Central Counterparty Clearing: Incentives, Market Discipline and the Cost of Collateral*

Thorsten V. Koepl
Queen’s University

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Abstract

This paper analyzes central counterparty (CCP) clearing – defined as the diversification of default risk through novation – in the presence of moral hazard that aggravates counterparty risk. Collateral is costly and serves two purposes: it insures against default and provides incentives to avoid moral hazard. When counterparty risk is not directly observable, I show that central clearing can lead to higher collateral requirements for two different reasons. First, a CCP offering diversification of risk cannot selectively forgo incentives for transactions that optimally use collateral only for insurance. Second, requiring more collateral for better incentives can reduce market liquidity and adversely affect market discipline which is a substitute for collateral. Hence, while CCP clearing can lower default through higher collateral requirements, there is a feedback effect through a fall in market liquidity that amplifies collateral costs. As a consequence, CCP clearing is likely to offer net benefits only in markets that are sufficiently liquid and where the potential for endogenous risk taking is not too severe.

Keywords: CCP Clearing, Counterparty Risk, Moral Hazard, Incentives vs. Insurance

JEL Classification: G32, G38, D82, D83

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1 Introduction

Risk management practices of financial institutions have been deemed insufficient in the aftermath of the financial crisis. One particular area of concern is the low level of collateral applied to secure over-the-counter (OTC) derivatives transactions. While trading in such instruments has risen sharply over the last decade, a total exposure of about $2trn in derivative exposures are not at all collateralized or “under”-collateralized.¹ The common policy response to this problem is to move these transaction under the umbrella of a clearinghouse that would offer central counterparty (CCP) clearing and ensure that “proper” collateral is posted in these transactions. Large dealer banks were quick to point out that – while the lack of collateralization need not represent an inefficiency – such an omnipresent rise in collateral requirements would lead to a significant increase in the costs of OTC trading while not dramatically improving overall risk exposures.²

This paper asks whether CCP clearing leads to an increase in collateral costs and, if so, whether default risk decreases sufficiently so that such clearing is indeed beneficial for financial markets. I start from the premise that clearing through a CCP pools and thereby diversifies counterparty risk by interposing itself as the sole buyer and seller of any contract traded on a financial market – a process called novation. Since diversification of default risk and individual posting of collateral are substitutes for counterparty risk insurance, CCP clearing would lead automatically to lower collateral requirements (see Koeppl and Monnet (2010)).³ Hence, it seems puzzling from this perspective that market participants fear an increase in overall collateral costs.

I show in this paper that collateral can also serve as an incentive device to keep counterparty risk in check. With bilateral clearing, collateral costs can then be low for two very different reasons. They can be low, because it is too expensive to set such incentives; or, they can be low, because there is a cheaper, alternative mechanism – which I call market discipline – that is a substitute to provide incentives. As a consequence, CCP clearing can cause increases in collateral requirements. Again, the reason is twofold. First, I demonstrate that a CCP cannot selectively forgive incentives for some transactions, when counterparty risk is not perfectly observed. Second, as CCP clearing enforces proper incentives across all

¹See for example Cecchetti, Gyntelberg and Hollander (2009) or Singh (2010)).
²Singh (2010) gives a back-of-the-envelope calculation of about $220bn in additional collateral requirements. There are other indirect costs if one takes into account further measures like restrictions on rehypothecation.
³There could be other savings offered by multilateral netting and netting across different financial products or by more efficient collateral management (see for example Checcetti, Gyntelberg and Hollander (2009)). These services could lower both the amount and the unit cost of collateral.
transactions through higher collateral requirements, it reduces surplus for some transactions rendering them non-profitable. This lowers markets liquidity and adversely affects market discipline. As a result, all transactions require now even higher collateral in order to set proper incentives. Hence, this paper is novel in that it formally compares the costs and the benefits of CCP clearing along the dimension of costly collateral and the incentives for risk taking. Moreover, as an important contribution it establishes a feedback effect whereby CCP clearing amplifies collateral requirements and their costs through a reduction in market liquidity.

To be more specific, consider a situation where a trade occurs with some exogenously given risk of default. This can be viewed as risk associated with the transaction itself. Suppose further that there is moral hazard in the sense that a trading party has a private (and non-contractible) benefit from increasing this risk. This can be regarded as counterparty specific default risk. Collateral is costly, but serves two purposes. First, it can act as a prepayment – essentially insuring against the exogenous risk of default. Alternatively, it can provide incentives to keep counterparty risk low. I label a trade insurance contract when collateral mainly serves the function of insurance and label it incentive contract when collateral is mainly used as an incentive device. I show that when collateral is costly, it can be (privately) optimal for the contracting parties to use an insurance contract that requires low collateral at the expense of higher default risk: as moral hazard increases, a transaction achieves higher surplus when reducing collateral, while incurring some default risk. As a result, collateral levels and default risk are negatively correlated, so that low collateral postings are always a sign of higher default risk.

For any given contract – insurance or incentive – CCP clearing can still lower collateral requirements in this environment, as it diversifies default risk through novation. It simply offers cheaper insurance against default. This implies that with CCP clearing collateral is not used for insurance at all, but only for incentives. I show, however, that the CCP cannot clear insurance contracts and incentive contracts at the same time, whenever the counterparty risk associated with transactions is private information for the contracting parties. The intuition for this result is as follows. Suppose that all transactions need to be cleared through a CCP. The CCP observes transaction prices and sets collateral based on this information. For incentive contracts, the price contains information concerning the default risk. I find that the

\[4\] Of course, I omit here the issue that a certain level of counterparty risk that is acceptable for the counterparties to a trade might not be optimal from a societal perspective. This can be due to contagion, knock-on effects outside asset markets or leveraged positions in derivatives transactions among other reasons.

\[5\] I am abstracting here from the issue whether regulators indeed can force trades to be cleared through a central counterparty (see the discussion in Koeppel, Monnet and Temzelides (2011)).
CCP can set higher collateral requirements for more risky transactions. Insurance contracts, however, are taking place at a fixed price that is not informative about the underlying moral hazard and require zero collateral. This fixed price, however, corresponds to a price that is also associated with transactions based on an incentive contract. Hence, the CCP needs to choose what type of contract to implement with its collateral policy.

When the CCP decides to have only incentive contracts take place, default risk in transactions will be low. However, this policy will also increase collateral requirements for transactions that were formerly using insurance contracts. Since the costs of these contracts go up, the surplus might not be sufficient anymore for the transaction to be carried out. This implies that with CCP clearing there could be a fall in market liquidity.

In my environment, market liquidity plays a key role for market discipline to act as a substitute for collateral in order to provide incentives. Suppose there are trading frictions that make long-term relationships attractive. More specifically, consider a search friction: when losing a trading partner it takes time to engage in a new trade. This search cost can thus be interpreted as a proxy for how easy it is to transact in the market, which in turn is a function of liquidity in the market. When search costs are low, the threat of terminating a relationship in response to moral hazard by the counterparty can be credible. It is cheaper to search for a new trading partner than to be exposed to the default risk from the transaction. Consequently, low collateral postings are not necessarily an indication of large default risk anymore. In other words, collateral could be low, but at the same time default risk could also be low as market discipline reduces moral hazard, and, thus the need for collateral as an incentive device.

CCP clearing can then have unintended consequences for incentive contracts. As the CCP prevents insurance contracts to take place, it can lower market liquidity as pointed out earlier. This can erode market discipline, in the sense that punishing moral hazard is not credible anymore. Collateral requirements then need to increase for all incentive contracts, just to keep default risk on all these transactions unchanged – which is inefficient as collateral is costly. Consequently, CCP clearing might make some transactions with high default risk unprofitable, but at the same time it might also reducing the surplus on other transactions where default risk is low to begin with.

There are several conclusions to be drawn for the current policy discussion. First, zero or

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6 There might be other reasons to believe that a CCP will decrease liquidity. CCPs for example are likely to set strict membership requirements so that only high quality counterparties have access to it in order to avoid adverse selection. Moreover, trades outside formal clearing arrangements are likely to face additional costs in the form of capital charges, thereby further reducing liquidity in the market (see BIS (2011)).
low collateral cannot be necessarily interpreted as insufficient risk management. It might be the case that they simply reflect market discipline or a more efficient contract design of the counterparties. Second, introducing a CCP can increase collateral costs without improving surplus for transactions. Third, even if the social costs of default exceed private ones, the decision whether to introduce a CCP or not must not only consider the impact on the cost of collateral, but also the impact of such a move on market liquidity and trading dynamics. A fall in market liquidity could weaken market discipline causing an unnecessary increase in collateral without actually lowering default risk for most transactions. Hence, my work calls for a more careful investigation of how changes in financial markets infrastructure and risk management influence the exposure to default risk as well as market liquidity.

The economic modeling of CCP clearing is relatively thin. My analysis here is mainly built on the framework of Koeppl and Monnet (2010) that stresses novation and mutualization of losses as the key ideas of how CCP clearing affects trade and welfare. As such it abstracts from other benefits such as netting (see Duffie and Zhu (2009)) or information dissemination (see for example Archaya and Bisin (2010)). A recent contribution by Carapella and Mills (2011) exhibits information insensitivity of securities as a key mechanism of CCP clearing. Most interestingly, this last paper also establishes a link between collateral in the form of margin calls and CCP introduction and design. The remainder of the paper is organized as follows. Section 2 describes the model. The next two sections deal with the optimal contract choice and what CCP clearing can achieve when counterparty risk is not publicly observable. Section 3 contains the static analysis, while Section 4 deals with a dynamic extension to investigate the role of market liquidity for market discipline. The last section concludes with some comparative statics.

2 Model

I follow a simplified version of the set up by Koeppl and Monnet (2010) to formalize bilateral trading of customized financial contracts. The key difference is that there are (i) no aggregate shocks and (ii) that there is no retrading ex-post, but these elements are not essential for this analysis. I add a moral hazard feature along the lines of Holmström and Tirole (1997) where some people can take an action that yields a private benefit which cannot be contracted upon directly.

More formally, there are two dates $t = 0$ and $t = 1$. There are two types of people, farmers and bakers, both of measure 1. There are also two different goods in the economy, wheat
and gold. Farmers can produce a specialized type of wheat for a particular baker. They can produce either one unit or none for bakers, and production takes time. The farmer has to produce the wheat in \( t = 0 \) for consumption by the baker in period \( t = 1 \). Since wheat is specifically produced for a baker, it cannot be retraded at \( t = 1 \). Bakers can produce gold in both periods which can be stored across periods.

Farmers preferences are described by

\[
 u_F(q, x) = -\theta q + u(x) \tag{1}
\]

where \( q \) is the amount of wheat produced – either 0 or 1 – and \( x \) is the amount of gold consumed in period \( t = 1 \). For most of the paper, we assume that \( \theta \) is sufficiently small, so that farmers have an incentive to produce the special wheat for a baker. The baker’s preferences are given by

\[
 u_B(q, x_1, x_2) = -\mu x_1 - x_2 + vq \tag{2}
\]

where \( v \) is the (fixed) utility obtained from \( q \) units of wheat. The baker can produce gold either in period 1 or 2. However, early production of gold implies an additional cost, since we assume that \( \mu > 1 \).

There are two complications. First, bakers can die with probability \( \epsilon \in [0, 1) \) after \( t = 0 \). If a farmer has produced specific wheat for a baker, he will not be able to deliver it against a payment in gold. Second, bakers can engage in an activity that delivers some private benefit \( B > 0 \) at \( t = 0 \), but increases their probability of dying. We denote this decision by \( \lambda_B \in \{0, 1\} \). In particular, we assume that if a baker engages in the activity he will go bust with probability \( \rho \) conditional on not dying. Hence, if \( \lambda_B = 1 \), the probability of dying increases to \( \epsilon + (1 - \epsilon)\rho \), where \( \rho \in (0, 1) \). I interpret the variable \( B \) as idiosyncratic counterparty risk. Note that it is inherently related to the counterparty of a transaction, but not to the transaction itself. To the contrary, the exogenous default probabilities \( \epsilon \) and \( \rho \) are constant across transactions and, hence, can be seen as linked to the type of transaction.

Trading is organized as follows. At \( t = 0 \), a farmer meets a baker and offers a contract \((p, k) \in \mathbb{R}^+_2 \). The variable \( p \) formalizes a total payment in gold by the baker upon delivery of wheat in \( t = 1 \). The variable \( k \) describes a prepayment in \( t = 0 \) when the farmer undertakes production. This allows us to interpret the relationship as a forward contract where the farmer asks for collateral \( k \) to safeguard against the risk that the baker dies. The final payment is then net of collateral \( p - k \).\footnote{The fact that wheat is specifically produced for a particular baker captures the notion of a non-standardized derivative transactions where replacement costs are – here infinite.}
the farmer, so that the contract cannot be contingent on it. Finally, if the baker survives, the contract is settled in net terms in period $t = 1$. That is the farmer delivers wheat against the net payment of $p - k$, i.e. there is perfect enforcement of the contract.

To summarize, we have a basic problem of moral hazard that leads to counterparty risk. $B$ is valuable for the baker, but decreases the expected surplus from a trade for a baker. We assume for reasons of tractability that bakers need to receive at least an expected surplus of $c > 0$ from trading wheat with a farmer, where

$$v(1 - \epsilon) \geq \frac{B}{\rho} \geq c. \quad (3)$$

As will become clear later, this restriction ensures that the baker prefers taking on the additional risk given the surplus $c$ from trading with the farmer and that any contract features a total payment that exceeds collateral postings ($p \geq k$). The latter restriction ensures that there is net settlement of contracts not in default.

3 Collateral: Incentives vs. Insurance

3.1 Bilateral Clearing with One Period Contracts

We solve first for the contract when farmers can make a take-it-or-leave-it offer $(p, k)$ to the baker. Interpreting the prepayment $k$ as collateral, it can take on two roles. First, it can be used to provide incentives, with the baker putting up a bond that prevents the farmer from taking on excessive default risk. But it also insures the farmer against the default risk by bakers independent of $\lambda_B$. Hence, collateral serves a dual role as an insurance and incentive device, while controlling the farmer’s default risk.

To make this more precise, the incentive constraint for the baker not to realize the private benefit $B$ is given by

$$-\mu k + (1 - \epsilon)(v - p + k) \geq -\mu k + B + (1 - \rho)(1 - \epsilon)(v - p + k) \quad (4)$$

or

$$v - p + k \geq \frac{B}{(1 - \epsilon)\rho}. \quad (5)$$

The baker weighs the expected benefit from obtaining the wheat $v - p + k$ against the gain from obtaining the (risk-weighted) benefit $B$. Note that when making a decision about $B$,
collateral $k$ is sunk. Hence, an increase in collateral $k$ relaxes the constraint as it increases the benefit from settling the contract with the farmer. It is in this sense that collateral provides incentives. The baker needs to receive a minimum expected surplus from the contract given by

$$-\mu k + (1 - \epsilon)(v - p + k) \geq c.$$  

Any contract that satisfies these two constraint is called an *incentive contract*.

Alternatively, the contract could violate the incentive constraint (5). In order for the baker to have an incentive to set $\lambda_B = 1$ it must be the case that

$$v - p + k \leq \frac{B}{(1 - \epsilon)\rho}.$$  

The participation constraint then becomes

$$-\mu k + (1 - \epsilon)(1 - \rho)(v - p + k) \geq c.$$  

Note that the expected surplus from the contract is lower for the baker. This implies that given the outside option $c$ for the baker, the surplus from obtaining wheat must be larger than with an incentive contract. We call such a contract an *insurance contract*.

A farmer will choose between the two type of contracts to maximize his expected utility which is given (net of production costs) by

$$\left(\epsilon + (1 - \epsilon)\rho\lambda_B\right)u(k) + \left(\epsilon + (1 - \epsilon)(1 - \rho\lambda_B)\right)u(p).$$  

The participation constraint will be binding for both type of contracts. The reason is straightforward. If the participation constraint were not binding for an incentive contract, the farmer could increase collateral $k$. This relaxes the incentive constraint and increases utility. For an insurance contract, the farmer could increase $p$ which also relaxes the insurance constraint. The incentive constraint is then given by

$$k \geq \frac{1}{\mu} \left(\frac{B}{\rho} - c\right)$$

whereas the insurance constraint yields

$$k \leq \frac{1}{\mu} \left(\frac{B(1 - \rho)}{\rho} - c\right).$$  

As a result, collateral posted in an insurance contract is always lower than in an incentive
contract. Furthermore, an insurance contract will only be feasible if \( B \geq c\rho/(1 - \rho) \).

The trade-off between the two contracts is then clear for the farmer. Offering a contract with incentives has two benefits. First, it lowers the default probability to \( \epsilon \). Second, it increases the expected surplus from the contract for the baker, thereby enabling the farmer to charge a higher price \( p \). An incentive contract, however, requires collateral to be posted. Collateral is costly here as requiring it reduces the total amount of gold \( p \) that a farmer can require for his special wheat. The cost arises from the fact that \( \mu > 1 \). Hence, when the risk-weighted private benefit \( B/\rho \) is sufficiently high, the farmer might find it optimal to forego the incentives and offer an insurance contract. Of course, with an insurance contract, the farmer allows the baker to engage in the risky activity \( B \). But such a contract might help to save on collateral costs. This is formalized in the next proposition.

**Proposition 1.** The optimal incentive contract is given by a fixed level of collateral \( k^* \) for \( B \in [c\rho, B^*] \) and an increasing level of collateral \( k_{INC}(B) \) for \( B \in [B^*, \rho v(1 - \epsilon)] \).

The optimal insurance contract is given by an increasing level of collateral \( k_{INS}(B) \) for \( B \in [c\rho, B^{**}] \) and a fixed level of collateral \( k_0 \) for \( B \in [B^{**}, \rho v(1 - \epsilon)] \).

The collateral posted in an incentive contract is strictly higher than in an insurance contract, i.e. \( k_{INS}(B) < k_{INC}(B) \).

**Proof.** Consider first an incentive contract. Since the participation constraint must be binding, we have that the price is given by

\[
p = v - \frac{c}{1 - \epsilon} - k \left( \frac{\mu}{1 - \epsilon} - 1 \right).
\] (12)

It follows that the optimal contract when the probability of default is given by \( \epsilon \) is described by

\[
\epsilon u'(k) + (1 - \epsilon)u'(p) \left( 1 - \frac{\mu}{1 - \epsilon} \right) + \lambda = 0,
\] (13)

where \( \lambda \) is the multiplier on the incentive constraint. Denoting \( k^* \) the solution to this equation when \( \lambda = 0 \), it follows directly that there exists a cut-off level \( B^* \) such that the incentive constraint is binding if and only if \( B \geq B^* \).

For the insurance contract, it must again be the case that the participation constraint is binding. Consider again the first-order condition

\[
u'(k) (\epsilon + (1 - \epsilon)\rho) + (1 - \epsilon)(1 - \rho)u'(p) \left( 1 - \frac{\mu}{(1 - \epsilon)(1 - \rho)} \right) + \lambda_{NN} - \lambda_{INS} = 0,
\] (14)
where \( p = v - \frac{c}{(1-\epsilon)(1-\rho)} - k \left( \frac{\mu}{(1-\epsilon)(1-\rho)} - 1 \right) \) and the Lagrange multipliers are on the constraints

\[
\begin{align*}
    k &\geq 0 \\
    k &\leq \frac{1}{\mu} \left( \frac{B(1-\rho)}{p} - c \right),
\end{align*}
\]

respectively.

Inspection of the two constraints implies immediately that the constraint set is empty for \( B < c\rho/(1-\rho) \). Furthermore, if \( u'(0) \) is sufficiently small, \( \lambda_{NN} > 0 \) so that \( k = 0 \) for all \( B \). Otherwise, collateral increases according to the second constraint (\( \lambda_{NN} = 0 \) and \( \lambda_{IC} > 0 \)).

Suppose next that the insurance constraint is binding at \( B = \rho(1-\epsilon)v \) for the optimal insurance contract. Then, we have that \( p = k \) and \( \lambda_{INS} > 0 \). But this contradicts the first-order condition, since \( \mu > 1 \). Hence, \( k < p \) and collateral remains constant for large enough values of \( B \).

The final statement is obvious from the incentive and insurance constraints.

Figure 1 summarizes this result. Optimal collateral policies for incentives and insurance contracts are driven by the incentive and insurance constraints. The slope of these constraints reflect the cost of collateral \( \mu > 1 \) and \( \frac{\mu}{1-\rho} \). As long as these constraints are not binding, the contract chooses the optimal level of collateral that equates the farmer’s marginal utility for default and no default by the baker. For low levels of \( B \), an insurance contract is not feasible. Then, if collateral is used at all, it will increase. When moral hazard is severe (i.e. \( B/\rho \) is large), the first-best level of collateral in an insurance contract becomes feasible and collateral remains insensitive to the degree of moral hazard. Note that in this case \( k < p \), so that an insurance contract is feasible for any level of \( B > c\rho/(1-\rho) \). To the contrary, an incentive contract features first a constant level of collateral which is the unconstrained optimal level given the exogenous default probability \( \epsilon \). As the moral hazard becomes more severe, we have that the collateral must be increased to prevent it. Eventually, collateral becomes so high that \( p = k \), a situation of complete prepayment at \( B/\rho = v(1-\epsilon) \).

The choice of contract for the farmer is then driven by the collateral costs of the contract. With an incentive contract, for low levels of moral hazard, the utility for the farmer is independent of \( B/\rho \), while it decreases as collateral begins to rise. For an insurance contract exactly the opposite is the case. As collateral cannot be too high, insurance against default by the baker is not at its optimal level. Hence, the farmer's utility rises as moral hazard becomes more severe eventually leveling out at some level. This is summarized as follows.
Corollary 2. The farmer prefers an incentive contract if and only if the level of moral hazard $B$ is sufficiently low.

In conclusion, low levels of collateral can go hand-in-hand with contracts where farmers optimally insure against moral hazard. Hence, low collateral levels can indeed be associated with higher default risk. Nonetheless, this is fully efficient from the perspective of the two parties writing the contract, as it maximizes surplus from the contractual relationship.

3.2 CCP Clearing

I now introduce CCP clearing along the lines of Koeppl and Monnet (2010). A CCP imposes collateral requirements taking the terms of trades of any transaction as given and novates the transactions: it collects all collateral and payments for settlement paying it out as an average to farmers. To be more specific, the CCP can only observe the price of a transaction, but not the level of moral hazard $B$ associated with the transaction. Its policy is to set collateral as a function of the transaction price, i.e. $k(p)$.

\footnote{I do not allow the CCP to use a direct mechanism, where it sets both the price and the collateral for a transaction. Such a mechanism would directly determine the terms of trade and, thus, would go beyond...}
default; i.e., $\epsilon > 0$.

Farmers make a take-it-or-leave-it offer to bakers in terms of only $p$ taking the CCP’s collateral schedule $k(p)$ as given. Hence, when making their offer they are aware of what collateral will be requested by the CCP for the transaction at price $p$. I assume that the CCP pools all the payments associated with a single transaction price. For the collateral policy $k(p)$ the revenue of the CCP associated with transactions at price $p$ is given by

$$R(p) = (\epsilon + (1 - \epsilon)\rho \lambda B) k(p) + ((1 - \epsilon)(1 - \rho \lambda B)) p.$$ (17)

Hence, the CCP receives payments form of collateral $k(p)$ at $t = 0$ and net settlement $p - k(p)$ at $t = 1$ from the non-defaulted contracts. The CCP’s payment to the farmers is then equivalent to this revenue $R(p)$.\(^9\) Hence, as we will discuss below, CCP clearing allows a diversification of counterparty default risk.

We show next that the CCP can set up an incentive compatible collateral schedule that fully reveals $B$ which is private information in the from of incentive contracts. For this purpose, recall that a contract with $\overline{B} = \rho c$ has no incentive problem and without collateral ($k = 0$) its price will be given by

$$\overline{p} = v - \frac{c}{1 - \epsilon}.$$ (18)

**Proposition 3.** The linear collateral schedule

$$k(p) = \left(\frac{(1 - \epsilon)}{\mu - (1 - \epsilon)}\right) (p - p)$$

is incentive compatible and uniquely implements incentive contracts for all $B$; i.e. given this policy there exists a unique, strictly decreasing function $p(B) : [cp, v(1 - \epsilon)\rho] \rightarrow \mathbb{R}_+$ for transaction prices such that there are only incentive contracts and default occurs with clearing. Also, I do not allow farmers to make a choice about clearing bilaterally or centrally (see for example Koeppel et al. (2011)).

\(^9\)The CCP could alternatively pool all payments it receives and make payments proportional to the value $p$ of a contract. The CCP’s revenue is then given by

$$R = (\epsilon + (1 - \epsilon)\rho \lambda B) k^{\text{CCP}} + ((1 - \epsilon)(1 - \rho \lambda B)) \int p(i) di.$$

and payouts equal to

$$\frac{p(i)}{\int p(i) di} R^{\text{CCP}}.$$  

While payouts are still strictly increasing in $p$, there would be cross-subsidization between low and high surplus transactions which would complicate the analysis further.
probability $\epsilon$ for all transactions.

Proof. Fix $B \in [c\rho, v(1-\epsilon)\rho]$. There exists a unique price $p_B$ such that

$$(1-\epsilon)\rho(v - p_B + k(p_B)) = B$$

$$(1-\epsilon)(v - p_B + k(p_B)) - \mu k(p_B) = c,$$

i.e., there is a unique incentive contract that extracts all surplus for the farmer given $B$.

Now consider a farmer facing a baker with the degree of moral hazard given by $B$. If there is no default, his payoff from negotiating a contract with price $p$ is given by

$$R(p) = \epsilon k(p) + (1 - \epsilon)p.$$

Since $\mu > 1$, the farmer’s pay-off is increasing in $p$. Hence, he would like to negotiate the highest price. Consider any price $p > p_B$. The collateral policy $k(p)$ implies that

$$v - p + k(p) < v - p_B + k(p_B) = \frac{B}{(1-\epsilon)\rho}.$$

Hence, the incentive constraint is violated, so that there will be default by the baker. By construction of the mechanism we also have that

$$(1-\epsilon)(1-\rho)(v - p + k(p)) - \mu k(p) < (1-\epsilon)(v - p_B + k(p_B)) - \mu k(p_B) = c$$

so that a baker would not accept such an offer. This implies that the farmer cannot offer a price higher than $p_B$. Since he does not want to offer a price lower than $p_B$, the mechanism implements incentive contracts for all levels of $B$. \qed

The intuition for this result is straightforward. The collateral schedule simply mirrors the participation constraint for bakers when there is no moral hazard. Hence, lower prices require higher collateral. This seems strange at first sight, but recall that trading surplus is decreasing with the moral hazard problem as one requires collateral to give incentives which is costly. Farmers will choose prices so as to maximize their payments from the CCP subject to be able to write a contract with the baker. Hence, by design, the CCP forces all contracts to lay on the participation constraint associated with no moral hazard for the baker.

Farmers would like to get the highest averaged payment from the CCP. Since the revenue $R(p)$ is strictly increasing in $p$ when $\lambda_B = 0$, they would like to transact at the highest price.
Larger levels of moral hazard are associated with higher collateral requirements and lower prices for incentive contracts. Hence, farmers would like to lie downward; i.e. they would like to report lower levels of $B$. However, this is not feasible, since this implies $\lambda_B = 1$ with bakers receiving not enough surplus for entering into the contract. Furthermore, the collateral schedule then replicates exactly the incentive constraint for all values of $B$.

To conclude, CCP clearing can perfectly control moral hazard through its collateral policy, while still being able to diversify the remaining, exogenous default risk. When doing so, the CCP can elicit – through its collateral policy – the revelation of varying counterparty risk (as given by $B$) in privately negotiated transactions. This implies that, by using exclusively incentive contracts, a CCP could minimize counterparty risk for all levels of $B$. What it cannot do, however, is to allow both types of contracts at the same time whenever the degree of moral hazard cannot be directly observed.

### 3.3 Incentive vs. Insurance Revisited

To be more precise about the last point, consider an insurance contract for a given value of $B$. If the CCP could observe $B$, the best insurance contract would be one without collateral for the farmer. Collateral is an imperfect and costly substitute to pooling idiosyncratic default risk. As soon as there is a possibility to diversify such risk through CCP clearing, it is not optimal to require collateral anymore ($k_0(B) = 0$ for all $B$). Hence, with CCP clearing, the payoff of the farmer from an insurance contract would be given by

$$R_0 = (1 - \epsilon)(1 - \rho)p_0 = (1 - \epsilon)(1 - \rho)\left(v - \frac{c}{(1 - \epsilon)(1 - \rho)}\right),$$

where we have used the optimal price charged by the farmer. It is then straightforward to compare incentive and insurance contracts under CCP clearing.

**Proposition 4.** Fix the level of moral hazard $B$. The optimal contract for a transaction with CCP clearing has lower collateral requirements than without CCP clearing. It is an incentive contract if and only if

$$B \leq \rho \left(\bar{B} \frac{\mu}{\mu - 1} + c\right).$$

**Proof.** That collateral is lower follows immediately from the fact of a non-random payoff for farmers, so that collateral is not needed for insurance against default. To obtain the equality,
we can simply compare the farmer’s payoffs across the two contract. The farmer prefers an incentive contract if and only if

\[
R(p_B) = ck(p_B) + (1 - \epsilon)p_B \geq (1 - \epsilon)(1 - \rho)p_0 = R_0
\]

\[
eck(p_B) + (1 - \epsilon)\left[v + k(p_B) - \frac{\mu}{1 - \epsilon}k(p_B) - \frac{c}{1 - \epsilon}\right] > (1 - \epsilon)(1 - \rho)v - c
\]

\[
\bar{B} = \rho(1 - \epsilon)v > k(p_B)(\mu - 1).
\]

Taking into account the CCP’s policy, we directly obtain the condition in the proposition.

For farmers the optimal collateral policy depends again on the degree of moral hazard \(B\) and cost of collateral. With CCP clearing, insurance contracts become even more attractive relative to incentive contracts for high levels of \(B\). While diversification lowers collateral costs for the former type of contract, collateral for incentive contracts is unchanged with central clearing above some threshold for moral hazard. Still, farmers facing sufficiently low levels of moral hazard would always like to write an incentive contract. We show next, that this implies a conflict of interest for the CCP whenever there are multiple values of \(B\) which cannot be directly observed by the CCP.

**Proposition 5.** Suppose \(\mu(1 - \rho) > 1\). There is no collateral policy that simultaneously implements optimal incentive and insurance contracts for all levels of \(B\).

**Proof.** Recall first that there is a unique collateral policy that implements incentive contracts and that the optimal collateral policy for an insurance contract is given by \(k_{INS} = 0\).

Let \(\bar{B} \geq \left(\frac{\mu - 1}{\mu(1 - \rho) - 1}\right)\rho c\), so that there is some level of \(B\) for which an insurance contract is preferred. I first show that there is another level of \(B\) such that the price associated with an incentive contract is equal to the price \(p_0\).

Using \(p_B = p_0 = v - \frac{c}{(1 - \epsilon)(1 - \rho)}\), the level \(B\) associated with such an incentive contract needs to satisfy

\[
c = (1 - \epsilon)(v - p_B + k(p_B)) - \mu k(p_B)
\]

\[
c = \frac{c}{1 - \rho} - \frac{1}{\mu} \left(\frac{B}{\rho - c}\right)(\mu - (1 - \epsilon))
\]

\[
B = \left[1 + \left(\frac{\rho}{1 - \rho}\right)\left(\frac{\mu}{\mu - (1 - \epsilon)}\right)\right] \rho c > \bar{B}.
\]
It is straightforward to verify that $B < B$. Hence, whenever farmers prefer the insurance contract for some level of $B$, there exists a lower, feasible level of moral hazard for which the incentive contract has the same price as the insurance contract.

I show next that it is always the case that farmers facing moral hazard level $B$ prefer the incentive contract. This is the case if and only if

\[
(1 - \epsilon)\rho \geq k(p_B)(\mu - 1)
\]

\[
\bar{B} \geq \left(\frac{\mu - 1}{\mu}\right)\left(\frac{B}{\rho - c}\right)
\]

\[
\bar{B} \geq \left(\frac{\mu - 1}{\mu}\right)\left(\frac{\rho}{1 - \rho}\right)\left(\frac{\mu}{\mu - (1 - \epsilon)}\right)c
\]

\[
\bar{B} \geq \left(\frac{1}{1 - \rho}\right)\left(\frac{\mu - 1}{\mu - (1 - \epsilon)}\right)c
\]

It is again straightforward to check that this inequality is satisfied given the condition on $B$.

Hence, there are at least two different levels of $B$ for which the price negotiated by the farmer is the same, but who require two different collateral policies, $k(p_0) > 0$ and $k_0 = 0$. This completes the proof.

With CCP clearing, all insurance contracts have the same price independent of $B$. This price, however, coincides with a price that is also associated with an incentive contract arising from a transaction with a lower $B$. In this circumstance, to implement both contracts, the CCP will observe a price $p$, but needs to implement two different collateral policies for this price depending on the underlying $B$ of the transaction. In other words, it cannot determine whether the transaction stems from an insurance contract for high counterparty risk or from an incentive contract with low counterparty risk.

As a consequence, when counterparty risk is not observable, the CCP needs to decide between implementing incentive contracts or insurance contracts; it cannot do both. If the CCP wants to minimize counterparty risk, it needs to opt for only incentive contracts. Such a policy, however, might prevent certain transactions from taking place.

**Corollary 6.** There exists a threshold level of production costs $\bar{\theta}(B)$ above which farmers facing moral hazard $B$ will not engage in incentive contracts.

Importantly, CCP clearing with incentive contracts can increase collateral costs for trans-
actions with high counterparty risk $B$, which might have used insurance contracts before.\footnote{CCP clearing could also lead to a fall in unit costs of collateral, i.e. $\mu_{CCP} < \mu$. Here, for incentive contracts the degree of moral hazard, $B/\rho$ determines only the overall cost of collateral, $\mu k$. Hence, a fall in unit costs is exactly compensated by an increase in collateral requirements with the consequence that overall collateral costs cannot increase with CCP clearing, but need not necessarily fall either.}

This can crowd out such transaction. Of course, this transaction would still be profitable using insurance contracts with CCP clearing. This will become important in the next section which analyzes the interaction between market liquidity, optimal collateral requirements and CCP clearing.

4 CCP Clearing and Market Liquidity

4.1 Search Costs and Repeated One-Period Contracts

To work towards a notion of market liquidity, I extend the static framework to a dynamic one. Consider a dynamic version of the economy. Farmers and bakers are randomly matched. When matched, they stay together and contract until either the baker dies or the farmer terminates the relationship. When a baker dies, he is replaced by a new baker. We restrict attention again to one period (static) contracts\footnote{This is a reasonable assumption when thinking about financial trades.} where the farmer agrees to produce one unit of wheat in exchange for a contract $(p,k) \in \mathbb{R}^2_+$. The contract specifies again an upfront payment or collateral and the total price for the wheat. The future is discounted at a rate $\beta \in (0,1)$. To simplify the analysis further, we also assume in the dynamic economy that $\epsilon = 0$. Hence, if there is no moral hazard, there will be no default. This shuts down the channel of novation for incentive contracts. Consequently, CCP clearing adds direct value through novation only for insurance contracts.

The timing in each period is as follows. First, all matched farmers make a take-it-or-leave-it offer to bakers. Next, the bakers chose their action $\lambda_B$. Then the farmer makes a decision to terminate the relationship or not, expressed as $\lambda_F \in \{0,1\}$. If he does so ($\lambda_F = 1$), he and the baker conditional on surviving are matched with new counterparties next period with probability $\sigma \in (0,1)$ which for now is exogenous. If he does not ($\lambda_F = 0$) and the baker dies, he is not matched with a baker next period. Finally, bakers die or survive and the contract is executed for the period.

I first assume that the farmer cannot observe the baker’s decision $\lambda_B$ to realize the private benefit $B$. I will relax this assumption in the next section. As a consequence, while the
farmer knows the value $B$, his decision cannot condition on the baker’s action $\lambda_B$. However, the baker will anticipate the decision of the farmer whether to continue the relationship or not and accordingly chooses $\lambda_B$. We again require that the expected surplus from the contractual relationship is $c$ for the baker so that the participation constraint for the baker becomes now

$$V_1^B = -\mu k + (1 - \rho \lambda_B) \left[ (v - p + k) + \beta \left( \lambda_F V_0^B + (1 - \lambda_F) V_1^B \right) \right] \geq c \quad (20)$$

where $V_i^B$ is the value function for the baker when he is in a match ($i = 1$) or not ($i = 0$) next period. Furthermore an incentive contract has to satisfy the incentive constraint

$$\rho \left[ (v - p + k) + \beta \left( \lambda_F V_0^B + (1 - \lambda_F) V_1^B \right) \right] \geq B. \quad (21)$$

Using the participation constraint this can again be written more compactly as

$$k \geq \frac{1}{\mu} \left( \frac{B}{\rho} - c \right). \quad (22)$$

Similarly, an insurance contract needs again to satisfy the constraint

$$k \leq \frac{1}{\mu} \left( \frac{B(1 - \rho)}{\rho} - c \right). \quad (23)$$

Hence, the constraints remain unaltered compared to the static analysis.

To characterize the subgame-perfect Nash equilibria of this game of imperfect information, take the contract $(p, k)$ as given. Again, denote $V_i^F$ as the value function for the farmer depending on whether he is in a match or not. Since the terms of the current one-period contract are sunk, the farmer will continue the relationship as long as

$$(1 - \rho \lambda_B) \beta V_1^F + \rho \lambda_B \beta V_0^F \geq V_0^F. \quad (24)$$

Hence, the farmer compares the expected value from continuing the relationship and potentially postponing the search for a counterparty by one period with the value of terminating the relationship and searching immediately. Since the farmer will be in a match next period only with probability $\sigma$ again whenever he terminates the relationship, his value function of
being without a match at the end of the period is given by

\[
V_0^F = \beta \left( \sigma V_1^F + (1 - \sigma)V_0^F \right) 
= \frac{\beta \sigma}{1 - \beta(1 - \sigma)} V_1^F
= \beta \chi V_1^F,
\]

(25)

where \( \chi = \frac{\sigma}{1 - \beta(1 - \sigma)} < 1 \). Using this expression in the condition for the farmer to continue the relationship, we obtain

\[
[(1 - \rho \lambda_B) - \sigma] \beta (V_1^F - V_0^F) \geq 0.
\]

(26)

This yields the following proposition.

**Proposition 7.** Suppose risk taking \((\lambda_B)\) is not observable. If \(\rho \leq 1 - \sigma\), a long-term relationship is always maintained independent of moral hazard and the contract choice. For \(\rho > 1 - \sigma\) whether the relationship is maintained depends on the form of the contract: for incentive contracts it is, but for insurance contracts it is not.

Whether the relationship is maintained or not depends on the search costs – the probability of not finding a counterparty, \(1 - \sigma\) – vs. the risk of default, \(\rho\). This is intuitive. When there is default and the farmer has not terminated and searched for a new counterparty, he will not have a transaction next period. Hence, he will make a choice that gives him the highest probability of having a match next period independent of the type of contract. Not too surprisingly, when search costs are large (\(\sigma\) is low) one would see only long-term contractual relationships. Interestingly, when search costs are relatively low, we find that insurance contracts take place only in short-lived relationships, while collateral is used as an incentive device in long-term relationships.\(^{12}\)

I turn now to a discussion of the optimal choice of contract \((p, k)\). The analysis is identical to the one for the static problem. Note first that the value for an unmatched baker is given by

\[
V_0^B = \frac{\beta \sigma}{1 - \beta(1 - \sigma)} V_1^B = \beta \chi V_1^B.
\]

(27)

\(^{12}\)Introducing an exogenous default probability \(\epsilon > 0\) would yield an additional third region where for \(\sigma \in (1 - \epsilon, 1)\) all relationships are short-term, while the cut-off value for the other regions would change to \((1 - \epsilon)(1 - \rho)\).
Since the participation constraint for the baker is always binding, we have that

\[
V_1^B = \mu k + (1 - \rho \lambda_B) [(v - p + k) + \lambda_F V_0^B + (1 - \lambda_F)\beta V_1^B]
\]

\[
= -\mu k + (1 - \rho \lambda_B) [(v - p + k) + (\lambda_F \chi + (1 - \lambda_F)) \beta V_1^B]
\]

\[
= \frac{1}{1 - (1 - \rho \lambda_B)\beta (\lambda_F \chi + (1 - \lambda_F))} [-\mu k + (1 - \rho \lambda_B)(v - p + k)]
\]

\[
= \frac{1}{\delta_B(\lambda_B, \lambda_F)} [-\mu k + (1 - \rho \lambda_B)(v - p + k)].
\] (28)

Hence for any level of collateral \( k \), we have then that the contract price is given by

\[
p = v - \frac{c}{(1 - \rho \lambda_B)} \delta_B(\lambda_B, \lambda_F) - k \left( \frac{\mu}{(1 - \rho \lambda_B)} - 1 \right). \] (29)

The optimal contract is qualitatively the same as in the static environment.\(^\text{13}\) The choice of collateral and type of contract maximizes again the farmer’s expected utility subject to giving bakers an expected life-time utility of \( c \). The additional term \( \delta_B(\lambda_B, \lambda_F) \) takes the role of an (endogenous) discount factor for bakers that summarizes the impact of the decisions by both contracting parties on how long the relationship will last.

Similarly, we have for the farmer’s utility

\[
V_1^F = ((1 - \rho \lambda_B)u(p) + \rho \lambda_B u(k)) + \lambda_F V_0^F + (1 - \lambda_F)\beta ((1 - \rho \lambda_B)V_1^F + \rho \lambda_B V_0^F)
\]

\[
= ((1 - \rho \lambda_B)u(p) + \rho \lambda_B u(k)) + [\lambda_F \beta \chi + (1 - \lambda_F)\beta ((1 - \rho \lambda_B) + \rho \lambda_B \beta \chi)] V_1^F
\]

\[
= \frac{1}{1 - \beta [\lambda_F \chi + (1 - \lambda_F) ((1 - \rho \lambda_B) + \rho \lambda_B \beta \chi)]} ((1 - \rho \lambda_B)u(p) + \rho \lambda_B u(k))
\]

\[
= \frac{1}{\delta_F(\lambda_B, \lambda_F)} ((1 - \rho \lambda_B)u(p) + \rho \lambda_B u(k)),
\] (30)

where \( \delta_F(\lambda_B, \lambda_F) \) is again an endogenous discount factor. Since the contract structure remains the same, it follows immediately that the analysis from the static environment carries over here. In particular from the incentive and insurance constraint, we again have that – for a given level of moral hazard – collateral is lower in an insurance contract. The next proposition characterizes the choice of contract and collateral further, where I interpret the probability of matching, \( \sigma \), as a parameter reflecting liquidity in the market. Hence, the next result links the costs of collateral to market liquidity and default risk.

**Proposition 8.** *Insurance contracts become more attractive as market liquidity increases,*

\(^{13}\)Note that incentive contracts are now feasible for \( B \in [\rho c, \rho (v + \beta c)] \). This allows us to simply redefine \( \tilde{v} = v + \beta c \) to maintain the same analysis as in the static set-up.
i.e. \( \partial V^F_1(INS)/\partial \sigma \geq 0 \).

For all \( \sigma \), we have that \( k^{INS}(B) < k^{INC}(B) \), that is incentive contracts have more collateral for any given level of moral hazard. Furthermore, if \( \rho > (1 - \sigma) \), collateral in the optimal insurance contract increases with market liquidity, i.e. \( \partial k^{INS}(B)/\partial \sigma \geq 0 \).

**Proof.** The value of an incentive contract for the farmer is independent of the search friction \( \sigma \), since \( \lambda_F = 0 \) for such a contract. The value of an insurance contract, however, is given by

\[
V^F_1(INS) = \frac{1}{\delta_F(\lambda_B, \lambda_F)} \left( (1 - \rho)u(p) + \rho u(k) \right).
\]

Suppose first that \( \rho < (1 - \sigma) \). We then have that

\[
\delta_B(1, 0) = 1 - \beta \left( 1 - \rho \right) \\
\delta_F(1, 0) = 1 - \beta \left( 1 - \rho (1 - \beta \chi) \right)
\]

with the price being independent of \( \sigma \)

\[
p = v - \frac{c}{1 - \rho} \left( 1 - \beta (1 - \rho) \right) - k \left( \frac{\mu}{1 - \rho} - 1 \right).
\]

Since \( \partial \chi/\partial \sigma > 0 \), the result follows for this case.

Suppose next that \( \rho > (1 - \sigma) \). Then, we get

\[
\delta_B(1, 1) = 1 - \beta (1 - \rho) \chi \\
\delta_F(1, 1) = 1 - \beta \chi
\]

with the price of the optimal insurance contract depending on \( \sigma \) according to

\[
p = v - \frac{c}{1 - \rho} \left( 1 - \beta (1 - \rho) \chi \right) - k \left( \frac{\mu}{1 - \rho} - 1 \right).
\]

The result follows then from the envelope theorem, as the discount factor \( \delta_F(1, 1) \) is decreasing in \( \sigma \) and the price \( p \) is increasing in \( \sigma \).

The first result follows directly, since we have that

\[
0 \leq k^{INS}(B) \leq \frac{B(1 - \rho)}{\rho} - c < \frac{B}{\rho} - c = k^{INC}(B)
\]
independent of $\sigma$.

Collateral levels are independent of $\sigma$ unless the insurance constraint is not binding. The first-order condition for the optimal collateral choice in an insurance contract is given by

$$-\left(\mu - (1 - \rho)\right)u'(p) + \rho u'(k) = 0$$

whenever the insurance constraint does not bind and $k^{INS}(B) > 0$. Using the expression for $p$ when $\rho > (1 - \sigma)$ and applying the implicit function theorem delivers the final result.

The intuition for this result is straightforward. When market liquidity increases, the implicit cost for farmers of engaging in short term relationships are small, as it is easy to find a new transaction. However, for $\rho > (1 - \sigma)$, there is a second effect. High market liquidity allows the farmer to extract more resources from the baker, as trading frictions have declined. This is reflected in larger collateral postings and – on the margin – a higher price for the contract. The reason is, however, a bit an artefact of our set-up. In order to provide a fixed surplus $c$ to the baker, the per period surplus needs to be higher when there is a risk of not having a transaction next period. Hence, with lower search frictions, one needs to provide less surplus. As a consequence, one can require more collateral despite its deadweight cost.

To summarize. When markets are not liquid, the attractiveness of an insurance contract is small – unless the costs of collateral are large. Nonetheless, less liquid markets can see lower collateral postings whenever the optimal contract structure does not rule out default. Hence, when insurance contracts are preferred over incentives ones, low liquidity in markets can lead to low levels of collateral postings.

4.2 Market Discipline

I now assume that the farmer can observe the baker’s action $\lambda_B$ in order to capture the flavor of market discipline: farmers continue the relationship until they discover moral hazard; then, they terminate the relationship.\(^{14}\) Such a strategy must be again subgame-perfect for the farmer; in other words, the threat to terminate a relationship upon observing moral hazard needs to be credible.

\(^{14}\)We could interpret this framework as allowing the farmer to pay a fixed cost $q_M > 0$ in order to observe the decision $\lambda_B$. This cost would simply increase the upfront costs of entering a contract and producing wheat. Hence, participation of the farmer would not be an issue for the analysis and the only question would be whether there is an incentive to monitor.
We again solve backwards. From condition (26), it is clear that for $\sigma < 1 - \rho$ continuing the relationship ($\lambda_F = 0$) is a strictly dominant strategy for the farmer independently of the contract choice $(p, k)$. Hence, I assume that $1 > \sigma > 1 - \rho$ for this section.

Note first that in an insurance contract, we get $\lambda_F = 1$. For an incentive contract, the continuation decision for the farmer can now depend directly on the observed action $\lambda_B$. It follows immediately from condition (26) that in an incentive contract the farmer will continue the relationship if and only if there is no engaging in moral hazard, or

$$\lambda_F(\lambda_B) = \begin{cases} 1 & \text{if } \lambda_B = 1 \\ 0 & \text{if } \lambda_B = 0. \end{cases} \quad (31)$$

Hence, with an incentive contract the farmer can credibly threaten to terminate the relationship whenever there is moral hazard. The reason is simple. When continuing, the farmer faces an increased default risk $\rho > 0$ which outweighs the risk of not finding a new counterparty for next period. I call this punishment strategy market discipline.

I show next that market discipline is a substitute to collateral, as it relaxes the incentive constraint. Using the punishment strategy (31), the baker’s incentive constraint is now given by

$$-\mu k + (v - p + k) + \beta V_1^B \geq -\mu k + B + (1 - \rho) ((v - p + k) + V_0^B). \quad (32)$$

Hence, the baker will choose $\lambda_B = 0$ as long as

$$\rho ((v - p + k) + \beta V_1^B) + (1 - \rho) (\beta V_1^B - V_0^B) \leq B. \quad (33)$$

Using the participation constraint on the equilibrium path where there is no default,\textsuperscript{15} one obtains

$$k \geq \frac{1}{\mu} \left( \frac{B}{\rho} - c - \left( \frac{1 - \rho}{\beta} \right) (\beta V_1^B - V_0^B) \right) = \frac{1}{\mu} \left( \frac{B}{\rho} - c - \left( \frac{1 - \rho}{\beta} \right) (1 - \chi) \beta c \right) \quad (34)$$

where we have used the fact that

$$V_0^B = \beta \chi V_1^B = \beta \chi c. \quad (35)$$

\textsuperscript{15}We do not consider social norms here, where a one-time deviation is followed by a global punishment in newly formed matches with the particular baker who deviated.
Hence, credible punishment as reflected by \((1 - \chi) > 0\) can relax the incentive constraint with the result that it is possible to save on collateral in an incentive contract. Hence, with market discipline the incentive constraint is given by

\[
k \geq \max \left\{ 0, \frac{1}{\mu} \left( \frac{B}{\rho} - c - \left( \frac{1 - \rho}{\rho} \right) (1 - \chi) \beta c \right) \right\}
\]  
(36)

while the insurance constraint satisfies

\[
0 \leq k \leq \frac{1}{\mu} \left( \frac{B(1 - \rho)}{\rho} - c - \left( \frac{1 - \rho}{\rho} \right) (1 - \chi) \beta c \right).
\]  
(37)

Figure 2 summarizes this discussion. Compared to the static contract (see Figure 1), collateral requirements shift down in the presence of market discipline for any level of moral hazard. However, now the lowest level of collateral is always achieved by an incentive contract that relies exclusively on market discipline for incentives.

**Proposition 9.** Suppose \(\rho > 1 - \sigma\) and \(\lambda_B\) is observable. If the degree of moral hazard \(B/\rho\) is sufficiently close to 0, the optimal contract is an incentive contract and features market discipline with optimal collateral being \(k = 0\).
Increases in market liquidity tighten the incentive constraint, but relax the insurance constraint. Insurance contracts thus become more attractive as market liquidity increases.

Proof. The first result follows directly from the fact that $\mu > 1$ and $k = 0$. The second result follows from the fact that $\partial \chi / \partial \sigma > 0$. The utility for incentive contracts then falls with $\sigma$, as

$$p = v - c(1 - \beta) - (\mu - 1)k$$

and collateral is weakly increasing with $\sigma$. Finally, the value of an insurance contract is given by

$$V_F^{INS} = \frac{1}{1 - \beta \chi} (\rho u(k) + (1 - \rho)u(p)).$$

Since higher $\sigma$ relaxes the insurance constraint, the final result follows.

This is interesting from two reasons. First, if the exogenous default probability is low ($\epsilon = 0$), collateral savings are large with the implication that collateral requirements could even drop to 0 for $B$ sufficiently close to $\rho c$. Second, if liquidity is relatively high in the market ($\rho > 1 - \sigma$), punishment is a credible (and cheap) incentive mechanism making short-term insurance contracts less likely. As a consequence, collateral is low, but so is default risk. This is very different from the static problem we analyzed earlier.

What is interesting here is that lower liquidity increases the cost savings associated with market discipline. The reason is straightforward. Lower $\sigma$ increases the costs associated with punishment in the form of a short-term contract for the baker. Hence, market discipline is stronger in a market that is less liquid. Still insurance contracts can be better, if the moral hazard problem becomes more severe and $\mu$ is relatively large.

The key message to take away from this section is that low collateral postings need not be a sign of the contracting parties incurring larger default risk. Collateral could be low precisely because market discipline achieves incentives in a cheaper way for the same level of default risk. Note that a prediction of the model is that collateral postings should be lower in markets that are less liquid, but rely on incentive contracts. Hence, low collateral does not necessarily mean higher default risk.

4.3 CCP Clearing and Endogenous Market Liquidity

I investigate next whether CCP clearing can have a detrimental effect on market activity through the cost of collateral that such clearing entails. It is straightforward to check that a
CCP can again not implement incentive and insurance contracts simultaneously as pointed out in the static version of the model. Suppose then a CCP opts for implementing incentive contracts to rule out default. This will push up the costs of some transactions that had lower collateral requirements associated with insurance contracts. As a consequence, market liquidity will fall when these transactions yield not enough surplus anymore. For a large enough decline \((1 - \sigma_{CCP} > \rho > 1 - \sigma)\), this will undermine market discipline for all transactions.

I assume again that \(\lambda_B\) is observable and that \(\rho > 1 - \sigma\), but now moral hazard levels are distributed in the economy according to some distribution function \(F(B)\) on the support \([B, \Bar{B}]\). I assume that a baker that dies is replaced by a new baker with exactly the same characteristic as the one that died. This renders the distribution of bakers time invariant.

The rest of the environment stays the same.

Denote \(B \subset [B, \Bar{B}]\) the set of moral hazard levels such that – conditional on meeting a new baker – there is a contract with surplus greater than \(\theta\), the production cost for the farmer. Consider a farmer that is searching for a trading partner. His value function is given by

\[
V^F_0 = \beta \sigma \mathcal{P}(B \in B) E_B[V^F_1(B)] + \beta (1 - \sigma \mathcal{P}(B \in B)) V^F_0, \tag{38}
\]

where \(E_B\) is the expectation operator with respect to the set \(B\). Let \(\bar{\sigma} = \sigma \mathcal{P}(B \in B)\) be the probability of engaging in a new contract with a baker. Observe that this is now an endogenous variable depending on which moral hazard levels offer positive surplus when contracting. The value of an unmatched farmer is then given by

\[
V^F_0 = \frac{\bar{\sigma}}{1 - \beta (1 - \bar{\sigma})} \beta E_B[V^F_1(B)] = \chi \beta E_B[V^F_1(B)]. \tag{39}
\]

Being matched with a baker of type \(B\), the farmer continues the relationship with the baker if and only if

\[
\rho \lambda_B \beta V^F_0 + (1 - \rho \lambda_B) V^F_1(B) = V^F_0. \tag{40}
\]

We then have the following proposition which is immediately obtained by using \(V^F_0\) in the expression above.

**Proposition 10.** Suppose \(B\) is distributed according to a distribution function \(F\) on \([B, \Bar{B}]\).
The farmer has the following optimal strategy:

\[
\lambda_F = 0, \quad \text{if} \quad \frac{V_F^F(B)}{E_B[V_F^F(B)]]} \geq \alpha(1)
\]

\[
\lambda_F = 0 \quad \text{iff} \quad \lambda_B = 0, \quad \text{if} \quad \alpha(1) \geq \frac{V_F^F(B)}{E_B[V_F^F(B)]]} \geq \alpha(0)
\]

\[
\lambda_F = 1, \quad \text{if} \quad \alpha(0) \geq \frac{V_F^F(B)}{E_B[V_F^F(B)]]},
\]

where

\[
\alpha(\lambda_B) = \left(\frac{1 - \rho \beta \lambda_B}{1 - \rho \lambda_B}\right) \chi.
\]

This generalizes the analysis of the previous section. When the current value of a relationship \( B \) is sufficiently large relative to finding a new trading partner, it is a strictly dominant strategy to continue independent of the one-period contract used. The opposite is true if the current value is low. In an intermediate region, we again obtain market discipline: the farmer breaks off the relationship if and only if the baker engages in moral hazard. Hence, in this region the one-period contract choice drives the continuation decision. For tractability, I will restrict the support of the level for moral hazard and impose a slightly more restrictive condition on the matching and default probabilities \( \sigma \) and \( \rho \).

**Assumption 11.** There are only two levels of moral hazard, \( B_L < B_H \) and the probability of matching satisfies \( \rho > 1 - \sigma \).

These restrictions allow me to construct an intuitive example where CCP clearing is detrimental for all transactions in the sense that it lowers surplus for all transactions while not lowering default probabilities for any transaction that remains with positive surplus. In my analysis, the optimal contracts for the two levels of moral hazard are an incentive contract governed by market discipline and an insurance contract, respectively. The next proposition establishes the optimality of such contracts whenever the cost of collateral relative to the probability of default is sufficiently large.

**Proposition 12.** If \( \sigma \) is sufficiently close to 1 and

\[
v > c \left(\frac{\mu - (1 - \rho)}{(\mu - 1)(1 - \rho)}\right),
\]

27
then there exist \( B_L < B_H \) such that the optimal contract for \( B_L \) features market discipline while the one for \( B_H \) is an insurance contract.

Proof. The proof is by construction. From the previous section it is clear that the optimal one-period contracts for any given \( B \) remain unchanged. I will construct levels of \( B_L \) and \( B_H \) such that the optimal contracts feature market discipline and insurance, respectively.

I first show that an insurance contract dominates for \( \bar{B} \) if \( \sigma \) is sufficiently close to 1. By the definition of \( \bar{B} \), we have for an optimal incentive contract that \( p^* = k^* \). At \( \bar{B} \) an insurance contract is feasible and the optimal insurance contract is weakly better than one with \( k = 0 \) and \( p_0 = v - \frac{c}{1 - \rho} \). The condition in the proposition then ensures that \( p_0 > p^* \). The farmer prefers an insurance contract at \( \bar{B} \) as long as

\[
\frac{1}{1 - \beta \chi} [\rho u(k_0) + (1 - \rho) u(p_0)] \geq \frac{1}{1 - \beta} u(p^*(\bar{B}))
\]

\[
1 - \beta (1 - \sigma) \geq \frac{u(p^*(\bar{B}))}{\rho u(k_0) + (1 - \rho) u(p_0)}.
\]

If \( \sigma \) large enough this inequality holds, as the LHS is smaller than 1 as shown above.

Next, I show that there is a level \( B_L < \bar{B} \) such that an incentive contract yields the same payoff for the farmer than the optimal insurance contract at \( \bar{B} \). For \( B_L \) sufficiently close to \( \bar{B} = c \), we have that \( k = 0 \). Then, \( p^*(\bar{B}) > p_0 > k_0 \) and \( V_{1F}(\bar{B}) \) strictly exceeds \( V_{1F}(\bar{B}) \). The values for an optimal insurance and incentive contract \( V_{1F}(INC) \) and \( V_{1F}(INS) \) are continuous in \( B \). The result then follows as \( V_{1F}(INC) \) is strictly decreasing, while \( V_{1F}(INS) \) is strictly increasing.

Finally, I verify that the condition for market discipline at \( B_L \) is satisfied. Since \( B_H \) are short-term relationships, punishing moral hazard by terminating the relationship causes a permanent switch to transactions with high moral hazard. Market discipline is fulfilled if and only if

\[
\chi \left( \frac{1 - \rho \beta}{1 - \rho} \right) \geq \frac{V_{1F}(B_L)}{V_{1F}(B_H)} \geq \chi.
\]

Note that \( \chi < 1 \) and that we can choose \( B_L \) and \( B_H \) such that \( V_{1F}(B_L)/V_{1F}(B_H) \) are sufficiently close to 1. The result then follows from the fact that

\[
\chi \left( \frac{1 - \rho \beta}{1 - \rho} \right) > 1 \text{ if and only if } \rho > 1 - \sigma.
\]

This completes the construction of optimal contracts at \( B_H \) and \( B_L \). \( \square \)
The intuition for this result is simple. The one-period contract problem remains unchanged from the previous analysis, with collateral requirements again given by equations (36) and (37) (see Figure 2). Hence, at $\bar{B}$ the one-period payoff for an insurance contract is larger than the one for an incentive contract irrespective of market discipline. When it is very likely to find a transaction next period – $\sigma$ is sufficiently close to 1 –, the optimal contract is insurance and the relationship is short-lived. To ensure market discipline for the level $B_L < B_H \leq \bar{B}$, it must be the case that

$$\alpha(1) \geq \frac{V^I_F(B_L)}{V^I_F(B_H)} \geq \alpha(0),$$

since all new matches will be at $B_H$, the only short-lived contracts. Assumption 11 and the fact that $\chi < 1$ ensure that this condition is satisfied, whenever we can find some level $B_L$, where the payoff for the farmer is close enough to the one offered by the insurance contract at $B_H$.

Figure 3 clarifies this last condition by plotting the value for incentive and insurance contracts as a function of moral hazard $B$. Note that the value of incentive contracts is decreasing
in $B$, while the value of insurance contracts is strictly increasing. As shown in the graph, as long as insurance contracts are preferred at $B_H$, there must exist some level $B_L < B_H$, where an incentive contract yields the same payoff for farmers.

The figure shows further that ruling out insurance contracts with CCP clearing will depress surplus, potentially rendering transaction for $B_H$ infeasible (as shown by the level $\theta_{\text{crit}}$ where incentive contracts with market discipline become infeasible for $B_H$). The associated drop in liquidity can then interfere with market discipline resulting in an increase in collateral requirements for CCP clearing and a reduction of surplus for all remaining transactions in the market. With only two levels of moral hazard, the drop in liquidity is severe: transactions with $B_H$ are not any longer carried out. Hence, threatening to terminate a relationship as a punishment for increasing risk on a transaction is not credible anymore. This can be summarized as follows.

**Corollary 13.** If the value of an incentive contract is below $\theta$ at $B_H$, CCP clearing with no default can lead to a reduction in market liquidity and an increase in collateral requirements.

**Proof.** Suppose the value of an incentive contract at $B_H$ satisfies

$$V_1^F(INC) < \theta,$$

so that there is no surplus from the transaction. For market discipline to work, we need that

$$\lambda \left( \frac{1 - \rho \beta}{1 - \rho} \right) V_1^F(B_H) \geq V_1^F(B_L).$$

However, $V_1^F(B_H) = 0$, since the surplus from transacting is negative. Hence, $\lambda_F = 0$ is a strictly dominant strategy for contracts with $B_L$. 

Under what circumstances is CCP clearing likely to undermine market discipline? First, insurance contract must be optimal for some levels of moral hazard. Insurance contracts are likely to occur whenever the underlying risk associated with the transaction, $\rho$, is small and the cost of collateral, $\mu$, is high (see the condition of Proposition 12). Second, market liquidity needs to be large enough for market discipline to play a role. Referring back to the condition in Assumption 11, the discount factor for the financial transaction environment is likely to be close to 1 which makes a large liquid market a precondition for market discipline as shown in the previous section. Finally, one needs sufficiently large heterogeneity in moral hazard risk to cause CCP clearing to have negative effects on market discipline.
5 Conclusions

This paper is novel in that it provides a first formal analysis of how central clearing aimed at reducing default risk will influence trading costs in financial markets and, hence, market liquidity. Most importantly, it develops a basic feedback mechanism between costly collateral and market liquidity.

I have shown that low collateral levels need not be correlated with high counterparty risk. Instead, they could be a result of market discipline which is an alternative to collateral. Notwithstanding, it can also be efficient for the contracting parties to save on collateral, whenever its cost is high and the problem of endogenous risk taking severe. In such an incidence, collateral might serve only as an insurance device so that default risk and collateralization are positively related.

A CCP is unlikely to perfectly observe the degree of endogenous risk taking in markets. Hence, observed contract terms will not make it possible to design collateral requirements that rule out default, while avoiding inefficiently high collateral levels for some transactions. Consequentially, one would expect a fall in market liquidity as a response to CCP clearing. This fall can impair discipline in markets, where trading partners break off trading relationship when they suspect or detect risk taking by their counterparties. This is the feedback effect that forces CCPs to increase collateral levels further.

My analysis has abstracted of course from other benefits of central clearing, such as information generation or netting of exposures. However, as I have pointed out, such benefits need not necessarily be provided by a CCP whose primary function is transferring risk from market participants and redistributing it\textsuperscript{16} – as modelled in this paper. Still, we need to understand the potential costs of central clearing better. My findings shift the focus within this discussion on the costs of collateral for financial transactions as well as the impact of clearing arrangements on liquidity in financial markets – a useful step in determining under what circumstances CCP clearing is useful and efficient.

\textsuperscript{16}An information warehouse can take on the role of information generation and dissemination, while multilateral netting arrangements have taken place outside formal clearinghouses (see Kroszner (1999) on ring netting).
References


