Convertible bonds and bank risk-taking

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Abstract

We study the effect of going-concern contingent capital on bankers’ risk choices. A possible conversion reduces risk-shifting by forcing deleveraging in high leverage states, when risk shifting incentives are worse. By diluting returns in high states, the additional equity reduces endogenous risk shifting. Optimally designed convertible debt set a trigger value and an amount which trades off the risk reduction against its dilution effect. Contingent capital may be less risky in equilibrium than traditional bank debt, as its lower priority is compensated by reduced endogenous risk. We show that its effectiveness in risk reduction depends critically on the informativeness of the trigger. Adopting a noisy market trigger produces excess conversion (type II error), while a regulated (accounting) trigger converts too infrequently (type I error) because of forbearance.

Key words: Risk shifting; Financial Leverage; Contingent Capital

JEL Classification: G13, G21, G28
1 Introduction

In the recent crisis, bank losses spilled over to the real economy, causing disruption and output losses. This has highlighted the need for more bank capital, which had fallen at historical lows. Higher own capital ensures more risk bearing by shareholders and thus less incentives for excess credit risk. The new Basel III capital ratios may be satisfied only by common equity, yet there is support for allowing contingent capital to count for extra buffers, such as those for SIFI. This is a form of long term debt (called also contingent convertible, or CoCo bonds) which automatically converts to equity upon some trigger. It was originally proposed by Flannery (2002) and Kashyap et al. (2008), and several contributions have discussed its design in terms of reducing risk shifting incentives (ex ante benefits) as well as ex post financial distress costs (Pennacchi (2011), McDonald (2011), Albul et al. (2010), Sundaresan and Wang (2010)). However, so far all formal work assumes asset risk is exogenous, thus unaffected by the introduction of Coco bonds. To understand the ex ante risk reducing incentive, we study a model of "going-concern" contingent capital, which may convert in a timely fashion, ahead of distress.\footnote{In contrast, gone-concern contingent capital converts into equity only upon bank insolvency, when equity is worthless (so called bail-in capital). This protects other lenders and reduce distress costs, but does not have any preventive effect on risk taking.}

In our model, asset choices are endogenous, as they reflect bankers’ risk taking incentives. The basic result is that the chance of conversion strongly reduces risk shifting incentives in high leverage states. The intuition is that conversion dilutes high returns, discouraging gambling.

The main result is that CoCo effectiveness depends on an appropriate trigger, one that delivers deleveraging just when this is most valuable, when risk incentives deteriorate under high leverage. There are clear trade offs in CoCo design. A higher trigger leads to more frequent conversion and a higher equity content. A first effect is to decrease leverage often, ensuring greater loss bearing. A second effect is to dilute equity returns more often. We show how a very high trigger level, and a large CoCo amount, become counterproductive, as too frequent conversions reduce bankers’ incentives for value enhancing effort. As a result, there is an optimal trigger level and amount for contingent capital, even in the absence of bankruptcy costs.

How well does contingent capital compare with straight equity? Our approach allows to compute an equivalence level. Considerably more CoCo debt needs to be issued to substitute for lower equity. Intuitively, the ratio declines as trigger precision improves.

Because risk is endogenous, CoCo bonds may be in equilibrium less risky than conventional bonds. They are incorrectly considered a package of conventional bonds and a short position in a put option. But this neglects their risk-reducing effect, which reduces the value of their short put position.
Our model focuses on asset value triggers, as conversion based on endogenous stock prices suffers from multiple equilibria (Sundaresan and Wang, 2010). In the main extension, we compare triggers based on regulated measures, such as accounting values of book equity, vis-a-vis a noisy market price. We find that market triggers produce excess conversions but create more equity, as regulated (accounting) triggers convert too infrequently because of forbearance (type I error). At present, all outstanding contingent capital converts on book equity. Such a regulated trigger limits conversions by reducing type II errors (converting when not necessary), at the cost of introducing forbearance. So a key advantage of a trigger based even on a noisy market price is that it eliminates discretion on conversion which produces too many type I errors, namely not converting when necessary.

In section 2, we present the basic model. Section 3 shows how CoCo design affects the banker’s risk-taking incentives. Section 4 compares the risk-reducing effect of CoCos against equity and convertible debentures converted at will, which also have been proposed as a solution to risk-shifting (Green, 1984). Section 5 compares market and regulated triggers, and Section 6 concludes. All proofs are in Appendix.

2 The Model

2.1 The Timeline

The sequence of events is:

- at $t = 0$: The banker has a stock of loans with initial value of 1, funded by equity of amount $1 - D$ and debt $D$. The debt may include an amount of $C$ convertible bonds and $D - C$ in deposits.

- at $t = 1$: Asset values are subject to an exogenous shock $\zeta$, distributed uniformly over $[-\delta, \delta]$. The interim asset value is observable by the banker. The banker chooses risk control effort, which affects asset risk and value at $t = 2$. After that, precise information about the interim asset value is revealed to the market with probability $\varphi$. Conversion occurs if the asset value is below the trigger value.

- at $t = 2$: The final value of assets is realized, and all payoffs are distributed.

The sequence of events is presented in Figure 1.

2.2 Agents

In the basic model, there is one active agent in the model: the banker/bank owner. Borrowers are price-takers, so lending is represented as an asset choice by the banker. Depositors are insured and passive. Conversion of CoCo is automatic once the value of assets falls
below the trigger value.

The banker and investors are risk-neutral and rational. The banker chooses either to exert effort to control credit risk \((e = 1)\) or not \((e = 0)\). Effort is costless, and result in better credit quality (higher mean and lower risk).

The banker’s payoff is the value of the original bank equity at \(t = 2\).\(^2\)

2.3 Information and Investment Technology

The bank has an exogenous amount of debt \(D\), which includes only deposits, if no convertible bonds are issued. The deposit rate is normalized to zero. Bank deposits are insured, and the banker enjoys limited liability. The banker invests capital \(1 - D\) at \(t = 0\), so as to satisfy an exogenous capital requirements of \(1 - D\). The assets are not risk-weighted. The initial assets value at \(t = 0\) is 1, so there is no excess capital. Interest rate is zero.

At \(t = 1\), asset value equals \(1 + \zeta\), where \(\zeta\) is an exogenous shock uniformly distributed over \([-\delta, \delta]\). The banker observes the exact realization of interim assets value \(V_1\), and thus the interim leverage \(D_v\). We denote the realization of \(V_1\) as \(v \in [1 - \delta, 1 + \delta]\). We assume that no bank equity may be raised at time \(t = 1\) if leverage turns out to be high.

Next the banker makes a choice whether to exert effort to control the riskiness of bank loans. Depending on his choice, asset values at \(t = 2\) may have two outcomes, safe or risky. If the banker exercises risk control (effort \(e = 1\)), it produces a safe payoff with gross

\(^2\)We assume the bank manager is the sole shareholder, to focus on the interaction of the share price and risk-taking incentives, rather than on the agency conflict between the manager and the shareholder.
return 1. Alternatively, when \( e = 0 \), the banker chooses a risky credit strategy.\(^3\)

The value of risky assets at \( t = 2 \) equals \( v + \varepsilon \), where \( \varepsilon \) follows a distribution \( F(\varepsilon) \) with density function \( f(\varepsilon) \), mean \( E(\varepsilon) = -z \), and standard deviation \( \sigma \). Thus, the riskier strategy yields a lower mean payoff \( v - z \) and has a higher volatility \( \sigma \) relative to the safer outcome.\(^4\)

After the risk choice is made, the value \( v \) is revealed with probability \( \varphi \) to all investors. A riskier strategy may enhance equity in high leverage states. To ensure bank solvency under a safe strategy, we assume that the maximum interim asset drop never fully wipes out equity, namely \( 1 - \delta - D \geq 0 \). We discuss later relaxing this assumption (see Appendix).

### 2.4 Convertible Capital Design

The bank may be required to issue an amount of \( C \) of convertible bonds.

In our model (as well as in the proposed legislation), contingent capital is automatically converted into equity when the interim asset value hits some pre-specified trigger level \( v_T \). This may occur at time \( t = 1 \) or \( t = 2 \). Initially, we treat here the trigger as exogenous. We restrict the trigger value to be below the initial asset value, else conversion is triggered immediately.

CoCo bonds substitute a part of its deposits, which drop to \( D - C \). As a result, the initial ratio of debt to equity does not change.

To simplify the analysis, the interim coupon rate is normalized to zero.

In the basic model we assume that it is mandatory for the banker to issue CoCos. However, later we show that the banker never issues CoCos voluntarily at \( t = 0 \).

The conversion ratio, modelled along existing CoCo bonds, is the ratio of nominal value over the trigger asset value minus debt \( v_T \): \( d = \frac{C}{v_T - D} \).\(^5\) After conversion, the amount of shares is \( d + 1 \). Note that the banker is never wiped out, unless the value of CoCos is also zero. The payoff structure is presented in the Graph 2.

We consider now what CoCo design improves banker’s risk incentives. Intuitively, the trigger should induce CoCo conversion in banks where interim leverage is high enough to

\( ^3 \)The distinction safe-risky is meant to distinguish between moderate, properly priced credit risk and a riskier gamble with lower economic value.

\( ^4 \)As a result, the distribution of asset return in the safe outcome has second-order dominance relative to risky outcome, though not first-order dominance.

\( ^5 \)This is consistent with existing CoCo issues. The fixed conversion ratio produces value redistribution at conversion. We will show later how on average, CoCo holders get less than face value.
create poor risk control incentives, but could not dilute equity in well capitalized banks.

2.5 Results

2.5.1 The risk taking incentive

The banker bases her risk decision on the expected payoff, conditional on being solvent. For some realization of asset value the bank will default, forcing a payment by the deposit insurance fund.

The expected payoff from risky asset is the payoff conditional on the bank being solvent ($V_2 > D$):

$$
(1 - F(D - v)) \cdot \mathbb{E}(V_2 - D|V_2 - D > 0) = \int_{D-v}^{\infty} (V_2 - D) f(\varepsilon) d\varepsilon
$$

(1)

The bank’s payoff from the risky asset is the sum of its unconditional mean $\mathbb{E}(V_2 - D) = v - z - D$ (which may be negative) and a measure of the right tail return in solvent states $\Delta(v) \geq 0$.

$$
(1 - F(D - v)) \cdot \mathbb{E}(V_2 - D|V_2 - D > 0) = v - D - z + \Delta(v)
$$

(2)

Here $\Delta(v)$ is the value of the put option (also called Merton’s put) enjoyed by shareholders under limited liability. It measures the temptation of the banker to shift risk, defined as the return difference between a risky and safe strategy for the banker:

$$
(1 - F(D - v)) \cdot \mathbb{E}(V_2 - D|V_2 - D > 0) - (v - D) = -z + \Delta(v)
$$
From now on we refer to the return $\Delta(v)$ as a measure of risk shifting incentives. Its value depends on the specific distribution of asset risk.

**Lemma 1** *If the risky payoff is normally or uniformly distributed, risk shifting incentives $\Delta(v)$ are monotonically increasing and convex in leverage.*

**Lemma 2** *Risk shifting incentives increase with a higher volatility of risky asset $\sigma$.*

Without making a specific assumption on $f(\varepsilon)$, we assume that it has a similar structure of the risk incentive function as for normal or uniform distribution.

**Assumption 1** *Risk shifting incentives $\Delta(v)$ are an increasing and convex function of leverage $D_v$: $\Delta'(v) \leq 0, \Delta''(v) \geq 0$. $\Delta(v)$ are increasing with $\sigma$: $\Delta'(\sigma) \geq 0$.*

![Risk Incentives](image)

**Figure 3:** Risk incentives under Gaussian risk distribution

For a normal distribution, risk shifting incentives are given by:

$$\Delta(v) = (v - D - z) \cdot \left[ \Phi \left( \frac{v - D - z}{\sigma} \right) - 1 \right] + \sigma \cdot \phi \left( \frac{v - D - z}{\sigma} \right)$$

(3)

### 2.5.2 Bank risk without convertible bonds

First, we consider the risk choice of the banker in the absence of convertible bonds $C = 0$.

The banker compares the payoff from the risky and the safe asset. The banker’s program is:

$$\max_e e \cdot (v - D) + (1 - e) \cdot (v - z - D + \Delta(v))$$

s.t. $e \in \{0, 1\}$

(4)
Under the Assumption 1, the optimal effort choice by the banker takes the form:

$$e = \begin{cases} 
1 & \text{if } v \geq \Delta^{-1}(z) \equiv v^* \\
0 & \text{otherwise}
\end{cases}$$

We denote as $v^* \equiv \Delta^{-1}(z)$ the cut-off interim asset value, above which the banker exerts effort without conversion. At $v = v^*$ the net present value of the the banker’s choice of a risky lending strategy is zero.

For normal distribution function the cut-off interim asset value $v^*$ is given implicitly by:

$$(v - D - z) \cdot \left[ \Phi \left( \frac{v - D - z}{\sigma} \right) - 1 \right] + \sigma \cdot \phi \left( \frac{v - D - z}{\sigma} \right) = z$$

**Proposition 1** If at the interim period leverage is low ($v \geq v^*$), the banker exerts effort in order to control risk. If $v < v^*$, she does not. Moreover, the ex ante probability that the banker will choose at $t = 1$ to control risk ($1 - \delta - \frac{v^*}{2\sigma}$) decreases with the volatility of risky asset $\sigma$.

Note that the revelation of information does not have any effect on the banker’s risk incentives, as disclosure does not change leverage.

### 3 Optimal CoCo design

This section studies how the banker’s incentives change if the bank issues convertible bonds, and solves for their optimal trigger level. Later we study the effect of the amount of CoCo debt $C$.

#### 3.1 Optimal trigger value

The trigger value $v_T$ is initially set lower than the initial book value 1, else there is immediate conversion at time 0. If $v > v_T$, conversion does not occur. If $v \leq v_T$, conversion occurs, provided the asset value is revealed.

We show next that inducing conversion for banks which do not have risk shifting incentives does not contribute to efficiency.

**Corollary 1 (to Proposition 1)** Setting a trigger asset value higher than $v^*$ does not change risk incentives for low leverage banks (with $v \geq v^*$).

This enables us to restrict the range of trigger values to the interval $v_T < \min[v^*; 1]$. 
**Assumption 2** The trigger asset value \( v_T \) is such that no conversion is triggered upon the revelation of an interim value \( v \geq v^* \), so that \( v_T \leq v^* \).

We later show that his is efficient, as dilution which does not affect risk incentive may be counterproductive.

Consider now the banker’s choice:

\[
\max_e \left( e \cdot \left[ (v - D) \cdot (I(v \geq v_T) + (1 - \varphi) \cdot I(v < v_T)) + \right] \right.
\]

\[
\left. \left[ \frac{v - D + C}{d + 1} \cdot \varphi \cdot I(v < v_T) \right] + \right)
\]

\[
(1 - e) \cdot \left[ (v - z - D + \Delta(v)) \cdot (I(v \geq v_T) + (1 - \varphi) \cdot I(v < v_T)) + \right]
\]

\[
\left. \left[ \frac{v - z - D + C + \Delta(v + C)}{d + 1} \cdot \varphi \cdot I(v < v_T) \right] \right)
\]

\[
\text{s.t. } e \in \{0, 1\}
\]

(7)

where \( I(\cdot) \) is an indicator function, and \( d = \frac{C}{v_T - D} \) is the conversion ratio.

Graph 4 shows how the effort choice depends on the interim asset value.

![Figure 4: Risk incentives](image)

There are two critical interim asset values. The first is \( v^* \), threshold for effort when no conversion takes place. The second is \( v_C^* \), the value of interim assets above which the introduction of CoCos improves effort. It is easy to see that \( v_C^* \) is in the range \([v^* - C, v^*]\) and increases with the probability of information revelation \( \varphi \) (see Graph 5).

Intuitively, risk incentives may improve with CoCos only if \( \varphi > 0 \), that is, if the trigger is informative about poor incentives and forces recapitalization in the right states.
Lemma 3  The introduction of CoCos improves effort for banks with $v^*_C \leq v \leq v_T$. Banks with extremely high leverage $v < v^*_C$ do not change their effort choice since their risk-shifting return is too large. Banks with $v > v_T$ are not affected.

A bank with $v < v^*_C$ has such high leverage that CoCos can not improve its risk-shifting incentives. However, if CoCos are large enough ($v^*_C < 1 - \delta$), this range does not arise, and all banks with $v < v_T$ have incentives to contain asset risk.

In general, the effort choice is:

\[
\begin{align*}
\text{If } v^*_C < v_T, e &= \begin{cases} 
1 & \text{if } v^* \leq v \leq 1 + \delta \\
0 & \text{if } v_T < v < v^* \\
1 & \text{if } v^*_C \leq v \leq v_T \\
0 & \text{if } 1 - \delta \leq v < v^*_C 
\end{cases} \\
\text{If } v^*_C \geq v_T, e &= \begin{cases} 
1 & \text{if } v \geq v^* \\
0 & \text{otherwise}
\end{cases}
\end{align*}
\]

(8)

where equation (9) defines the critical value $v^*_C$:

\[
\frac{\varphi}{d + 1} \cdot \Delta(v + C) + (1 - \varphi) \cdot \Delta(v) - z \left(1 - \varphi + \frac{\varphi}{d + 1}\right) = 0
\]

(9)

Note that the difference $\frac{v_T - v^*_C}{28}$ measures the expected improvement in risk incentive $E(\Delta e)$, induced by CoCos. It is strictly increasing in the interval $[v^*_C, v_T]$.

Proposition 2  The trigger value is optimally set at $v_T = v^*$, which maximizes the expected effort $\frac{v_T - v^*_C}{28}$ for a given amount of CoCos $C$.

Graph 4 shows that unless the trigger $v_T$ is chosen optimally, risk incentives are not necessarily monotonic in $v$. If the trigger is too high (above $v^*$), CoCos will not affect effort. But if it is too low (below $v^*$), there will be no conversion for relatively high levered banks, which is clearly inefficient. Intuitively, it is easier to induce effort for higher $v$, so it cannot be efficient to allow effort to fall as $v$ increases.
As a result, the optimal trigger value guarantees the monotonicity of incentives with respect to leverage, as shown in Graph 6.

The optimal trigger value depends on $v^*$ and $v^*_C$, which reflect the risky opportunities available to the banker. A higher asset volatility increases the risk shifting return, which becomes attractive to the banker for a larger range of interim values $v$. Intuitively, the trigger value should be raised to correct weaker incentives when asset values are riskier in a mean-preserving sense.

Lemma 4 A higher asset volatility requires that the trigger value be raised to maintain risk-shifting incentives.

3.2 Optimal amount of Contingent Capital

Having set $v_T$, we now seek to optimize risk incentives by varying the amount of CoCos.

Convertible bonds have two effects on the banker’s effort for low interim asset values $v \leq v^*$. We can separate two effects: a risk-shifting and a dilution effect.

Proposition 3 The reduction in the banker’s equity increases effort if risk-shifting is most severe, i.e. when $\Delta(v + C) > z$. When risk-shifting incentives are less strong, the dilution of the banker’s profits by conversion may discourage effort.

The risk-shifting effect arises because the chance of conversion reduces the banker’s share of high payoffs from a risky strategy, and induces bank to avoid risk shifting. Note that this result match the intuition in Green’s (1984) model of convertible debt. However, here conversion is automatic and occurs earlier, before risk is fully realized.

Second, conversion leads to some reduction in the banker’s share in profits from safe strategy. This may reduce effort in low leverage banks.
When the amount of CoCos is so large that conversion exceeds what would be required to eliminate all risk shifting incentives, the dilution effect is excessive. Recall that effort is both risk-reducing and value increasing. Thus, the disincentivizing dilution effect is strongest for less levered banks \( v \geq v^*-C \), for which the risk shifting effect is limited.

This suggests there is an optimal amount of CoCo funding, which trades off reducing risk shifting while maintaining incentives for value enhancement. Graph 7 shows how the cut-off value \( v_C^* \) changes with CoCos under a uniform distribution for asset risk. A greatest risk reduction effect is achieved as \( v_C^* \) falls, as more banks are induced to control risk. The graph shows the dilution effect of CoCos on effort as the difference between \( v_C^* \) and \( v_C^* \) (without dilution). This second expression reflects only the risk-shifting effect.\(^6\)

![Graph 7: Threshold values of \( v_C^* \) with and without dilution effect](image)

Since expected effort \( E(e) \) is proportional the range of states \( v \) when the banker exerts effort, it equals \( \frac{1+\delta-v_C^*}{2}\delta \). In the Appendix we show how the increase in the amount of CoCos change the expected effort disentangling risk-shifting and dilution effect.

**Proposition 4** Expected effort increases with the amount of CoCos up to a threshold \( C^* \), and then declines. Thus, there exists an optimal amount of CoCos in terms of effort improvement.

\[
\Delta_C'(v + C^*)(C^* + v_T - D) - \Delta(v + C^*) + z = 0
\]  

(10)

Graph 8 shows how effort improvement changes with the amount of CoCos under the uniform distribution\(^7\).

\(^6\)The graph uses the parameter values \( D = 0.93, \sigma = 0.12, z = 0.04, \delta = 0.07 \). The trigger asset value is \( v_T = v^* = 0.95 < 1 \).

\(^7\)The graph uses the parameter values \( D = 0.93, z = 0.04, \delta = 0.07, \phi = 0.8 \).
Corollary 2 The amount of CoCos and trigger value act as substitutes in reducing risk.

Since the marginal rate of substitution between the amount of CoCos and the trigger value is not constant, we can see that the amount of CoCos and trigger value are imperfect substitutes.

\[
\frac{\partial C^*}{\partial v_T} = -\frac{\Delta'_C(v + C)}{\Delta''_C(v + C)(C + v_T - D)} > 0
\]  

Thus a lower trigger value can be compensated by a higher amount of CoCos to maintain risk incentives.

We next look at how key parameters on the economic environment (risky asset volatility \(\sigma\), probability of information revelation \(\varphi\)) affect the expected improvement in effort.

Proposition 5 For an exogenously given trigger value, the expected effort improvement \(\frac{v_T - \mu^C}{2\delta}\) decreases in the volatility of risky asset \((\sigma)\), since the risk shifting incentives grow with \(\sigma\).

Corollary 3 Higher \(\sigma\) implies a higher optimal trigger value: \(\frac{\partial v^*}{\partial \sigma} \geq 0\).

Corollary 4 A higher probability of information revelation increases the expected effort improvement \(\frac{\nu^* - \nu_C^*}{2\delta}\).

Figure 8: Effort improvement for different amount of CoCos
Clearly, if the state is never revealed $\varphi = 0$, convertible bonds never convert and thus do not change risk incentives. An increasing chance of conversion upon revelation of high leverage triggers conversion precisely when incentives are poor.

## 4 Extensions

### 4.1 Private choice to issue CoCo bonds

It is easy to show that banks will not be willing to issue CoCos voluntarily. Since deposits are guaranteed by the deposit insurance fund, they can be issued at par, whereas CoCos are risky. Moreover, CoCos force the banker to choose a safer strategy than she would prefer in some cases. This decreases the banker’s return for a range of intermediate value states.

Suppose the banker may choose between the issuing CoCos of amount $C$ at $t = 0$ or deposits of amount $C$. We show here that the banker has no incentives to issue CoCos.

Consider the payoff of the CoCo holders.

If the information about interim asset value $v$ is not revealed, CoCo holders’ payoff is similar to conventional bondholders. If $v \geq v^*_C$, CoCo holders get the face value of the bond $C$, since the bank invests in the safe strategy. If $v < v^*_C$, CoCo holders face the risk that bank won’t repay the value of the bond fully. The risk is not borne by deposit insurance. It is now fully priced.

It is easy to show that on average for $v < v^*$, CoCo holders get less than the face value of the bond $^9$, although post conversion they may enjoy a capital gain as shareholders.

Graph 2 show the payoff of the CoCo holders in highly leveraged banks ($v < v^*_C$).

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8We assume deposit insurance fees are not risk sensitive. In our approach, this is not easy, as risk is endogenous.

9In our setup, CoCo holder gets less than face value at conversion because of the fixed conversion ratio. The issue price will be set so that this loss is fully priced ex ante.
As a result, CoCos are sold at the discount on their face value. Their price equals to:

\[
P_C = \begin{cases} 
\phi \cdot [ & \text{Prob}(v > v^*) \cdot C + \text{Prob}(v^*_C < v \leq v^*) \cdot \frac{d}{d+1} \cdot \mathbb{E}(v - D + C | v^*_C < v \leq v^*) ] + \\
\text{Prob}(v \leq v^*_C) \cdot \frac{d}{d+1} \cdot \text{Prob}(V_2 > D - C) \cdot \mathbb{E}(V_2 - D + C | V_2 > D - C, v \leq v^*_C) ] + \\
(1 - \phi) \cdot [ & \text{Prob}(v \geq v^*_C) \cdot C + \text{Prob}(v < v^*_C) \cdot \mathbb{E}(B | v < v^*_C) ] 
\end{cases}
\]

(12)

where \( B \) is the value of a traditional bond of face value \( C \) for a risky bank:

\[
B = \text{Prob}(V_2 \geq D, v) \cdot C + \text{Prob}(D - C \leq V_2 < D, v) \cdot \mathbb{E}(V_2 - D + C | D - C \leq V_2 \leq D, v)
\]

(13)

Graph 9 shows that the discount is at minimum when the CoCo amount is optimal. The intuition is that at that point, the risk reduction is maximized, and the discount increases with the asset risk.

Figure 9: Price of CoCos as a percentage of face value

**Proposition 6** The banker never chooses to issue CoCos instead of deposits, since CoCos are riskier than deposits, and implied a higher cost of funding.\(^{10}\)

\(^{10}\)For certain parameters, CoCos may actually be riskless, if they always improve risk incentives (\( v^*_C \leq 1 - \delta \)).
Therefore, CoCos will be issued only if required by regulators. CoCos provide higher welfare, since the value of assets increases. The social welfare gain due to CoCos equals the range of states on which the inefficient risk outcome (which has a cost $z$) is avoided:

$$\frac{v^* - v_C^*}{2\delta} \cdot z$$

(14)

4.2 Convertible bonds versus Debt

Are CoCos cheaper than ordinary uninsured bond? There are two effects. CoCo holders face less protection when converted than traditional debt holders, but they induce safer asset choices. We are able to show that an optimal amount of CoCos under some parameter values represent a less risky security than traditional bank debt.

The difference in payoffs is shown in the Graph 10.

![Figure 10: Expected CoCo and debt value and bounds](image)

The value of a traditional bond with face value $C$ is:

$$P_B = \text{Prob}(v \geq v^*) \cdot C + \text{Prob}(v < v^*) \cdot E(B|v < v_C^*)$$

(15)
The price of CoCos may be higher than for a traditional bond, when asset risk and trigger precision are high and the amount of CoCos is chosen optimally (Graph 11)\textsuperscript{11}.

![Figure 11: CoCo price minus bank debt around $C^*$](image)

Note that when the asset risk increases, the optimal trigger price on CoCo bonds should be raised to adjust incentives. Traditional bond holders instead will passively bear the increased risk.

### 4.3 Contingent Capital versus Equity

What amount of contingent capital is required to substitute equity, to provide the same effort incentives?

Suppose the bank substitutes one unit of deposits by an extra amount of equity $\epsilon$, or by an amount $k\epsilon$ of CoCos. We solve for the level of $k$ which guarantees an equivalent improvement in risk incentives as with equity.\textsuperscript{12}

The banker chooses effort according to the schedule:

\[
e = \begin{cases} 
1 & \text{if } v \geq v^* - \epsilon \\
0 & \text{if } v < v^* - \epsilon 
\end{cases}
\]  

\textsuperscript{11}We use parameter values: $D = 0.93, z = 0.04, \delta = 0.07, \varphi = 0.8, \sigma = 0.14$

\textsuperscript{12}Note that after adding extra equity $\epsilon$, the bank has debt $D - \epsilon$, so the amount of equity in the interim stage is $v - D + \epsilon$. The bank operates with lower leverage.
The expected improvement in effort compared to basic model (24) is \( \frac{\epsilon}{2\delta} \), which reflects the increased range of asset values for which there are improved risk incentives. From earlier results, the improvement in effort achieved by CoCos is \( \frac{v^* - v^*_C}{2\delta} \).

So the condition \( v^* - v^*_C = \epsilon \) guarantees that the expected improvement in effort from introducing extra equity \( \epsilon \) and CoCos \( k\epsilon \) is the same.\(^{13}\)

**Proposition 7** The effect of CoCos on effort is in general weaker than that of equity, unless trigger is fully informative \((\varphi = 1)\). A higher amount of contingent capital is required to provide the same effort incentives.

**Lemma 5** The substitution ratio between extra equity and CoCos \( k \) decreases in a convex way with the probability of information revelation \( \varphi \). It reaches its minimum in the fully informative trigger \((\varphi = 1)\), when CoCos and equity provide an equivalent effort improvement.

Graph 12 shows the equivalence ratio is very sensitive to \( \varphi \). As \( \varphi \) approaches zero, the substitution ratio becomes infinite.\(^{14}\)

The key efficiency factor for CoCos depends on the precision of the trigger to signal a state where incentives are poor, relatively to equity which is always risk bearing. When the trigger is less precise, conversion takes less often when required. As a result, a larger amount of CoCos must be used.

How does asset risk \( \sigma \) affect the substitution ratio \( k \)? Intuitively, higher risk requires more CoCos. The substitution ratio increases with asset risk (for a given \( v_T \)) (see Graph 12).

### 4.4 Debt with warrants

In this section we compare the overall risk incentive of automatic conversion against the convertible bonds proposed by Green (1984).

Convertible bonds, freely convertible in shares at maturity, dilute higher risk-shifting payoffs, as investors always convert when asset value is high at maturity. This reduces the attractiveness of high risk strategies. However, it relies on the counterintuitive notion of increasing bank equity in the best states, as opposed to the worse states. Voluntary conversion bonds also do not protect depositors, once the bank defaults.

There are three differences between CoCos and convertibles. First, conversion is not automatic. Bondholders have an option to convert into some amount of shares \( w \). Second, the

\[^{13}\text{As before, we set the trigger value to insure monotonic incentives in } v, \text{ so } d = \frac{C}{v_T - D} = \frac{k\epsilon}{v_T - D}.\]

\[^{14}\text{The graph assumes an uniform distribution and parameters } D = 0.93, z = 0.04, \delta = 0.07, \epsilon = 0.001.\]
risky payoff in Green’s model reflect a mean preserving spread.\textsuperscript{15} Finally, conversion there occurs, if at all, only at the final stage $t = 2$, when asset values are symmetrically known.

We compare their effectiveness in containing risk choices and compute an equivalence ratio with CoCos.

Consider a bank with a face value $\epsilon$ of convertibles outstanding, and deposits $D - \epsilon$. Bondholders will convert into $w$ shares at $t = 2$ only if they are worth more than $\epsilon$, namely when $V_2 > D + \frac{\epsilon}{w}$.

It is easy to see once again how the banker chooses to control risk according to the schedule shown in Figure 13.

\begin{align*}
\Delta(v_G^*) - \frac{w}{w+1} \cdot \gamma(v_G^* + \epsilon) - z &= 0 \\
w \cdot (v_G^{**} - D - z + \Delta(v_G^{**}) - \gamma(v_G^{**} + \epsilon)) + \epsilon + z - \Delta(v_G^{**}) &= 0
\end{align*}

\textsuperscript{15}This could be easily introduced in our setting, provided we also add a (realistic) cost of bankruptcy.
The conversion ratio $w$ is set optimally to ensure monotonicity of bank incentives, such that $D + \frac{1}{w} = v_G^*$. As in the basic model, by assumption we ensure the monotonicity of effort incentives in $v$.

**Proposition 8** The effect from CoCos on effort is stronger than from Green’s convertibles, for a sufficiently informative trigger, and certainly when $\varphi = 1$, as a lower amount is required to provide the same incentives. The substitution ratio $k$ increases in a convex fashion with a lower trigger precision, and may become higher than 1.

## 5 Market versus Regulatory Trigger

A critical aspect of CoCo design focuses on whether the conversion trigger should be based on accounting or market measures of bank equity, or by regulatory discretion.

An accounting trigger may fail “to capture the true financial condition of the bank” (Duffie, 2010). On the other hand, regulators have become skeptical of the ability of market prices to signal risk since the crisis. Bank share prices may be considered too noisy for at least three reasons. Prices of highly leveraged banks may rationally trade high as shareholders benefit from large scale risk shifting. Banks may be very sensitive to irrational exuberance and panics alike. And finally, Sundaresan and Wang (2010) showed that conversion upon an endogenous market price produce multiple equilibria around the trigger price, because of the share price discontinuity caused by conversion.

Currently, all outstanding CoCo bonds are designed to convert on accounting thresholds (book equity over assets). Yet balance sheet measure do not accurately reflect actual values, and are to some extent manipulable. As we showed, bankers prefers to avoid equity dilution, as it reduces the bank put option value. Bank reporting needs therefore close monitoring by bank supervisors, who have a critical role in challenging accounting choices that flatter book equity. Yet regulators may defuse conversions to avoid market repercussions, and have been known to delay recognition of bank losses in the hope of a recovery. We compare market and book equity triggers, where a market price triggers automatic conversion while an accounting trigger is influenced by regulatory choice. We assume that market prices and regulatory assessments are equally noisy indicators of real asset values. An immediate result is that market triggers cause more unnecessary conversions, but help avoid regulatory forbearance, which fails to trigger necessary conversions.

Suppose that the trigger value is set optimally $v_T = v^*$, and the probability of revelation is $\varphi = 1$.

In the case of a regulatory trigger, at $t = 1$, the regulator observes a noisy signal of the interim asset value $\tilde{a} = v + \tilde{r}$ (where $\tilde{r}$ has zero mean and standard deviation $\sigma_r$), and decides whether to trigger conversion. As this occurs through a bank accounting statement, we assume that the banker observes the signal before making risk decision.
In the case of a market trigger, at $t = 1$, the market price is a noisy measure of true asset value $\tilde{p} = v + \tilde{m}$ (where $\tilde{m}$ has zero mean and standard deviation $\sigma_m$) and triggers conversion automatically if $\tilde{p} \leq v^*$. As conversion in this case is immediate, the banker must choose its risk profile before the price may trigger conversion.

We look at their relative efficiency when the two triggers uses equally noisy signals, assuming that $\tilde{r}$ and $\tilde{m}$ follow uniform distribution with support $[-\mu, \mu]$, where $\mu \geq C$.

We assume that any conversion at $t = 1$ causes a social cost $k$. This reflects a general loss of confidence which e.g. may affect bank funding conditions. In case of bank failure at $t = 2$ (when $V_2 < D - C$), a larger social cost $K$ is incurred. A default clearly causes a larger loss of confidence.

**Proposition 9** A market trigger produces more frequent conversion than a regulatory trigger, including in states when it is not necessary (type 1 error). Conversely, a regulatory trigger will convert less, and this may encourage more risk taking in banks with $v$ from $[v^* - C, v^*]$ (type 2 error). The net effect of a market trigger may be more risk reduction (and more equity in general) but higher conversion costs.

Figure 14 illustrates the different conversion decisions in terms of $\tilde{p}$ and $\tilde{a}$. Figure 15 summarizes the different bank incentives in terms of $v$.

\begin{equation}
\Delta(v^*_R + C) + F(D - C, v^*) \cdot K = k
\end{equation}

(19)

Regulatory conversion is triggered for $\max[v^*_C; v^*_R] \leq v < \min[v^*_R; v^*]$, where

\begin{equation}
\frac{\mu + v - v^*}{2\mu} \cdot [z - \Delta(v)] + \frac{\mu + v^* - v}{2\mu} \cdot \frac{z - \Delta(v + C)}{d + 1} = 0
\end{equation}

(20)

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The welfare gain is greater for the regulatory trigger if:

\[ z \cdot \left( \frac{v^*_R - (v^* - C)}{2\delta} - \frac{v^* - v^*_M}{2\delta} \right) + k \cdot \int_{v^* - C, v^*_R} \text{Prob}_R(\text{Conv}|v) \, dv + \int_{v^* - C, v^*_M} \text{Prob}_M(\text{Conv}|v) \, dv \geq 0 \]  

(21)

where \( \text{Prob}_M(\text{Conv}|v) \) and \( \text{Prob}_R(\text{Conv}|v) \) are a probabilities of conversion for a bank with the interim asset value \( v \) under market and regulatory triggers respectively.

Notice that the efficiency of the market trigger depends on the noisiness of the signal \( \mu \). A more precise market price provides better risk reduction.

A regulator finds it rational to avoid conversion at \( t = 1 \) when default is possible but unlikely, as long as the expected bankruptcy loss is lower than \( \bar{k} \):

\[ \bar{k} = \Delta(v^* + C) + F(D - C, v^*) \cdot K \]  

(22)

Intuitively, this will occur when regulatory estimates \( \tilde{a} \) are close to the threshold \( v^* \). In addition, a regulator chooses not to convert if interim leverage is so high that conversion does not improve incentives. In this latter case bank default is every likely at \( t = 2 \). When \( k < \bar{k} \), there is never regulatory forbearance, and a market trigger is always less efficient.

In conclusion, regulatory forbearance avoids conversions which are needed but costly, and others that will not restore incentives but would save depositor insurance funding.

6 Discussion and conclusions

The paper assesses the optimal design of bank contingent capital. The literature so far has relied on models where the asset choice is exogenous, so CoCo have no effect on risk

Our contribution is to study explicitly contingent capital’s effect on bank risk choices, a necessary feature for its optimal design and pricing.

We show that its effectiveness in controlling risk incentives and bankruptcy losses depends on the precision of its trigger in converting into equity in the worse incentive states, when leverage is very high. The intuition is that conversion contains risk shifting, as it dilutes high returns.

Our approach establishes how the optimal amount of CoCo and their trigger level trade off a risk reduction versus a dilution effect. It enables to assess what amount of CoCo produces an equivalent risk reduction as common equity, as well as freely convertible bonds. It helps clarify a key difference between bail in bonds, which convert in equity only in default, and going concern contingent capital which restore equity while the bank is still solvent. A one for one exchange ratio of CoCo for equity is equivalent in terms of loss absorption upon default. But once the risk prevention effect is taken into account, even optimally designed contingent capital is much less efficient than equity because of limited trigger precision, which does not ensure recapitalization in all states of excessive leverage.

We also explore the relative efficiency of different triggers, in a setting where both market and regulatory measures of leverage are noisy. We show that a market trigger produce more conversions, some unnecessary (type II error), and ensures on average a lower bank leverage. A book value trigger subject to supervisory discretion instead converts too infrequently, as it suffers from regulatory forbearance. Forbearance is likely to occur closer to the default threshold, as policymakers avoid an early conversion by gambling on asset value recovery. Regulatory incentives may also be very poor for the most leveraged banks, where incentives are not improved by conversion. For such banks more direct intervention is necessary. In conclusion, the relative merit of price versus accounting triggers depends on the relative cost of type I and type II errors, related to their informativeness in signalling the need to recapitalize.

The simplified framework allows to compare various proposals in terms of risk incentives. It echoes Flannery (2009), who argues that a stock price trigger with conversion at par avoids regulatory forbearance and reduces manipulation. It may justify the use of more signals to increase trigger precision. McDonald (2011) proposes a dual price trigger, where conversion occurs when the share price falls below the threshold, and a financial index value is low. This allows a bank to fail as long as there is no generalized financial distress, when it would have impact on confidence. The main advantage of these market based triggers is to require no regulatory involvement.
Future research should focus on better understanding the effect of CoCo on share pricing, which is distorted by risk shifting. Share prices increase with bank risk when leverage is high, which may explain why Lehmann shares peaked just a year before its default. For this reason, shareholder returns drop on conversion, creating multiple equilibria. This discontinuity, driven by the tendency of the share price to fall towards the trigger level once it comes in its neighborhood, is inappropriately named “death spiral”. Yet it comes from the corrective effect of CoCo conversion on an underlying distortion (ie, risk shifting), not from a distortion it introduces.

An open issue is whether potential CoCo conversion helps increase share pricing precision when leverage is excessive. Once CoCos are issued, the possibility of conversion may create downside risk. Were this to produce higher equity volatility, it would also enhance investor incentives to monitor bank risk.
7 Appendix

Relaxing the initial capital constraint

In our model we assume that for any interim asset value \( v \), book equity is non-negative. In this case the choice of the safe asset always provides the banker with a positive return, equal to \( v - D \). It is equivalent to the condition \( 1 - \delta - D \geq 0 \).

However, if initial capital is low (the banker observes interim asset value \( v < D \)) and this condition does not hold, the banker’s return to the safe asset changes and the banker has different incentives to exert effort.

In case if conversion is not triggered \( v \geq v_T = v^* \), the banker’s return from the safe strategy is zero, and then chooses \( e = 0 \).

If conversion is triggered \( v \leq v_T = v^* \), the choice of the banker depends on \( v \). If \( v < D - C \), the banker’s payoff from the safe asset is zero. If \( v \geq D - C \), the banker’s payoff is positive and equal to \( \frac{v - D + C}{d+1} \).

The banker’s program becomes:

\[
\max_e \cdot \{ I(v \geq D) \cdot \left[ (v - D) \cdot (I(v \geq v_T) + (1 - \varphi) \cdot I(v < v_T)) + \frac{v - D + C}{d+1} \cdot \varphi \cdot I(v < v_T) \right] \}
\]

\[
I(v < D) \cdot \frac{v - D + C}{d+1} \cdot \varphi \cdot I(D - C < v < v_T)
\]

\[
(1 - e) \cdot \left[ (v - z - D + \Delta(v)) \cdot (I(v \geq v_T) + (1 - \varphi) \cdot I(v < v_T)) + \frac{v - z - D + C + \Delta(v + C)}{d+1} \cdot \varphi \cdot I(v < v_T) \right]
\]

s.t. \( e \in \{0, 1\} \) \quad (23)

We solve the problem assuming that \( v_T = v^* \).

The banker’s incentives change when either two conditions hold: (1) \( v^* < D \) and (2) \( v^*_C < D - C \).

If we don’t impose any condition on \( v - D \) and the conditions defined above hold, the
banker’s effort choice is:

\[ e = \begin{cases} 
1 & \text{if } D < v \leq 1 + \delta \\
0 & \text{if } v^* < v \leq D \\
1 & \text{if } D - C < v \leq v^* \\
0 & \text{if } 1 - \delta < v \leq D - C 
\end{cases} \]  

(24)

As in the basic model, it is best to ensure monotonicity of \( e \) in \( v \). In order to incentivize the banker to exert effort when \( v^* < v \leq D \), the trigger value must be set as \( v_T = D \).

As a result, when \( v \) may be below \( D \), but \( \forall v : v \geq D - C \), the banker’s incentives don’t change if the trigger value is set optimally: \( v_T = D \). However, for all interim asset values \( v \) below \( D - C \), risk incentives for bank with \( v < D - C \) can not be improved.

Thus, our results will be valid for the weaker restriction of \( v \geq D - C \). This leaves open the possibility of losses for depositors as \( V_2 \) may be below \( D - C \).

**Proof of Lemma 1**

We consider two possible distribution of the asset value: normal and uniform.

In the first case let \( x = v - D + \varepsilon \) be normally distributed with mean \( v - D - z \) and variance \( \sigma^2 \). We refer to \( x \) as the difference between the value of assets and debt.

In the second case let \( x = v - D + \varepsilon \) be uniformly distributed with support \([v - D - z - \sigma\sqrt{3}, v - D - z + \sigma\sqrt{3}]\), so that mean is \( v - D - z \) and variance is \( \sigma^2 \).

We assume that the highest possible equity value when the bank takes the risky asset is positive, \( v - D - z + \sigma\sqrt{3} \geq 0 \). Otherwise, risky asset is never chosen. Moreover, the lowest possible capital value is negative \( v - D - z - \sigma\sqrt{3} \leq 0 \), else no risk shifting takes place.

The expected value of bank equity is the expected value of assets minus debt conditional on being solvent, multiplied by the probability of being solvent.

\[ (1 - F(0, v)) \cdot \mathbb{E}(x|x > 0, v) \]

For a normal distribution:

\[ (1 - F(0, v)) \cdot \mathbb{E}(x|x > 0, v) = \left(1 - \Phi\left(\frac{-(v - D - z)}{\sigma}\right)\right) \cdot \frac{\int_0^\infty x \cdot \frac{1}{\sigma} \cdot \phi\left(\frac{x - (v - D - z)}{\sigma}\right)dx}{1 - \Phi\left(\frac{-(v - D - z)}{\sigma}\right)} = \]

\[ \int_0^\infty x \cdot \frac{1}{\sigma} \cdot \phi\left(\frac{x - (v - D - z)}{\sigma}\right)dx = \]

\[ (v - D - z) \cdot \Phi\left(\frac{v - D - z}{\sigma}\right) + \sigma \cdot \phi\left(\frac{v - D - z}{\sigma}\right) \]
For a uniform distribution:

\[(1 - F(0, v)) \cdot \mathbb{E}(x|x > 0, v) = \int_{0}^{\infty} x \cdot \frac{1}{2\sigma\sqrt{3}} dx = \frac{(v - D - z + \sigma\sqrt{3})^2}{4\sigma\sqrt{3}}\]

The expected value of equity in the case of risky asset is by definition the sum of unconditional mean of the value of asset minus debt \(v - D - z\) and the risk taking incentives \(\Delta(v)\) (the put option enjoyed by shareholders).

Normal distribution:

\[\Delta(v) = (1 - F(0, v)) \cdot \mathbb{E}(x|x > 0, v) - (v - D - z) = (v - D - z) \cdot \left[ \Phi \left( \frac{v - D - z}{\sigma} \right) - 1 \right] + \sigma \cdot \phi \left( \frac{v - D - z}{\sigma} \right)\]

Uniform distribution:

\[\Delta(v) = (1 - F(0, v)) \cdot \mathbb{E}(x|x > 0, v) - (v - D - z) = \frac{(v - D - z - \sigma\sqrt{3})^2}{4\sigma\sqrt{3}}\]

Consider now how the risk shifting incentive changes with interim asset value \(v\). It is easy to show that under these distributions the derivative of the risk shifting incentive function with respect to \(v\) is negative.

Normal distribution:

\[\frac{\partial \Delta(v)}{\partial v} = \Phi \left( \frac{v - D - z}{\sigma} \right) - 1 + \frac{v - D - z}{\sigma} \cdot \phi \left( \frac{v - D - z}{\sigma} \right) - \frac{v - D - z}{\sigma} \cdot \phi \left( \frac{v - D - z}{\sigma} \right) = \Phi \left( \frac{v - D - z}{\sigma} \right) - 1 \leq 0\]

Uniform distribution:

\[\frac{\partial \Delta(v)}{\partial v} = \frac{2(v - D - z - \sigma\sqrt{3})}{4\sigma\sqrt{3}} \leq 0\]

Thus, the risk shifting incentive decrease with asset value \(v\), or capital \(v - D\).

The second derivative of function \(\Delta(v)\) with respect to \(v\) is positive.

Normal distribution:

\[\frac{\partial^2 \Delta(v)}{\partial v^2} = \phi \left( \frac{v - D - z}{\sigma} \right) \cdot \frac{1}{\sigma} \geq 0\]

Uniform distribution:

\[\frac{\partial^2 \Delta(v)}{\partial v^2} = \frac{1}{2\sigma\sqrt{3}} \geq 0\]

Thus, risk shifting incentives fall in a convex fashion with bank capital \(v - D\).
Proof of Lemma 2

We look at how risk incentives change when volatility of risky asset grows. The derivative of risk shifting function with respect to $\sigma$ is positive.

Normal distribution:

$$\frac{\partial \Delta(v)}{\partial \sigma} = -\frac{v-D-z}{\sigma^2} \phi \left( \frac{v-D-z}{\sigma} \right) + \phi \left( \frac{v-D-z}{\sigma} \right) + \frac{v-D-z}{\sigma^2} \phi \left( \frac{v-D-z}{\sigma} \right) = \phi \left( \frac{v-D-z}{\sigma} \right) \geq 0$$

Uniform distribution:

$$\frac{\partial \Delta(v)}{\partial \sigma} = -\frac{24}{48 \sigma^2} \left( v-D-z-\sigma \sqrt{3} \right) - 4\sqrt{3} \cdot (v-D-z-\sigma \sqrt{3})^2 = -\frac{(v-D-z-\sigma \sqrt{3}) \cdot (v-D-z+\sigma \sqrt{3})}{4\sqrt{3} \cdot \sigma^2} \geq 0$$

Thus, the risk shifting incentives increase with volatility of the risky asset.

And finally, we find the effect of difference in means of payoffs from safe and risky assets $z$ on the risk shifting incentives. The derivative of risk shifting function with respect to $z$ is:

Normal distribution:

$$\frac{\partial \Delta(v)}{\partial z} = \left[ \Phi \left( \frac{v-D-z}{\sigma} \right) - 1 \right] + \frac{v-D-z}{\sigma} \cdot \phi \left( \frac{v-D-z}{\sigma} \right) - \frac{v-D-z}{\sigma} \cdot \phi \left( \frac{v-D-z}{\sigma} \right) = \left[ \Phi \left( \frac{v-D-z}{\sigma} \right) - 1 \right] \geq 0$$

Uniform distribution:

$$\frac{\partial \Delta(v)}{\partial z} = -\frac{2(v-D-z-\sigma \sqrt{3})}{4\sigma \sqrt{3}} \geq 0$$

So, higher $z$ leads to higher risk shifting incentives.

Proof of Proposition 1

First, we show that indeed the banker with interim leverage $v > v^*$ exerts effort. The banker solves the problem (4). Her decision depends on the whether the risk shifting incentive is higher or lower than the difference in means from safe and risky payoff $z$. If $\Delta(v) \leq z$, the banker exerts effort. According to the Assumption 1, the risk shifting incentive function $\Delta(v)$ is decreasing in $v$. Then our condition $\Delta(v) \leq z$ implies that the
banker with interim asset value $v \geq \Delta^{-1}(z) \equiv v^*$ exerts effort.

Next we show that the probability that the banker controls risk $(1 - \delta - v^*)$ decreases with the volatility of risky asset $\sigma$. The probability of risk control negatively depends on the magnitude $v^*$. Remember that $v^*$ is derived from the condition $G(v, z, \sigma) \equiv \Delta(v) - z = 0$.

We find the effect of $\sigma$ on the critical value $v^*$ using the implicit function theorem and computing

$$\frac{\partial v}{\partial \sigma} = -\frac{\partial G/\partial \sigma}{\partial G/\partial v}$$

where

$$\frac{\partial G/\partial \sigma}{\partial G/\partial v} = \Delta'_\sigma(v) \geq 0$$

and

$$\frac{\partial G/\partial v}{\partial G/\partial \sigma} = \Delta'_v(v) \leq 0$$

As a result the derivative is $\frac{\partial v}{\partial \sigma} \geq 0$. The critical asset value $v^*$ becomes higher if $\sigma$ increases, since higher higher volatility provides larger risk-shifting benefits. Thus, the probability that the banker controls risk diminishes with $\sigma$.

Finally, note that the revelation of information does not have any effect on the banker’s incentives. The reason is that information revelation makes market participants informed about the interim asset $v$, but does not change the incentives of the banker, since market does not have an instrument to affect the banker’s payoff in case of high or low risk choice.

**Proof of Corollary 1**

Here we demonstrate that if the trigger value is set higher than low levered bank with $\geq v^*$ does not change the effort choice $e = 1$. To prove that we show the extreme case when $\varphi = 1$. It is sufficient to show that if for the certain information revelation the banker still exerts effort for $v \geq v^*$. The reason is that for lower $\varphi$ we have the banker is more reluctant to choose effort similar to case without CoCos (there is lower probability of conversion), i.e. $e = 1$.

The banker’s problem in this case $(\varphi = 1, v \geq v^*)$ is:

$$\max_e e \cdot \left( \frac{v - D + C}{d + 1} + (1 - e) \cdot \frac{v - z - D + C + \Delta(v + C)}{d + 1} \right)$$

s.t. $e \in \{0, 1\}$

The banker’s choice is then

$$e = \begin{cases} 1 & \text{if } v \geq \Delta^{-1}(z) - C \equiv v^* - C \\ 0 & \text{otherwise} \end{cases}$$

(25)
Note that the cut-off interim asset value is now lower than $v^*$. It means that banks with $v \geq v^*$ still chooses to exert effort independent of $\varphi$.

**Proof of Lemma 4**

We show how asset volatility affects the chosen trigger value schedule $v_T = v^*$. Remember that in the Proof of Proposition 1, we already demonstrated that critical value $v^*$ increases with the volatility of the risky asset $\sigma$. This result implies that the trigger value $v_T = v^*$ should be raised when volatility grows in order to avoid increased risk-shifting incentives.

**Proof of Proposition 3**

Consider the implicit function from (9), which defines the improvement in risk incentives (namely the lower threshold $v_C^*$ such that CoCos induce effort). Call it $G(v|C,d)$.

$$
\frac{\partial G}{\partial C} = \frac{\partial G}{\partial C}|_{d=\text{const}} + \frac{\partial G}{\partial d} \cdot \frac{\partial d}{\partial C}
$$

(26)

The effect of CoCos on $v_C^*$ is defined by the sign of $\frac{\partial G}{\partial C}$, since $\frac{\partial v}{\partial C} = -\frac{\partial G/\partial C}{\partial G/\partial v}$, where $\partial G/\partial v < 0$.

$$
\frac{\partial G}{\partial C}|_{d=\text{const}} = \frac{\varphi}{d+1} \cdot \Delta_C(v + C) < 0
$$

(27)

Second, conversion leads to some reduction in the banker’s share in profits. This may reduce effort in low leverage banks.

$$
\frac{\partial G}{\partial d} \cdot \frac{\partial d}{\partial C} = -\frac{\varphi(\Delta(v+C) - z)}{(d+1)^2} \cdot \frac{1}{v_T - D}
$$

$$
\begin{align*}
\frac{\partial G}{\partial d} \cdot \frac{\partial d}{\partial C} &\geq 0 & \text{if } v \geq v^* - C \\
\frac{\partial G}{\partial d} \cdot \frac{\partial d}{\partial C} &< 0 & \text{otherwise}
\end{align*}
$$

**Risk-shifting and dilution effects: Numerical example**

Consider a bank with debt $D = 95$ and initial assets $V_0 = 100$. The risky asset return $V_2$ follows the binomial distribution:

$$
V_2 = \begin{cases} 
  v + 5 & \text{with prob } \frac{1}{2} \\
  v - 10 & \text{with prob } \frac{1}{2}
\end{cases}
$$

Model parameters take values: $\varphi = 0.5, \delta = 5, z = 2.5, \sigma = 7.5$. 

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In the absence of CoCos bank with \( v < v^* = 100 \) does not control risk, i.e bank controls risk with probability 0.5.

Next we introduce CoCos of amount \( C_L = 2.5 \), and then show how the banker’s incentives change if the amount of CoCos increases up to \( C_H = 5 \). The trigger value is \( v^* = 100 \).

First, consider the case of \( C_L = 2.5 \). The conversion ratio is \( d_L = 0.5 \).

Payoff from \( e = 1 \) is:
\[
\varphi \cdot \frac{v - D + C_L}{d_L + 1} + (1 - \varphi) \cdot (v - D) = 0.5 \cdot \frac{v - 92.5}{1.5} + 0.5 \cdot (v - 95)
\]

Payoff from \( e = 0 \) is:
\[
\varphi \cdot \frac{1}{2} \cdot \frac{v + 5 - D + C_L}{d_L + 1} + (1 - \varphi) \cdot \frac{1}{2} \cdot (v + 5 - D) = 0.25 \cdot \frac{v - 87.5}{1.5} + 0.25 \cdot (v - 90)
\]

Bank with \( v > v_c^L = 99 \) chooses to control risk in the presence of CoCos \( C_L = 2.5 \), i.e bank controls risk with probability 0.6.

Second, consider the case of \( C_H = 5 \). We keep the conversion ratio constant in order to disentangle risk-shifting and dilution effects. Instead of \( d_H = 1 \) we use \( d_L = 0.5 \).

Payoff from \( e = 1 \) is:
\[
\varphi \cdot \frac{v - D + C_H}{d_L + 1} + (1 - \varphi) \cdot (v - D) = 0.5 \cdot \frac{v - 90}{1.5} + 0.5 \cdot (v - 95)
\]

Payoff from \( e = 0 \) is:
\[
\varphi \cdot \frac{1}{2} \cdot \frac{v + 5 - D + C_H}{d_L + 1} + (1 - \varphi) \cdot \frac{1}{2} \cdot (v + 5 - D) = 0.25 \cdot \frac{v - 85}{1.5} + 0.25 \cdot (v - 90)
\]

If the equity were not diluted, bank with \( v > v_c^H = 98 \) would choose to control risk in the presence of CoCos \( C_H = 5 \). The effort improvement would be 0.1 due to the increase in CoCos from 2.5 to 5. This risk reduction arises because the total value of equity increases when CoCos are added to equity. We refer to this effect as risk-shifting effect.

However, if conversion reduced banker’s share to \( \frac{1}{2} \) in the bank equity (i.e the conversion ratio is 1), the risk reduction becomes lower.

Payoff from \( e = 1 \) is:
\[
\varphi \cdot \frac{v - D + C_L}{d_L + 1} + (1 - \varphi) \cdot (v - D) = 0.5 \cdot \frac{v - 90}{2} + 0.5 \cdot (v - 95)
\]

Payoff from \( e = 0 \) is:
\[
\varphi \cdot \frac{1}{2} \cdot \frac{v + 5 - D + C_L}{d_L + 1} + (1 - \varphi) \cdot \frac{1}{2} \cdot (v + 5 - D) = 0.25 \cdot \frac{v - 85}{2} + 0.25 \cdot (v - 90)
\]
Because of higher conversion ratio $d_H$, CoCos dilute equity more. The effort improvement becomes lower as opposed to the case when the banker’s share in equity stays the same.

Bank with $v > v^*_C = 98.33$ chooses to control risk when CoCos of amount $C_H = 5$ dilutes equity. Since the safe payoff is more sensitive to the reduction of banker’s share than the risky payoff, dilution effect disincentives the banker to control risk. The dilution effect here is measured as the reduction in effort improvement of $0.033$.

Thus, risk shifting effect raises the probability of bank controlling risk to 0.7 (effort improvement of 0.1), whereas dilution effect reduces this probability to 0.667 (effort decrease by $-0.033$). Overall effect from increasing the amount of CoCos from 2.5 to 5 is the expected effort increase by 0.067, i.e by 11% (where risk-shifting effect accounts for 150% of total effort increase, and dilution effect - for $-50\%$).

**Proof of Proposition 4**

The maximum improvement in effort is achieved when threshold for bank with CoCos $v^*_C$ reaches its minimum. The condition for optimal amount of CoCos generating minimum $v^*_C$ (this increases the probability of bank exerting higher effort, and thus expected effort improvement) is:

$$\frac{\partial v^*_C}{\partial C} = 0$$

We use the implicit function theorem to compute this derivative:

$$\frac{\partial v^*_C}{\partial C} = -\frac{\partial F/\partial C}{\partial F/\partial v^*_C}$$

where

$$\frac{\partial F}{\partial C} = \frac{\varphi(v_T - D) (\Delta'_C (v + C) \cdot (C + v_T - D) - \Delta (v + C) + z)}{(C + v_T - D)^2}$$

$$\frac{\partial F}{\partial v} = \frac{\varphi(v_T - D)}{C + v_T - D} \cdot \Delta'_v (v + C) + (1 - \varphi) \cdot \Delta'(v)$$

The resulting condition is then:

$$\frac{\partial v^*_C}{\partial C} = -\frac{\varphi(v_T - D) (\Delta'_C (v + C) \cdot (C + v_T - D) - \Delta (v + C) + z)}{(C + v_T - D)^2} \cdot \left( \frac{\varphi(v_T - D)}{C + v_T - D} \cdot \Delta'_v (v + C) + (1 - \varphi) \cdot \Delta'(v) \right) = 0$$

From here the condition for the amount of CoCos $C$ guaranteeing the minimum $v^*_C$ is:

$$\Delta'_C (v + C^*)(C^* + v_T - D) - \Delta (v + C^*) + z = 0$$

\footnote{Further we use just $v$ instead of $v^*_C$.}
where \( v_T = v^* \).

Note that we treat the solution of this equation as the amount of CoCos providing the minimum \( v^*_C \), since we know that at \( C = 0 \), the function \( v^*_C(C) \) is decreasing, and at \( C = \infty \) it is constant. This suggests the existence of at least one minimum point.

\[
\lim_{C \to 0} \frac{\partial v^*_C}{\partial C} = -\frac{\varphi (\Delta'_C(v) \cdot (v_T - D) - \Delta(v) + z)}{(v_T - D) \cdot \Delta'_v(v)} \leq 0
\]

\[
\lim_{C \to +\infty} \frac{\partial v^*_C}{\partial C} = 0
\]

**Proof of Proposition 5**

We show the effect of volatility \( \sigma \) on the effort improvement \( \frac{v^* - v^*_C}{2\delta} \) upon assumption that the trigger value \( v_T \) is exogenous. We need to find the effect of \( \sigma \) on the critical value \( v^*_C \), i.e. compute \( \frac{\partial v^*_C}{\partial \sigma} \). Using the implicit function theorem, we define it as:

\[
\frac{\partial v^*_C}{\partial \sigma} = -\frac{\partial F}{\partial \sigma} \cdot \frac{\partial v^*_C}{\partial v}
\]

where we use the result \( \frac{\partial F}{\partial v} \leq 0 \) from the proof of proposition 4, and

\[
\frac{\partial F}{\partial \sigma} = \frac{\varphi}{d+1} \cdot \Delta'_\sigma(v + C) + (1 - \varphi) \cdot \Delta'_\sigma(v) \geq 0
\]

where we exploit the assumption 1 that \( \Delta'_\sigma \geq 0 \).

Thus, \( \frac{\partial v^*_C}{\partial \sigma} \geq 0 \).

Consequently, the effect of the asset volatility on the expected effort improvement \( \frac{v^* - v^*_C}{2\delta} \) is negative given that the trigger value is exogenously given.

**Proof of Corollary 3**

Next we examine the marginal effect from setting trigger value \( v_T = v^* \) on effort. This effect consists of two: the effect on \( v^* \) as an upper bound of interim asset value for which the conversion takes place, and the effect of \( v^* \) on the \( v^*_C \) via dilution ratio \( d \).

First effect is positive, as we already established in the proof of Proposition 1, that critical value \( v^* \) increases with the volatility of the risky asset \( \sigma \).

Second effect is also positive for the expected effort:

\[
\frac{\partial v^*_C}{\partial \sigma} = \frac{\partial v^*_C}{\partial d} \cdot \frac{\partial d}{\partial \sigma} = -\frac{\partial F}{\partial v} \cdot \frac{\partial F}{\partial \sigma} \cdot \frac{\partial d}{\partial \sigma} = -\varphi \Delta'_v(v + C) \cdot \Delta'_\sigma(v) \cdot \frac{v^*(v - D)^2}{(d+1)^2} \leq 0
\]
Since volatility increases trigger value $v^*$, the dilution ratio diminishes. This leads to the lower critical value $v^*_C$.

Thus, the marginal effect is positive, and setting trigger value to be $v^*$ reduces the negative effect of volatility on the expected effort (achieved with exogenous trigger price).

However, the sign of the overall effect is undefined and depends on the parameters:

$$\frac{\partial v^*-v^*_C}{\partial \sigma} = \frac{1}{2\delta} \cdot \left[ v^*_C \cdot \left( 1 + \frac{\varphi d(\Delta(v^*_C + C) - z)}{\partial F(v^* - D)(d+1)^2} \right) + \frac{\varphi \Delta'_\sigma(v^*_C+C)}{d+1} + (1 - \varphi)\Delta'_\sigma(v) \right]$$

As a result, the overall effect of $\sigma$ on effort may also become positive.

**Proof of Corollary 4**

Here we look at the effect of the higher probability of information revelation on the expected effort $\frac{v^*-v^*_C}{2\delta}$. The sign of the effect is opposite to the sign of the derivative $\frac{\partial v^*_C}{\partial \varphi}$, where $\frac{\partial F}{\partial v^*_C} \leq 0$

$$\frac{\partial F}{\partial \varphi} = \frac{\Delta(v + C) - z}{d + 1} \left( \frac{z}{\Delta(v)} \right) \leq 0$$

The derivative $\frac{\partial v^*_C}{\partial \varphi}$ is negative, therefore the effect of probability of information revelation on the expected effort is positive.

Note that critical asset value $v^*$ is not affected by $\varphi$ according to the results of Proposition 1.

**Proof of Lemma 7**

The banker’s program with extra equity is:

$$\max_e e \cdot (v - D + \epsilon) + (1 - e) \cdot (v - D + \epsilon - z + \Delta(v + \epsilon))$$

s.t. $e \in \{0, 1\}$

In order to compute the substitution ratio $k$, we use the condition for finding $v^*_C$:

$$G(v^* - \epsilon|k\epsilon, d) = 0$$
or equivalently
\[
\frac{\varphi}{d+1} \cdot \Delta[v^* + \epsilon(k - 1)] + (1 - \varphi) \cdot \Delta[v^* - \epsilon] - z \left(1 - \varphi + \frac{\varphi}{d+1}\right) = 0 \tag{29}
\]

Here we prove that \( k \geq 1 \). The proof is by contradiction. Assume that \( k < 1 \). We can rewrite condition (29) as
\[
\frac{\varphi}{d+1} \cdot \left(\Delta[v^* + \epsilon(k - 1)] - z\right) + (1 - \varphi) \cdot \left(\Delta[v^* - \epsilon] - z\right) = 0
\]

Note that \( \Delta[v^* - \epsilon] \geq z \), since banker with \( v < v^* \) does not exert effort. Since the whole expression is equal to zero, and the second term is non-negative, the first term should be non-positive. Hence,
\[
\frac{\varphi}{d+1} \cdot \Delta[v^* + \epsilon(k - 1)] - z \leq 0
\]
The expression above is non-positive only if
\[
\Delta[v^* + \epsilon(k - 1)] - z \leq 0
\]
The risk shifting incentive is smaller than or equal to \( z \) only if \( v \geq v^* \). And if \( k < 1 \), then \( v^* + \epsilon(k - 1) < v^* \). This is a contradiction. Consequently, it always holds that \( k \geq 1 \), and higher amount of CoCos is required to provide the same effect as equity.

**Proof of Proposition 5**

In order to show the effect of the probability of information revelation on the substitution ratio \( k \), we compute first and second derivatives of \( k \) with respect to \( \varphi \): \( \frac{\partial k}{\partial \varphi} \) and \( \frac{\partial^2 k}{\partial \varphi^2} \). We apply the implicit function theorem to the condition \( G(v^* - \epsilon|k\epsilon, d) = 0 \). We rewrite it using the fact that \( d = \frac{k\epsilon}{v^* - D} \):\[
\frac{\varphi \cdot (v^* - D)}{k\epsilon + v^* - D} \cdot \Delta[v^* + \epsilon(k - 1)] - z + (1 - \varphi) \cdot \Delta[v^* - \epsilon] = 0
\]

According to the implicit function theorem:
\[
\frac{\partial k}{\partial \varphi} = -\frac{\partial G(v^* - \epsilon|k\epsilon, d)/\partial \varphi}{\partial G(v^* - \epsilon|k\epsilon, d)/\partial k}
\]

where
\[
\frac{\partial G(v^* - \epsilon|k\epsilon, d)}{\partial k} = \frac{\varphi \cdot (v^* - D) \cdot \epsilon}{(k\epsilon + v^* - D)^2} \cdot \left(\frac{(k\epsilon + v^* - D) \cdot \Delta[v^* + \epsilon(k - 1)] - (\Delta[v^* + \epsilon(k - 1)] - z)}{\leq 0} \right)
\]
which is non-positive for infinitesimal \( \epsilon \).

\[
\frac{\partial G(v^* - \epsilon|k\epsilon, d)}{\partial \varphi} = \frac{(v^* - D)}{k\epsilon + v^* - D} \cdot (\Delta[v^* + \epsilon(k - 1)] - z) + \left( z - \Delta[v^* - \epsilon] \right) \leq 0
\]

Thus, the substitution ratio falls as probability of revelation rises \( \frac{\partial k}{\partial \varphi} \leq 0 \).

Next, consider the second derivative of substitution ratio with respect to \( \varphi \):

\[
\frac{\partial^2 k}{\partial \varphi^2} = -\left( \frac{(v^* - D)\epsilon}{(k\epsilon + v^* - D)^2} \cdot (\Delta^2_k[v^* + \epsilon(k - 1)] \cdot (k\epsilon + v^* - D) - (\Delta[v^* + \epsilon(k - 1)] - z)) \right) \geq 0
\]

This result implies that the substitution ratio \( k \) is decreasing and convex function of the probability of information revelation \( \varphi \).

**Proof of Proposition 8**

The banker’s payoff from the risky strategy is lower than in the case of non-convertible debt by the value of the call option held by the bondholders, which we denote as \( \frac{w}{w+1} \cdot \gamma(v + \epsilon) \):

\[
v - z - D + \Delta(v) - \frac{w}{w+1} \cdot \gamma(v + \epsilon)
\]

where the value of the call option:

\[
\frac{w}{w+1} \cdot \gamma(v + \epsilon) = \frac{w}{w+1} \cdot (1 - F(D + \frac{\epsilon}{w} - v)) \cdot E(V_2 - D|V_2 - D > \frac{\epsilon}{w})
\]

is positive and increasing in \( v \).

If the interim asset value is high \( v > D + \frac{\epsilon}{w} \), shareholders will choose to convert at the final date. The banker’s return from the safe strategy becomes then \( \frac{v - D + \epsilon}{w+1} \). If \( v \leq D + \frac{\epsilon}{w} \), the banker’s payoff from the safe strategy is the same as in the case of non-convertible debt \( v - D \).

The banker’s problem is:

\[
\max_e e \cdot \left[ (v - D) \cdot I(v \leq D + \frac{\epsilon}{w}) + \frac{v - D + \epsilon}{w+1} \cdot I(v > D + \frac{\epsilon}{w}) \right] + (1 - e) \cdot \left[ v - z - D + \Delta(v) - \frac{w}{w+1} \cdot \gamma(v + \epsilon) \right]
\]

s.t. \( e \in \{0, 1\} \)
The banker chooses effort according to the schedule:

If $v_G^{**} > D + \frac{\epsilon}{w}$, $e = \begin{cases} 1 & \text{if } v \geq v_G^{**} \\ 0 & \text{if } D + \frac{\epsilon}{w} \leq v < v_G^{**} \\ 1 & \text{if } v_G^{*} \leq v < D + \frac{\epsilon}{w} \\ 0 & \text{if } v < v_G^{*} \end{cases}$

If $v_G^{**} < D + \frac{\epsilon}{w}$, $e = \begin{cases} 1 & \text{if } v \geq D + \frac{\epsilon}{w} \\ 0 & \text{otherwise} \end{cases}$

(33)

Under Assumption ??, we show here that the equivalence ratio between CoCos and Green’s convertible bonds is lower than 1 ($k \leq 1$), which implies stronger effect on effort is produced by CoCos.

The condition for the equivalent effect from CoCos and Green’s convertibles is:

$$G(D + \frac{\epsilon}{w} | k \epsilon, d) = 0$$

or equivalently,

$$\frac{\varphi}{d + 1} \cdot (\Delta [D + \epsilon (k + \frac{1}{w})] - z) + (1 - \varphi) \cdot (\Delta [D + \frac{\epsilon}{w}] - z) = 0$$

(35)

Note that $D + \epsilon (k + \frac{1}{w}) \geq D + \frac{\epsilon}{w}$, when $k \geq 0$. For the equality (35) to hold, we need one part of the equation to be positive and another negative. $\Delta (v) - z$ is positive for $v < v^*$, and negative for $v > v^*$. This implies that $D + \epsilon (k + \frac{1}{w}) > v^*$ and $D + \frac{\epsilon}{w} < v^*$. This implies that for $\varphi = 1$, the equivalence condition is:

$$v^* - k \epsilon = D + \frac{\epsilon}{w}$$

$$k \geq \frac{v^* - D}{\epsilon} - \frac{1}{w}$$

(36)

We proof by contradiction. Assume that $k > 1$. Then it implies that $\frac{v^* - D}{\epsilon} - \frac{1}{w} > 1$, which is equivalent to $w(v^* - D - 1) < \epsilon$. This is contradiction, since $v^* < 1 \ (v_T < 1)$, $v^* - D - 1 < 0$, but $\epsilon > 0$ by construction. As a result, $k \leq 1$ for $\varphi = 1$.

**Proof of Proposition 9**

Consider the decision of the regulator on conversion at $t = 1$.

If the regulator observes $1 - \delta \leq v < v_C^*$ and triggers the conversion, the banker still chooses risky asset. The expected loss to the deposit insurance fund is

$$-Prob(V_2 < D - C, v) \cdot E(V_2 - D + C | V_2 < D - C, v)$$

(37)
and expected private cost to the regulator is

\[ k + \text{Prob}(V_2 < D - C, v) \cdot K \]

If the regulator does not trigger conversion, the risk choice of the banker does not change. However, this reduces the expected private cost of the regulator to

\[ \text{Prob}(V_2 < D - C, v) \cdot K \]  \hspace{1cm} (38)

As a result, regulator never triggers conversion for highly levered banks with interim asset values \(1 - \delta \leq v < v_C^*\).

If the regulator observes \(v_C^* \leq v < v^*\) and triggers the conversion, the banker chooses safe investment. The expected loss to the deposit insurance fund is 0, and expected private cost to the regulator is \(k\). If the regulator does not trigger conversion, the banker chooses risky strategy, and there is an expected loss to the deposit insurance fund as in (37). The regulator has an expected private cost as in (38).

Thus, the regulator triggers the conversion if

\[ \text{Prob}(V_2 < D - C, v) \cdot [-E(V_2 - D + C|V_2 < D - C, v) + c_2] \geq k \]  \hspace{1cm} (39)

where

\[ -\text{Prob}(V_2 < D - C, v) \cdot E(V_2 - D + C|V_2 < D - C, v) = \\
- (v - D - z - \text{Prob}(V_2 > D - C, v) \cdot E(V_2 - D + C|V_2 > D - C, v)) = \\
- (v - D - z - (v - D - z + \Delta(v + C))) = \Delta(v + C) \]

Thus, conversion is triggered for \(\max[v_C^*; v_R^*] \leq v < \min[v_R^*; v_C^*]\), where

\[ \Delta(v_R^* + C) + F(D - C, v) \cdot K = k \]  \hspace{1cm} (40)

For private cost of conversion \(k\) being high enough relative to the cost of bank failure, so that \(v_R^* \leq v^*\).

Consider now the risk incentives under the market trigger. Banker estimates the probability of conversion for a given \(v\).

\[ \text{Prob}_M(\text{conv}|v) = \begin{cases} 
1 & \text{if } v \leq v^* - \mu \\
\frac{v^* + \mu - v}{2\mu} & \text{if } v^* - \mu < v \leq v^* + \mu \\
0 & \text{if } v > v^* + \mu
\end{cases} \]

If \(\mu \geq C, v^* - C < v^* - \mu\), which means that banker won’t exert effort for \(v \in [1 - \delta, v^* - \mu]\) and also for \(v \geq v^* - C\) until the critical value \(v^* - C \leq v_M^* \leq v^*\) defined by:

\[ \frac{\mu + v - v^*}{2\mu} \cdot [z - \Delta(v)] + \frac{\mu + v^* - v}{2\mu} \cdot \frac{z - \Delta(v + C)}{d + 1} = 0 \]  \hspace{1cm} (41)
which is the solution of banker maximization problem with $v \in [v^* - \mu, v^* + \mu]$:

$$
\max e \cdot \left[ (v - D) \cdot \frac{\mu + v - v^*}{2\mu} + \frac{v - D + C}{d + 1} \cdot \frac{\mu - v + v^*}{2\mu} \right] + 
+ (1 - e) \cdot \left[ (v - D - z + \Delta(v)) \cdot \frac{\mu + v - v^*}{2\mu} + \frac{v - D + C - z + \Delta(v + C)}{d + 1} \cdot \frac{\mu - v + v^*}{2\mu} \right]
$$

(42)

Next, we show that market trigger produces more frequent conversion than a regulatory trigger, including in states where it is not necessary, or equivalently:

$$
\Delta_{MR} \int_{1+\delta}^{1-\delta} \text{Prob}(\text{Conv}|v) dv > 0
$$

(43)

$\Delta_{MR}$ is an operator of difference between welfare gains produced by market and regulatory trigger, $\text{Prob}(\text{Conv}|v)$ is a probability of conversion for a bank with the interim asset value $v$.

Consider the efficiency of conversion for a market trigger. Market converts CoCos if $v + \tilde{m} < v^*$.

This means that the probability of conversion for banks with $v > v^* + \mu$ is zero. For banks with $\max[1 - \delta, v^* - \mu] \leq v \leq \min[1 + \delta, v^* + \mu]$, the probability of conversion is $v^* + \mu - v^* 2\mu$.

CoCos in banks with $v < v^* - \mu$ are converted with certainty.

The expected cost of conversion is then

$$
k \cdot \int_{1+\delta}^{1-\delta} \text{Prob}_{MT}(\text{Conv}|v) dv = k \cdot [\mu + (v^* - \mu - (1 - \delta))]\]

(44)

Next, consider the regulatory trigger. CoCos with such a trigger are converted whenever $v^* - C \leq v + \tilde{\tau} \leq v^*_R$.

Expected cost of conversion $k \cdot \int_{1+\delta}^{1-\delta} \text{Prob}_{RT}(\text{Conv}|v) dv$ is:

$$
k \cdot \begin{cases} v^*_R - (1 - \frac{\delta}{2}) & \text{if } \min[v^* - v^*_R; v^*_R - (v^* - C)] \leq \mu < \frac{\delta}{2} \\
(1 - \frac{\delta}{2}) + v^*_R - \frac{\mu - (1 - \delta)}{4\mu} \cdot (v^*_R + \mu - 1) & \text{if } v^*_R - (1 - \delta) \leq \mu \leq v^*_R - (1 - \delta) \\
(1 - \frac{\delta}{2}) \cdot \frac{v^*_R - 1 + \frac{3}{2}\delta + \mu}{4\mu} & \text{if } \mu > v^*_R - (1 - \delta) \end{cases}
$$

The expected cost of conversion is always higher for the market trigger.
References


