictly higher payoff to the high-type securitiser, i.e. $U^*_H > U'_H$ when Eq. 5 holds.

A.7 Proof of Proposition 5

This proposition follows immediately from Eq. 17 and Proposition 4.

A.8 Proof of Proposition 6

Denote with $U_i(\lambda)$ the expected payoff obtained by the high-type securitiser in the least cost separating equilibrium, for a given foreclosure policy. In this equilibrium, the high-type securitiser chooses a security to offer at $t = 1$ to maximise her expected payoff, while preventing mimicking from the low type. Formally, $U_i(\lambda)$ is equal to the value of the optimisation programme Eq. 3, given $\lambda_H = \lambda$.

By definition of $\lambda^*_H$ as the optimiser of Eq. 3, $U_H(\lambda^*_H) > U_H(\lambda_H)$ for any $\lambda^+_H \neq \lambda_H$. Thus the screening effort $\gamma^*$ decreases as

$$
\gamma^*(U_H(\lambda_H), U_L^{FB}) < \gamma^*(U_H(\lambda^*_H), U_L^{FB}) \quad \forall \lambda_H \neq \lambda^*_H
$$

For efficiency, we only need to look at the securitiser’s expected payoff as the investors are always indifferent. The expected payoff is lower when $\lambda^*_H$ is replaced
with $\lambda_H$, i.e.

$$
\gamma^*(U_H(\lambda_H), U_L^{FB})U_H(\lambda_H) + [1 - \gamma^*(U_H(\lambda_H), U_L^{FB})]U_L^{FB} - \frac{1}{2}k^*\gamma^2(U(H(\lambda_H), U_L^{FB})
$$

$$
< \gamma^*(U_H(\lambda_H), U_L^{FB})U_H(\lambda_H) + [1 - \gamma^*(U_H(\lambda_H), U_L^{FB})]U_L^{FB} - \frac{1}{2}k^*\gamma^2(U_H(\lambda_H), U_L^{FB})
$$

$$
\leq \gamma^*(U_H(\lambda_H), U_L^{FB})U_H(\lambda_H) + [1 - \gamma^*(U_H(\lambda_H), U_L^{FB})]U_L^{FB} - \frac{1}{2}k^*\gamma^2(U_H(\lambda_H), U_L^{FB})
$$

The first inequality comes from $U_H(\lambda_H) < U_H(\lambda_H^*)$ and the second weak inequality follows from the definition of optimal $\gamma^*$. Finally, $\lambda_H^{FB}$ is one of the possible $\lambda_H \neq \lambda_H^*$ if and only if Eq. 5 holds.